Topology optimization of transient nonlinear structures—A comparative assessment of methods

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1 Topology optimization under transient nonlinear structural-mechanical behavior

Topology optimization considering transient nonlinear behavior of mechanical structures, e.g. automotive crash, remains a challenge in both the implementation as well as computational effort. In recent years, efficient optimization algorithms and increased computer technology has begun to allow the development of methodologies to examine optimal topology of structures undergoing such behavior.

Here, the topology optimization methodologies are categorized by the abstraction of the loads, from fully transient nonlinear structural-mechanical analysis to multiple static replacement loads to a single static replacement load. Several methods are investigated for varying loading conditions and degrees of nonlinear behavior: 1) the use of the algorithms based on hybrid cellular automata with full transient nonlinear finite-element analyses; 2) multiple static replacement loads with updates via transient nonlinear finite-element analysis; 3) multiple static replacement loads without updates; 4) single static replacement load. These will be introduced in the following.

Each method has limits to its validity and application. To assess this, representative validation cases showing typical behavior of automotive components in an automotive crash have been used here.

2 Structural mechanics of crash simulation

Nonlinear structural-mechanical analysis is carried out via the finite-element method and is typical in the assessment of vehicular crashworthiness (cf. Fig. 1).

Fig.1: Simulated crash of a space-frame structure

In this type of analysis the following governing equation—albeit with simplified notation—is solved for acceleration $\ddot{u}$ each time step:

$$M(u,t)\ddot{u}(t) + D(u,t)\dot{u}(t) + f^{int}(u,t) = f^{ext}(u,t), \quad (1)$$

where M is the mass matrix, D is the damping matrix, $f^{\mathbf{int}}$, is the internal force vector, $f^{\mathbf{ext}}$ is the external force vector, $\ddot{u}$ is nodal acceleration vector, $\dot{u}$ is nodal velocity vector and is nodal displacement vector. The all-important constitutive relationship (i.e. material model) is implicitly part of the internal force term. Compacter this will be expressed here as

$$M\ddot{u} + D\{\dot{u} + f^{int} = f^{ext}. \quad (2)$$

As this process is highly nonlinear in addition to being exacerbated by noise (numerical as well as physical) and bifurcations, no analytical gradient information is available, i.e. analytical design sensitivity with respect to topological or geometrical design variables. Analytical design sensitivities are the cornerstone of standard topology optimization and without this, further efficient methodologies are needed. Therefore, either the nonlinear simulations are reduced or efficient algorithms are utilized to make optimization of such structure feasible. To address the former, linear elastostatic approximations are used and, therefore, a short introduction of the linear analysis is given.

As seen below, this nonlinear process can in some cases be simplified to a linear elastostatic case of the familiar form

$$Ku = f^{ext}, \quad (3)$$

where K is the stiffness matrix, and the left-hand side is of course also equal to the internal force,

$$Ku = f^{int}. \quad (4)$$

The Dirichlet boundary conditions are often artificial in this abstracted form, i.e. there are no true fixed boundary conditions in the real crash event. Therefore, inertial relief is used to simulate this free condition. This is carried out in that a reduction after Guyan [1965] is performed from which an inertia load is calculated such that further boundary conditions are not necessary. For the full derivation of inertia relief analysis refer to the following references: [Nelson & Wolf 1977; Barnett & al. 1995; Pagaldipti & Shyy 2004; Liao 2011; N.N. 2014]

3 Methods for topology optimization in crashworthiness

Here methods of topology optimization under transient nonlinear loading are categorized in two general types: 1) Reducing or eliminating the number of nonlinear structural analyses and 2) efficient optimization methods. Both have advantages and disadvantages, and these will be discussed after a short introduction and example of each method.

3.1 Methods reducing or eliminating nonlinear structural analyses via replacement loading

In this category, the number of nonlinear structural-mechanical analyses is greatly reduced or eliminated and instead linear elastostatic replacement loading is used. These methods vary from a single load derived from the nonlinear analysis applied to a structure and then calculated with linear elastostatic finite-element method to several loads that can be further updated throughout the optimization process.

