# Implementation and Application of a new Plasticity Model in LS-DYNA including Lode Angle Dependence


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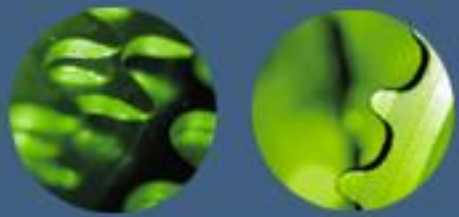
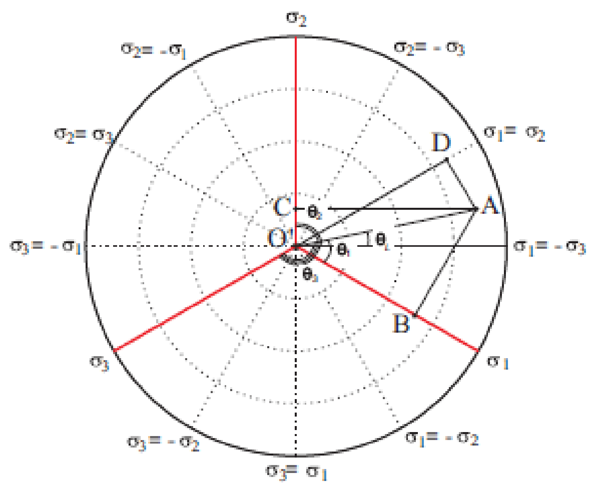
# OUTLINE

1. **Motivation**
   - Definition of the Lode angle and normalized third invariant
   - Materials with Lode angle dependent plasticity
   - Existing plasticity models with Lode angle dependency
2. **New plasticity model**
   - Base assumptions and desired features
   - Constitutive equations: yield function and flow rule
   - Graphic representation on the $\sigma_1$-$\sigma_2$ space
   - Numerical implementation
3. **Numerical examples**
   - Tensile stress states: notched and flat grooved specimens
   - Shear stress states: butterfly specimen
   - Compressive stress states: upset test
4. **Final remarks**



# 1. Motivation

## Lode angle definition




Taken from “Xue, L. (2007),Ductile Fracture Modeling – Theory, Experimental Investigation, and Numerical Verification, PhD thesis, Massachusetts Institute Technology, Cambridge, MA.”

- Geometrically, the Lode Angle is the “smallest angle between the line of pure shear and the projection of the stress tensor on the deviatoric plane” (L.Malcher et al.)

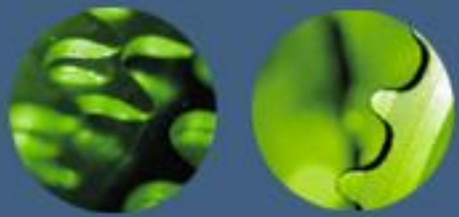
$$\theta_L = \tan^{-1}\left\{\frac{1}{\sqrt{3}}\left[2\left(\frac{s_2 - s_3}{s_1 - s_3}\right) - 1\right]\right\}, \theta_L = \left[0, \frac{\pi}{3}\right]$$

- The Lode Angle $\theta_L$ is relatated with the normalized third deviatoric stress invariant, $\xi$

$$\boxed{\xi} = \cos(3\theta_L) = \frac{27}{2}\frac{\det(\boldsymbol{s})}{\sigma_{eq}^3}, \xi = [-1,1]$$

- Hereinafter, the normalized third deviatoric stress invariant, $\xi$, will be denoted as Lode Angle.

**In LS-DYNA, the normalized third deviatoric stress invariant, $\xi$, is denoted as the “Lode Angle Parameter”**

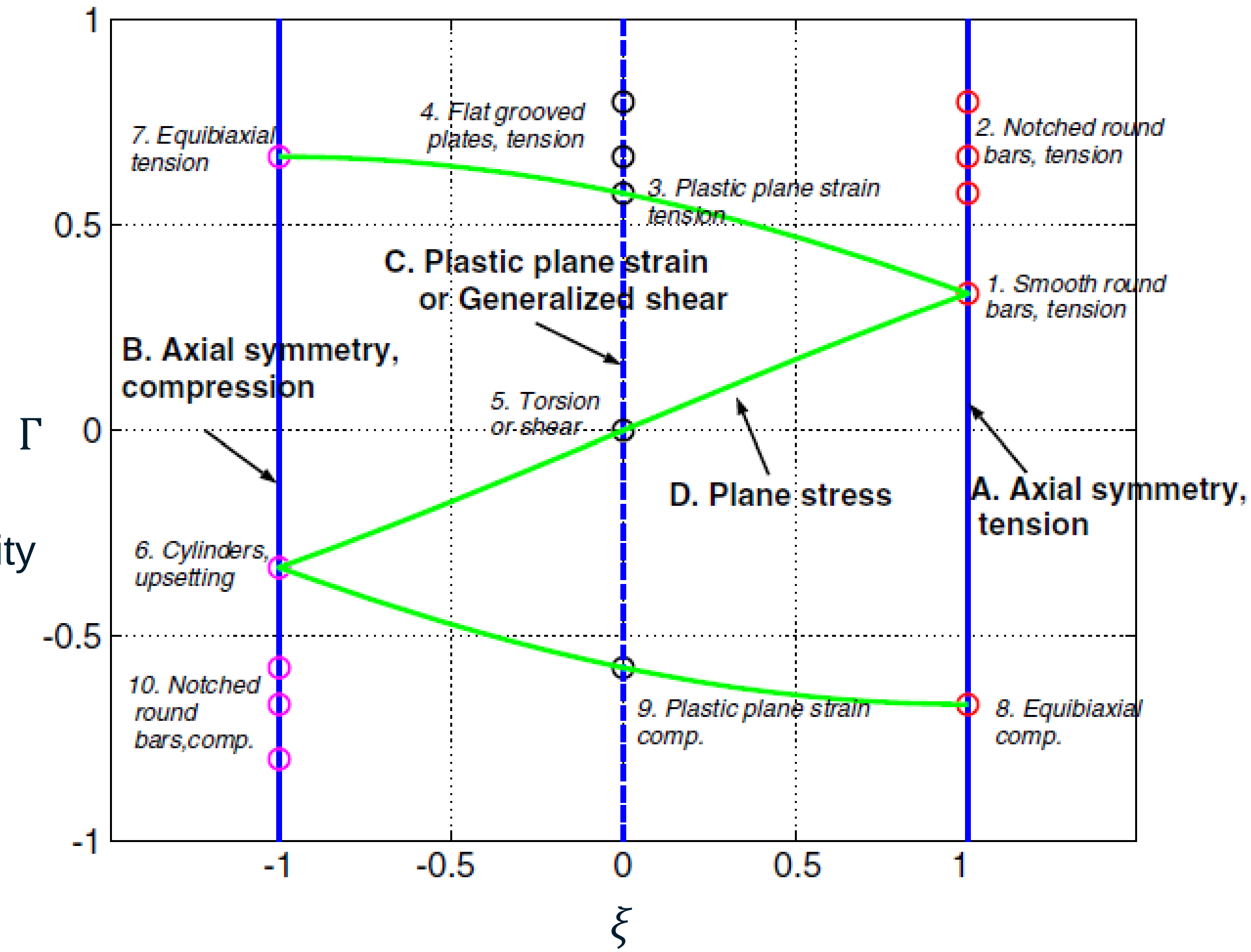


# 1. Motivation

## Lode Angle Definition, $\xi$

Stress Triaxiality

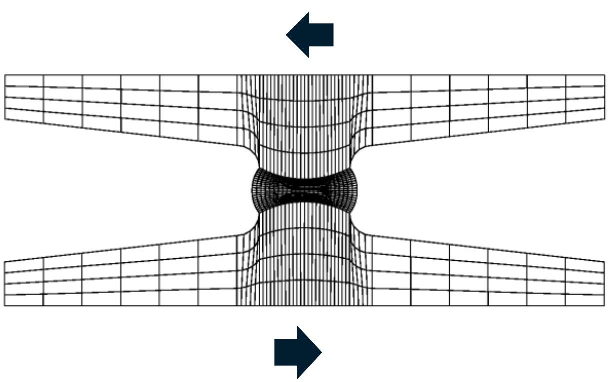
$$\Gamma = \frac{p}{\sigma_{eq}}$$

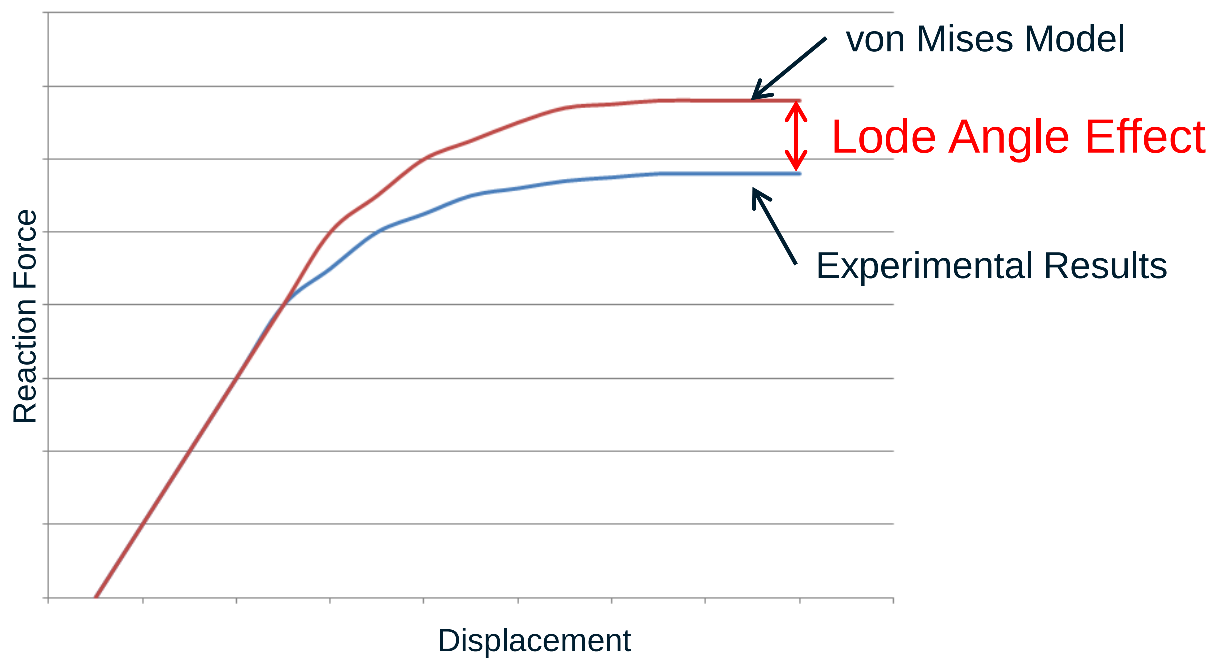


# 1. Motivation

## Aluminium alloys

- Recent experimental analysis have proved that the Lode Angle have a considerable effect on the stress-plastic relation of some aluminium alloys (Y. Bai et al., Mirone et al, X. Gao et al.)
- In contrast, they also concluded that the hydrostatic pressure has a negligible effect





# 1. Motivation

## Aluminium alloys

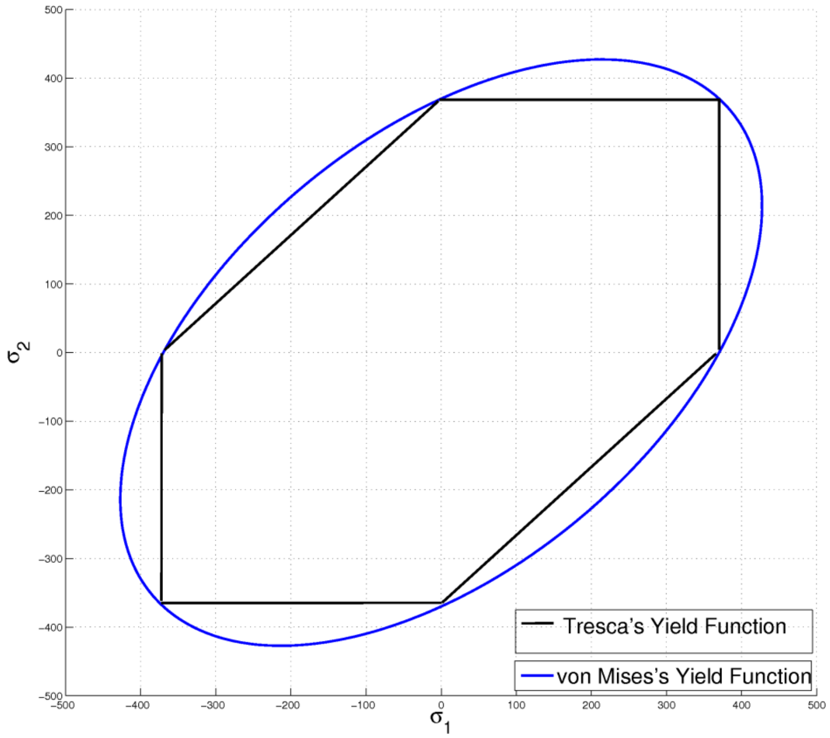
- However, the effect of the Lode Angle is not constant.
- It depends on the stress state!

| Stress State/Specimen | $\xi$ | Lode Angle Effect (according to experimental evidences) |
|---|---|---|
| 1. Tensile round bars | $\xi = 1$ | N |
| 2. Compression round bars | $\xi = -1$ | N |
| 3. Shear stress states | $\xi = 0$ | Y |
| 4. Plane Strain Specimen - Traction | $\xi = 0$ | Y |
| 5. Plane Strain Specimen – Compression | $\xi = 0$ | Y |
| 5. Bi-axial tension | $\xi = 1$ | N |
| 6. Bi-axial compression | $\xi = -1$ | N |

The maximum Lode Angle effect takes place when $\xi = 0$

# 1. Motivation

## Tresca's yield function




$$\Phi = (\sigma_{max} - \sigma_{min}) - \sigma_y$$

**Advantages:**

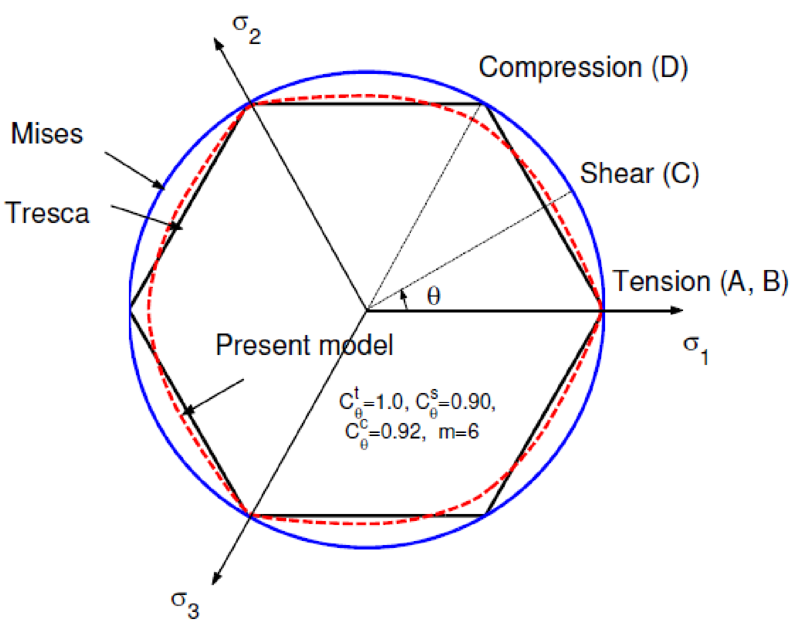
- It is a classical, well-established yield function

**Disadvantages:**

- The effects of the 3rd invariant are fixed
- The yield function is non-continuous, making its numerical implementation more difficult (directional derivatives are needed!)

# 1. Motivation

## Bai & Wierzbicki's yield function




Taken from "Bai, Y. (2008), Effect of loading history on necking and fracture, PhD thesis, Massachusetts Institute Technology, Cambridge, MA."

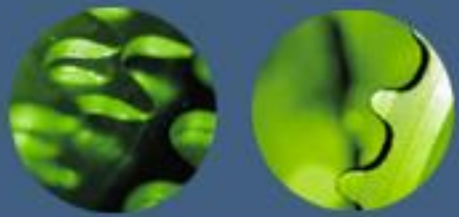
$$\Phi = \sigma_{eq} - \sigma_y[1 - c_\Gamma(\Gamma - \Gamma_0)]\left[c_\theta^s + (c_\theta^{ax} - c_\theta^s)\left(\phi - \frac{\phi^{m+1}}{m+1}\right)\right]$$

**Advantages:**

- The convexity of the yield function has to be ensured by a numerical parameter

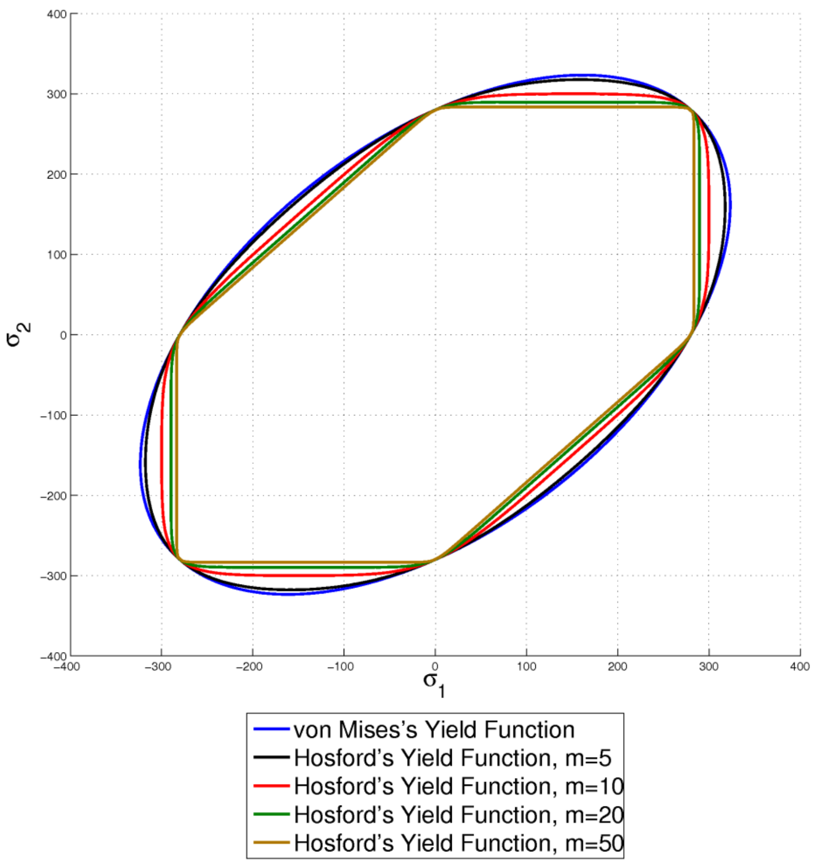
**Disadvantages:**

- 7 material parameters plus a hardening curve are required for calibration
- Isochoric plasticity is questionable
- Complex to implement



# 1. Motivation

## Hosford’s yield function




$$\Phi = |\sigma_1 - \sigma_2|^m + |\sigma_2 - \sigma_3|^m + |\sigma_3 - \sigma_1|^m - 2\sigma_y^m$$

**Advantages:**

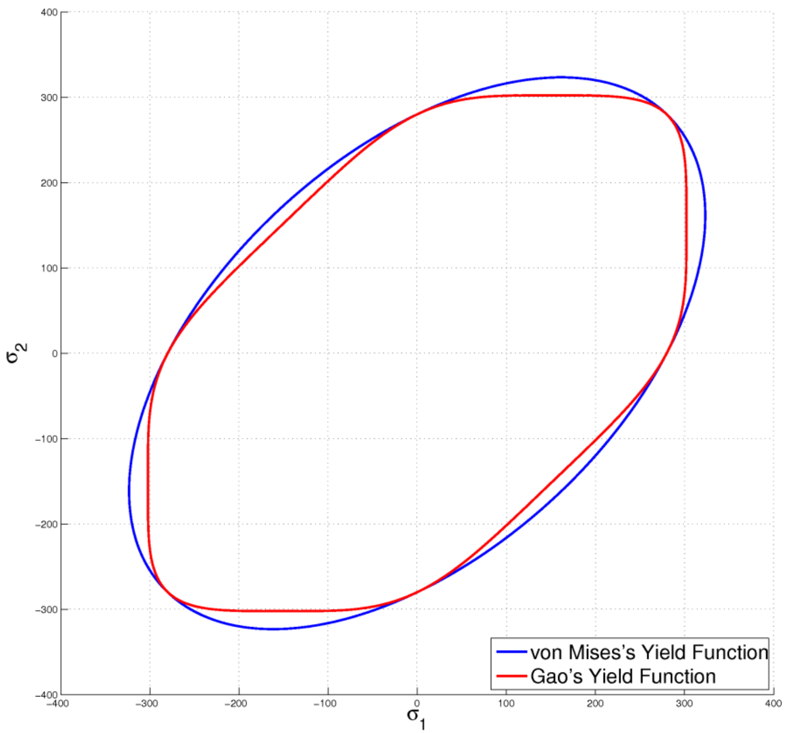
- It is available in LS-DYNA through MAT_36 if parameters are set to be isotropic

**Disadvantages:**

- Effect of 3rd invariant can be controlled by the exponent “m”, but the physical meaning of “m” is somewhat difficult to grasp
- Formulated in principal stress space


# 1. Motivation

## Gao's yield function




$a_1 = 0,\ b_1 - 60.75$ and $c_1 = 1.07$

$$\Phi = c_1\left(a_1 I_1^6 + 27 J_2^3 + b_1 J_3^2\right)^{1/6} - \sigma_y$$

$a_1$ and $b_1$ are material parameters and $c_1$ is a function of $a_1$ and $b_1$

**Advantages:**

- It is possible to control the effect of the third invariant

**Disadvantages:**

- In addition to the hardening curve, 6 more material parameters are required which have no physical meaning
- Non-associated model
- The yield function may be non-convex!

# 2. New Plasticity Model

## Desired features

- Starting from classical $J_2$ von Mises plasticity (*MAT_24)
- $J_2$ plasticity should be easily recovered if material is not dependent on $\xi$
- Easiness of calibration (as less new parameters as possible)
- Dependency of $\xi$ should be simple to grasp

## Main assumptions for the new model

- Material has a different yield stress under tensile and shear stress states
- The dependency of the yield stress in respect to $\xi$ is assumed quadratic

# 2. New Plasticity Model

## Proposed yield function

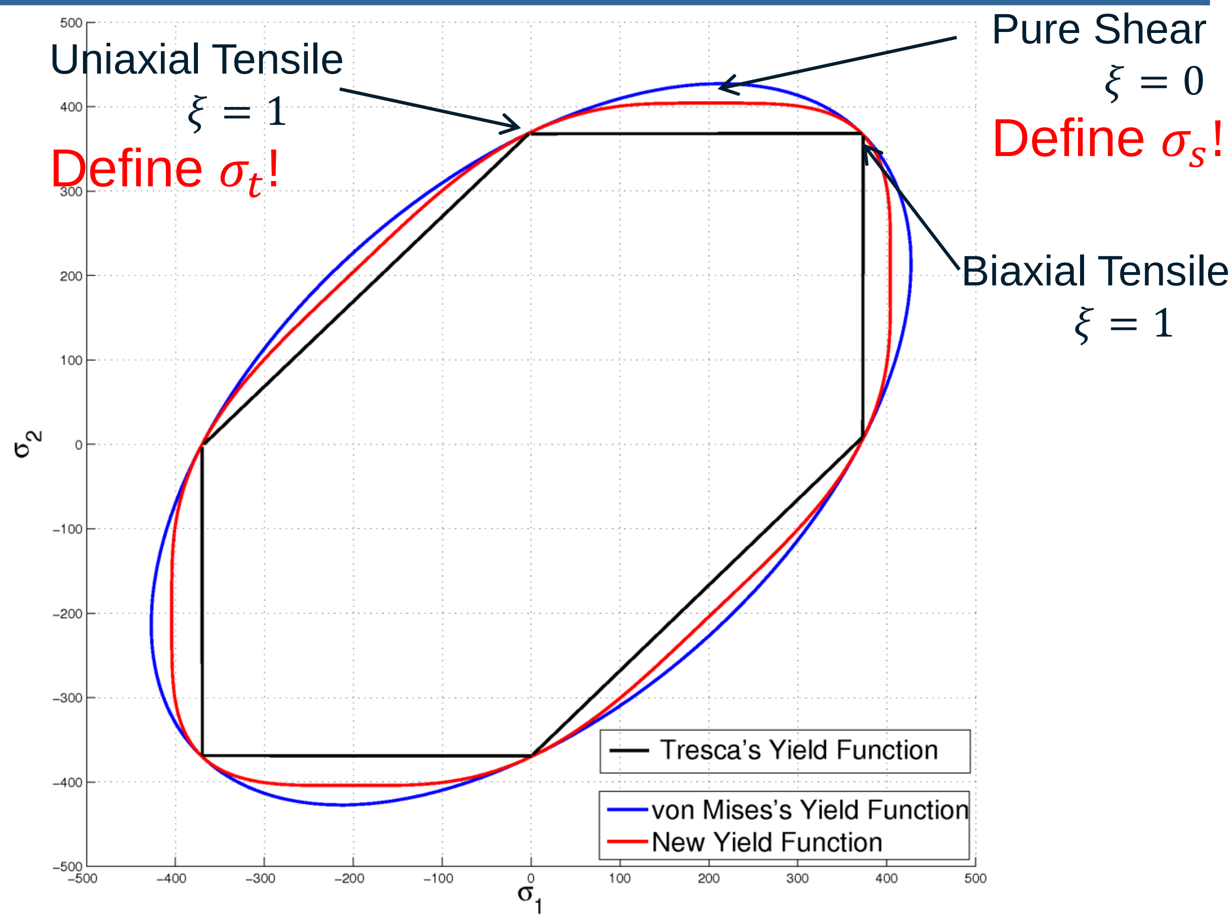
$$\Phi = \sigma_{eq} + (\sigma_t - \sigma_s)(1 - \xi^2) - \sigma_y$$

Parameters definition:

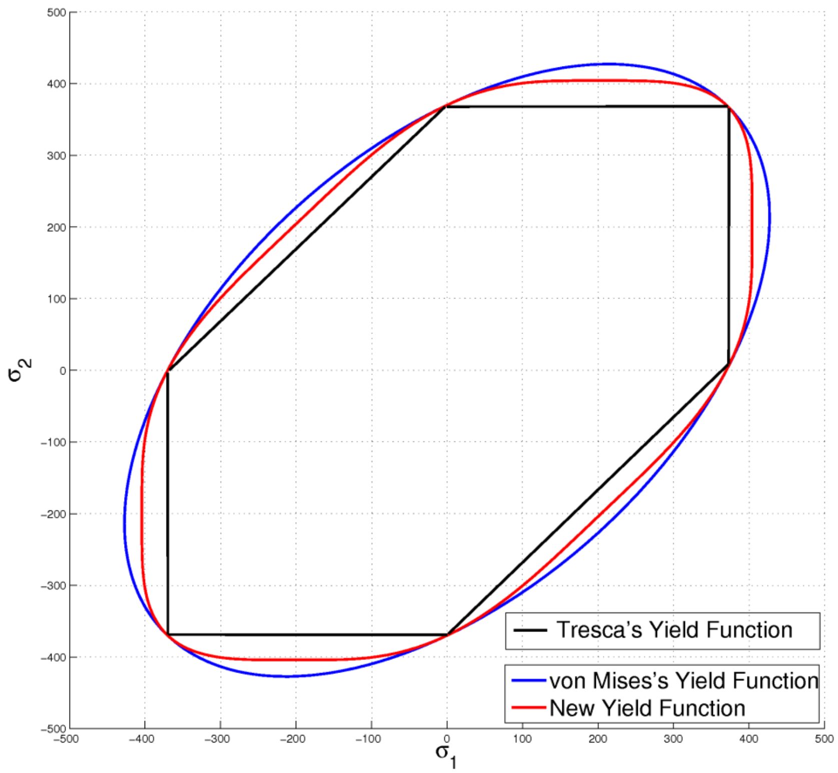
1. Hardening Curve: The same as used in von Mises Model

2. Tensile Yield Stress, $\sigma_t$. Determined through standard tensile test

3. Shear Yield Stress, $\sigma_s$. Determined through a shear test



## Proposed yield function




$$\Phi = \sigma_{eq} + (\sigma_t - \sigma_s)(1 - \xi^2) - \sigma_y$$

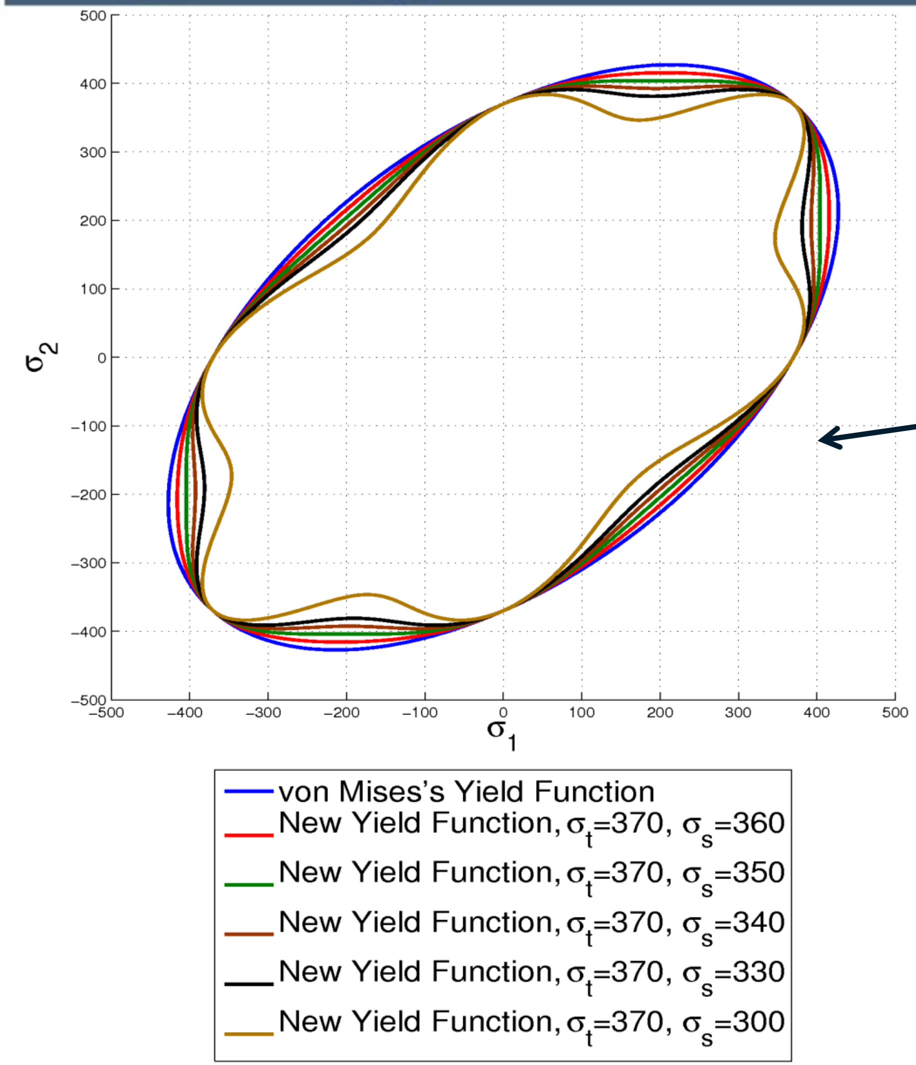
**Advantages**:

- Only one extra parameter in comparison to classical $J_2$ plasticity (*MAT_24)
- $\xi$-dependency is easier to grasp than other plasticity models
- Only two physical tests are required for calibration: tensile and shear test
- The yield function is always continuous
- The model is relatively simple to implement

**Disadvantage:**

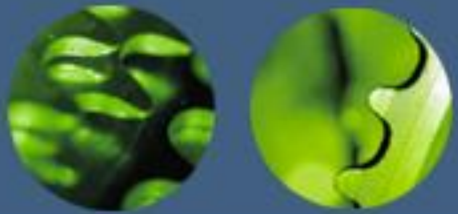
- Yield surface may be non-convex!

# 2. New Plasticity Model




Non-convexity may arise for large differences between $\sigma_t$ and $\sigma_s$!

$$\Phi = \sigma_{eq} + (\sigma_t - \sigma_s)(1 - \xi^2) - \sigma_y$$



# 2. New Constitutive Model: Constitutive relations

| Yield function |
|---|
| $$\Phi = \sigma_{eq} + (\sigma_t - \sigma_s).(1 - \xi^2) - \sigma_y = 0$$ |
| **Associative plastic flow** |
| $$\dot{\boldsymbol{\varepsilon}}^p = \dot{\gamma}\frac{\partial \Phi}{\partial \boldsymbol{\sigma}} = \left[\sqrt{\frac{3}{2}}\frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} - 2.(\sigma_t - \sigma_s)\xi\frac{\partial \xi}{\partial \boldsymbol{\sigma}}\right]$$ |
| **Plastic work equivalence** |
| $$\sigma_{eq}.\dot{\bar{\varepsilon}}^p = \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}^p$$ |

# 2. New Constitutive Model: Numerical implementation

**The following system of equations has to be solved:**

$$\begin{cases} res_{\boldsymbol{s}} = \boldsymbol{s}_{n+1} - 2G\boldsymbol{\varepsilon}_{n+1}^{e\,Trial} + 2G\Delta\gamma\left[\sqrt{\dfrac{3}{2}}\dfrac{\boldsymbol{s}_{n+1}}{\|\boldsymbol{s}_{n+1}\|} - 2.\,(\sigma_t - \sigma_s)\xi_{n+1}\dfrac{\partial\xi_{n+1}}{\partial\boldsymbol{\sigma}_{n+1}}\right] \\ res_{\boldsymbol{\varepsilon}^p} = \bar{\varepsilon}^p{}_{n+1} - \bar{\varepsilon}^p{}_n - \dfrac{\sigma_{n+1} : \Delta\boldsymbol{\varepsilon}^p}{\sigma_y\left(\bar{\varepsilon}^p{}_{n+1}\right)} \\ res_{\Delta\gamma} = \sigma_{eq_{n+1}} + (\sigma_t - \sigma_s).\,(1 - \xi_{n+1}^2) - \sigma_y\left(\bar{\varepsilon}^p{}_{n+1}\right) \end{cases}$$

**The consistent tangent operator (implicit analysis) reads:**

$$\boldsymbol{D}^{ep} = \frac{\partial\boldsymbol{\sigma}_{n+1}}{\partial\boldsymbol{\varepsilon}_{n+1}^{e\,Trial}} = 2G[\boldsymbol{A}]^{-1} : \left(\boldsymbol{I}_4 - \frac{1}{3}\boldsymbol{I}_2 \otimes \boldsymbol{I}_2\right) + K\boldsymbol{I}_2 \otimes \boldsymbol{I}_2$$

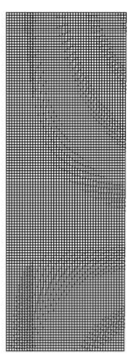
# 3. Numerical Results

| Material Properties | |
|---|---|
| Young Modulus | $E = 71.15\ GPa$ |
| Poisson’s Ration | $v = 0.3$ |
| Tensile yield stress | $\sigma_t = 370\ MPa$ |

## Tensile Stress States

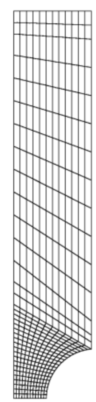
### Smooth bar

(Specimen used to calibrate the hardening curve)



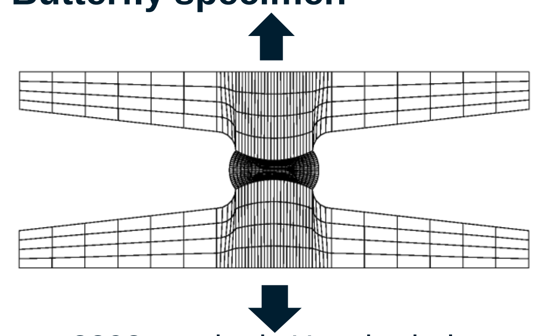
- Only a quarter was modelled
- 5760 Axisymmetric quadratic elements
- $D = 9mm$
- 5% reduction of section to trigger localization

### Flat grooved specimen



- Only a quarter was modelled
- 420 Axisymmetric quadratic elements
- Displacement Prescribed, $u = 0.4mm$
- $D = 5mm$, $R_{notched} = 1.59mm$, $l = 25mm$, $h = 50mm$

### Butterfly specimen

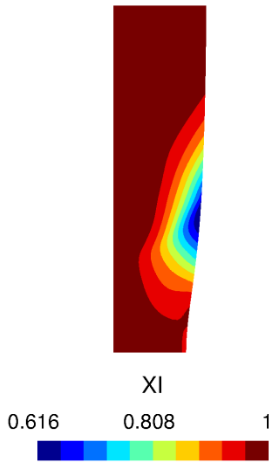


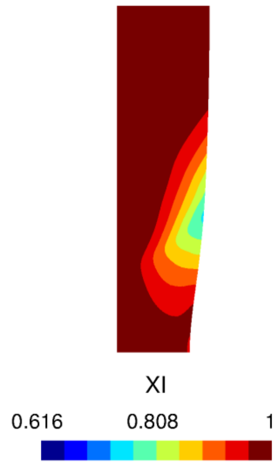
- 3392 quadratic Hexahedral elements
- Displacement Prescribed, $u = 1mm$
- Geometry taken from “Bai, Y. (2008), Effect of loading history on necking and fracture, PhD thesis, Massachusetts Institute Technology, Cambridge, MA.”

# 3. Numerical Results

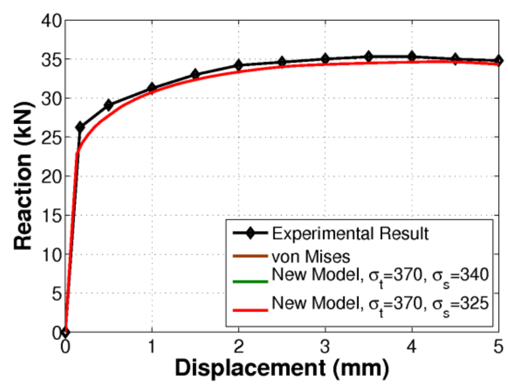
## Tensile Stress States: Smooth Bar

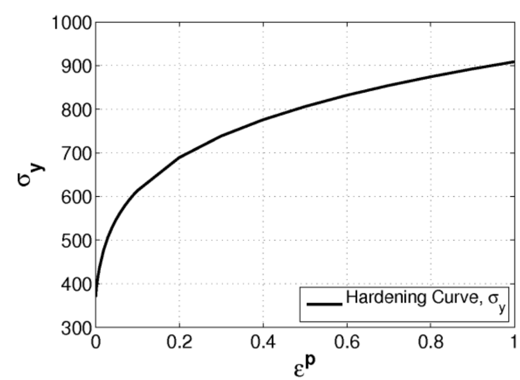
| Von Mises | New Model, $\sigma_s = 340$ | New Model, $\sigma_s = 325$ |
|---|---|---|





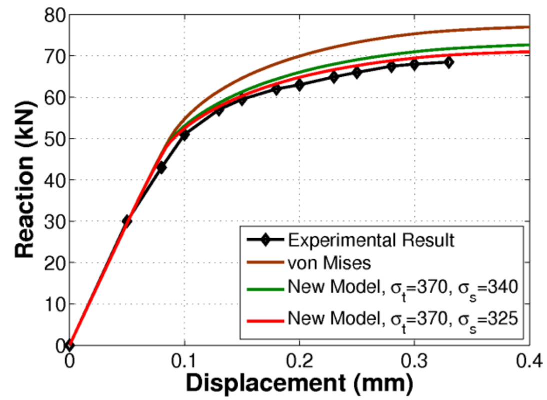



Hardening curve suggested by Bai et al.

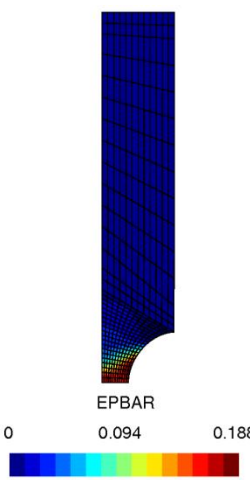
# 3. Numerical Results

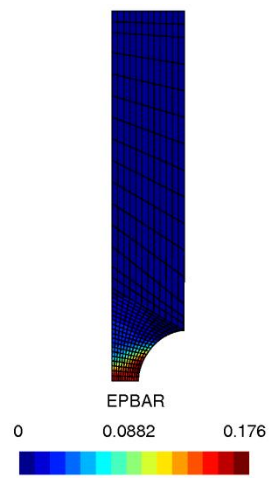
## Tensile Stress States: Flat Grooved, $R = 1.59mm$

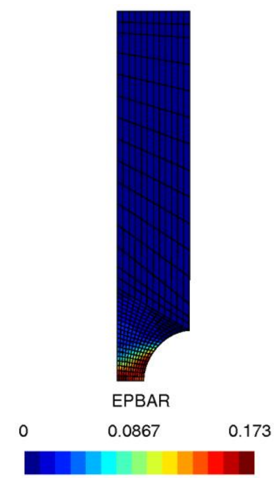


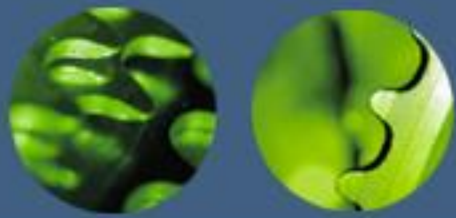

- The new model has the ability to properly capture the effects of $\xi$
- The appropriate value of $\sigma_s = 325\ MPa$
- Despite the slight non-convexity of the yield function for $\sigma_s = 325\ MPa$, no convergence problems were found → In fact, the convergence rate is practically quadratic

| Von Mises | New Model, $\sigma_s = 340$ | New Model, $\sigma_s = 325$ |
|---|---|---|



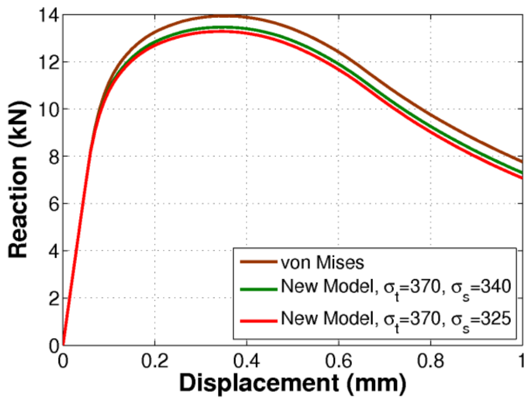





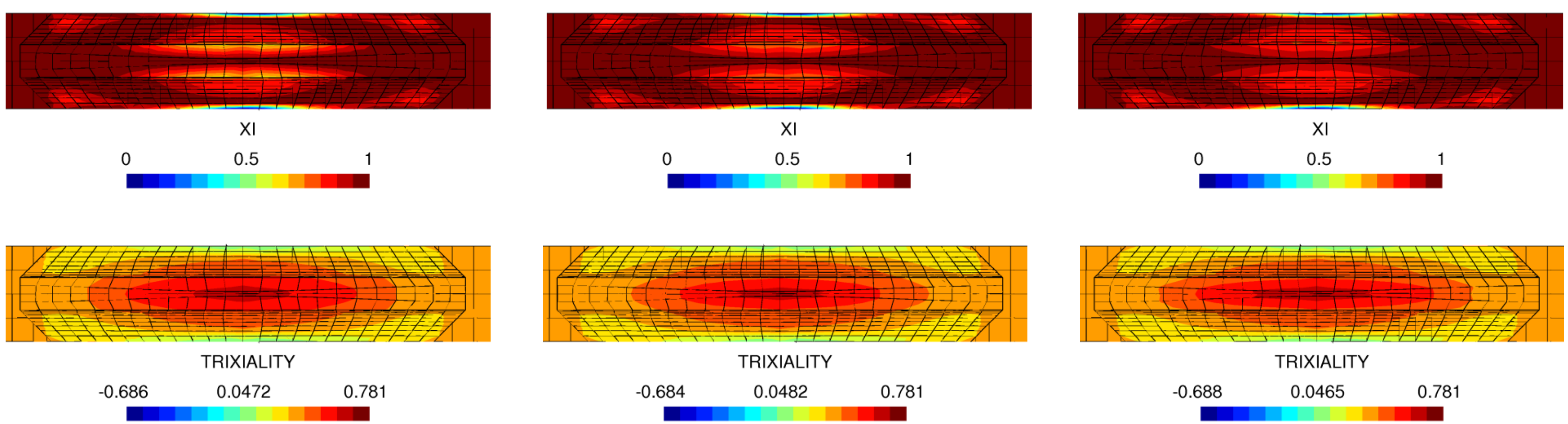
# 3. Numerical Results

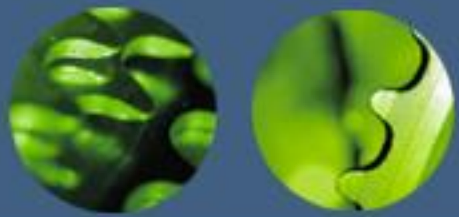
## Tensile Stress States: Butterfly Specimen




- The difference among the results is due to the fact that at the critical section $\xi$ is not equal to one
- Shear effects associated with average stress triaxialities take place at the critical section
- This is a consequence of the geometry

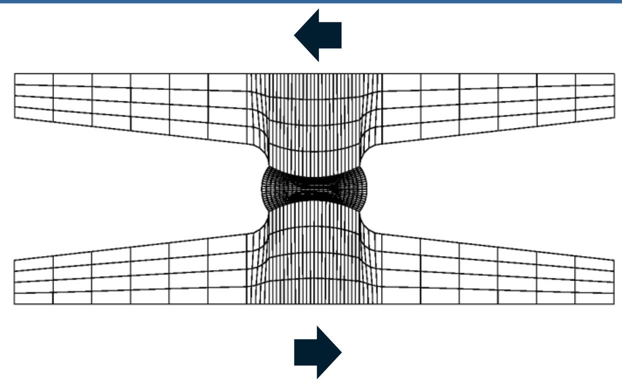
| Von Mises | New Model, $\sigma_s = 340$ | New Model, $\sigma_s = 325$ |
|---|---|---|

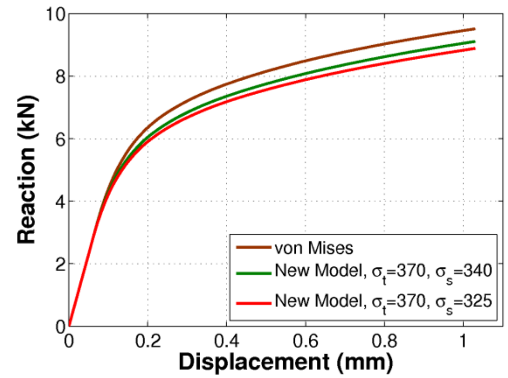




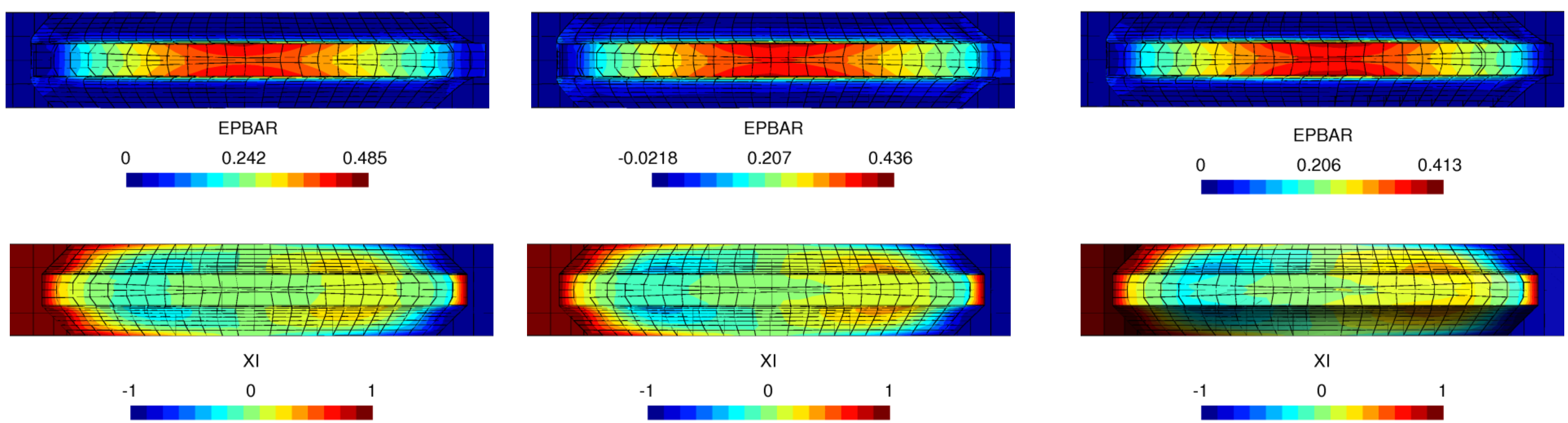
# 3. Numerical Results

## Shear Stress States: Butterfly Specimen



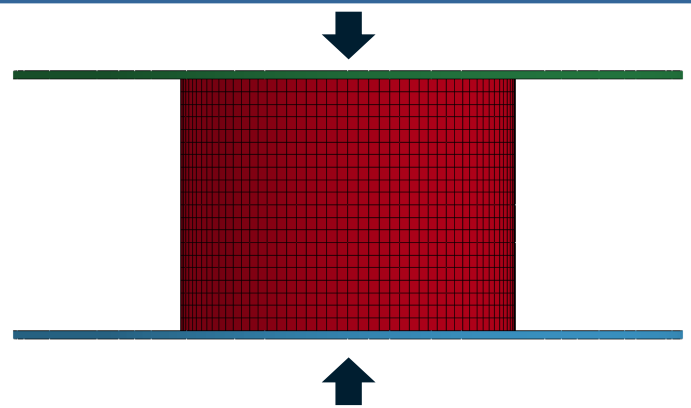



| Von Mises | New Model, $\sigma_s = 340$ | New Model, $\sigma_s = 325$ |
|---|---|---|

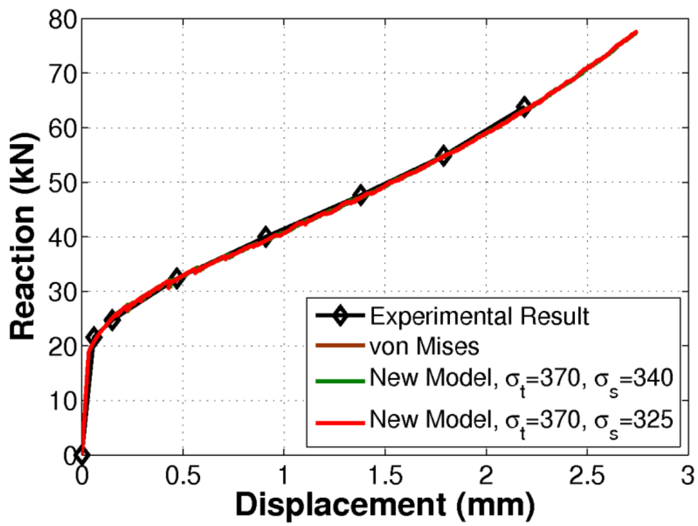


# 3. Numerical Results

## Compressive Stress State: Upset Test



- Number of Elements: 3253
- Friction coefficient: $\mu = 0.05$
- $D = 8mm,\ h = 6mm$

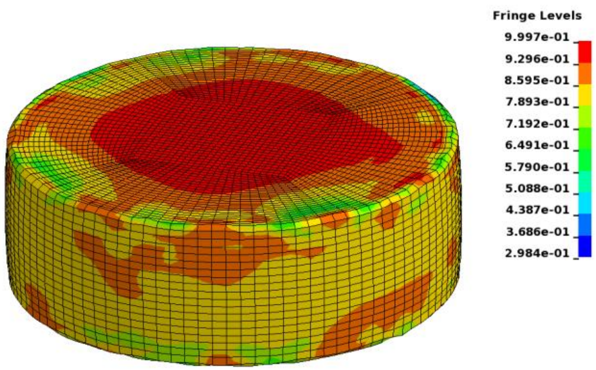


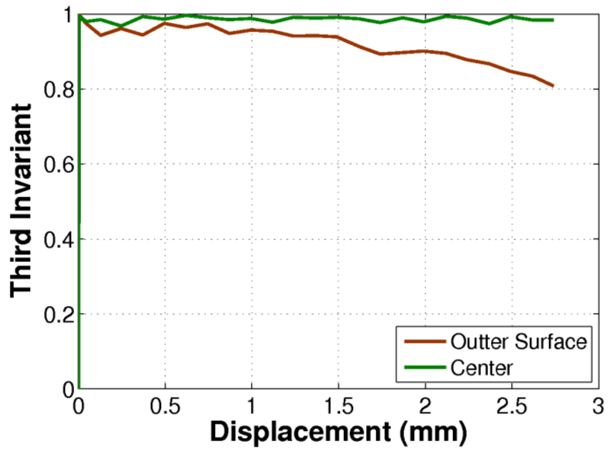

- There is no significant difference between the results provided by the von Mises and the new model
- Results do not change with varying $\sigma_s$
- The numerical and experimental results match perfectly

# 3. Numerical Results

## Compressive Stress State: Upset Test

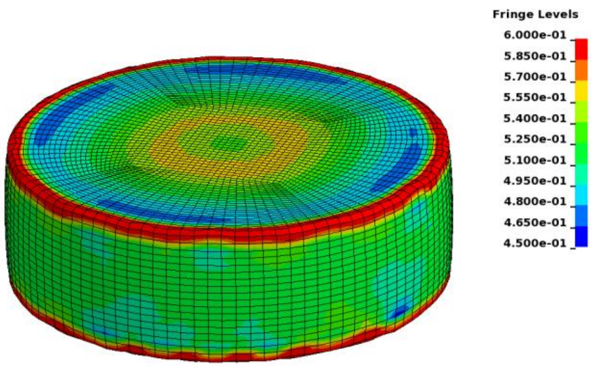
### Third Invariant, $\xi$

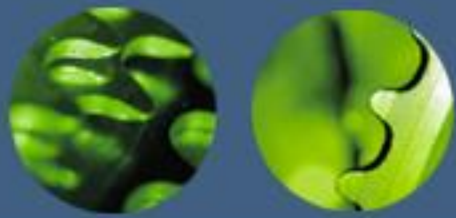



- Throughout the deformation process and at the center of the specimen, $\xi$ remains practically constant and equal to 1
- However, on the outer surface, $\xi$ is not constant, verifying a small decrease
- The evolution of $\xi$ on the outer surface of the specimen does not have any impact on the final result for different values of $\sigma_s$
- The results are in agreement with the main assumptions initially proposed

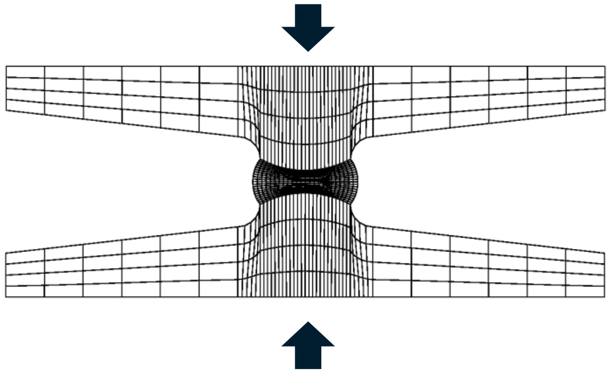
### Accumulated Plastic Strain

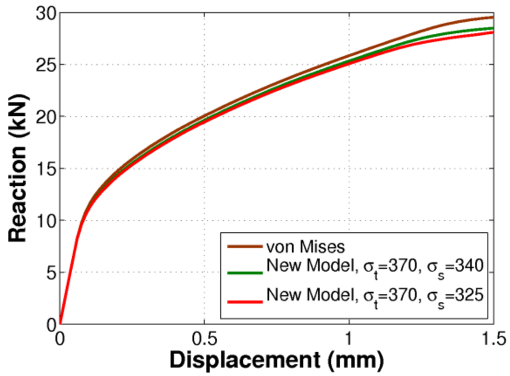




# 3. Numerical Results

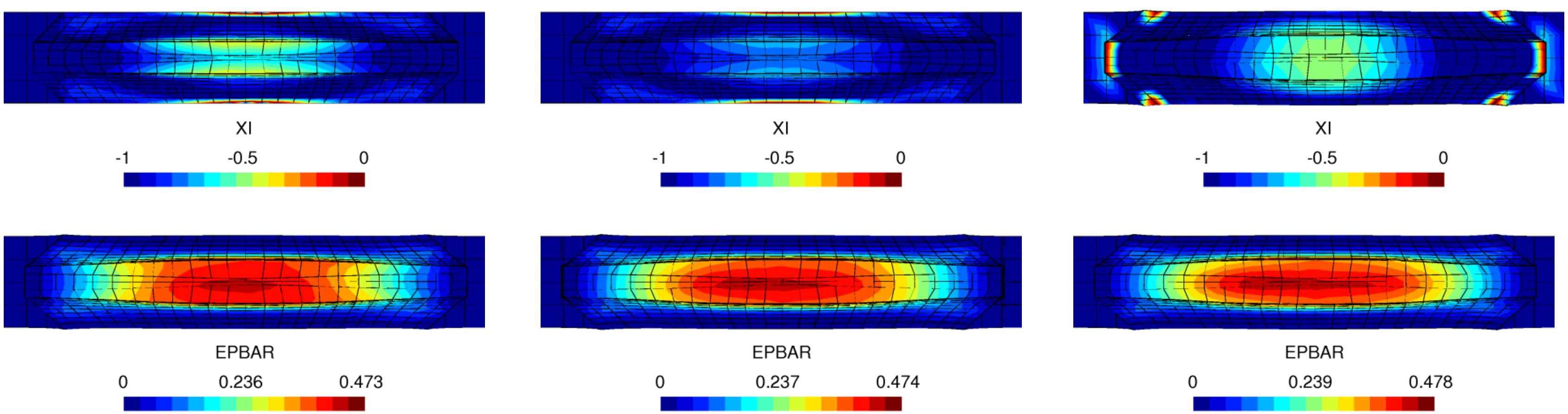
## Compressive Stress State: Butterfly Test






- There is no pronounced difference between the responses provided by von Mises and the new model
- In agreement with the initially desired features

| Von Mises | New Model, $\sigma_s = 340$ | New Model, $\sigma_s = 325$ |
|---|---|---|



# Final Remarks

- The proposed model is able to capture the effects of the third invariant at different stress states (tension, shear and compression)

- Only one extra material parameter is required by the proposed model when compared to classical von Mises plasticity; the effects of the additional material parameter are also easy to grasp

- From the computational point of view, the proposed model is simple to implement and also to understand; furthermore, it does not present convergence problems

- The initial desired features for the model were achieved


# Final Remarks

## Future work

- To perform an analytical analysis to find out the critical value of $\sigma_t - \sigma_s$ from which the resulting yield function is non-convex
- To define based on experimental tests how to define $\sigma_s$ (flat grooved specimen, pure shear test, …)
- To combine the new model with a damage/failure model (such as GISSMO through *MAT_ADD_EROSION)
- To use the new model to simulate components and structures in practical applications
- To better investigate the yield locus of aluminium alloys based on experimental evidence (is non-convexity perhaps a must in some cases?!)

# Thank You!

Visit the city of Porto in Portugal!

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