3.1.1 Single replacement load

The simplest method of this category is using a single replacement load that is applied to the structure. The structure is then analyzed using linear elastostatic finite-element method. Standard topology optimization is then carried out, typically with compliance as the objective function. Constraints such as maximum allowable displacements can be implemented. Although efficient and relatively straight-forward, this method obviously cannot correctly represent areas of plastic material behavior, large deformation and contact.

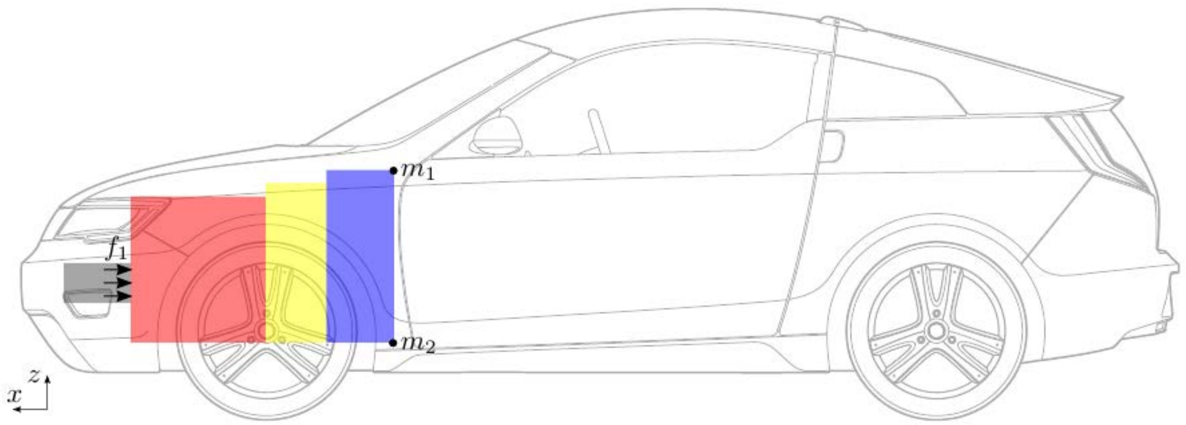

Fig.2: Load case for single replacement loading

This test example is based on the geometry of MUTE[1] and Visio.M[2], electrical vehicles developed by the Technische Universität München. A single force of 100 kN representing the mean force of the crash absorbers is loaded onto the geometry (Fig. 2) and the response is calculated via inertia relief with four mass points representing the mass of the vehicle.

This topology optimization and its structural-mechanical analysis is carried out using the software GENESIS[3,4]. The optimization problem is defined as minimum compliance design with no structural-mechanical constraints. A solid isotropic material and penalization approach is used to continualize the discrete [0, 1] nature (i.e. material is there or not there) of topology optimization. This problem is defined mathematically as following:

$$
\begin{aligned}
&\min\left\{u^T K u\right\} \\
&\text{where} \quad x \in [0,1] \\
&E_i = x_i E_{0,i} \\
&\rho_i = x_i \rho_{0,i} \\
&\text{such that} \\
&h(\rho) = m(\rho) - m_{\text{spec}} \\
&\text{governed by} \quad Ku = f^{ext},
\end{aligned}
\tag{5}
$$

with x being the design variables, ρ the density, E the elasticity module and ρ the equality constraint requiring the specified mass m_{spec}.

The optimization rapidly found an optimum and converged to this in eight iterations (Fig. 4). The optimal topology found is seen in Fig. 3. This represents the load paths for the stiffest structure for this linear load case. If the crash boxes were designed to absorb all crash energy, which of course is not usually the case, this would indeed be a proper basis for detailed design. Further, change in loading magnitudes and direction typical to an automotive crash has not be considered.

1 www.mute-automobile.de
2 www.visiom-automobile.de
3 www.vrand.com/Genesis.html
4 Version 13.1 in Linux

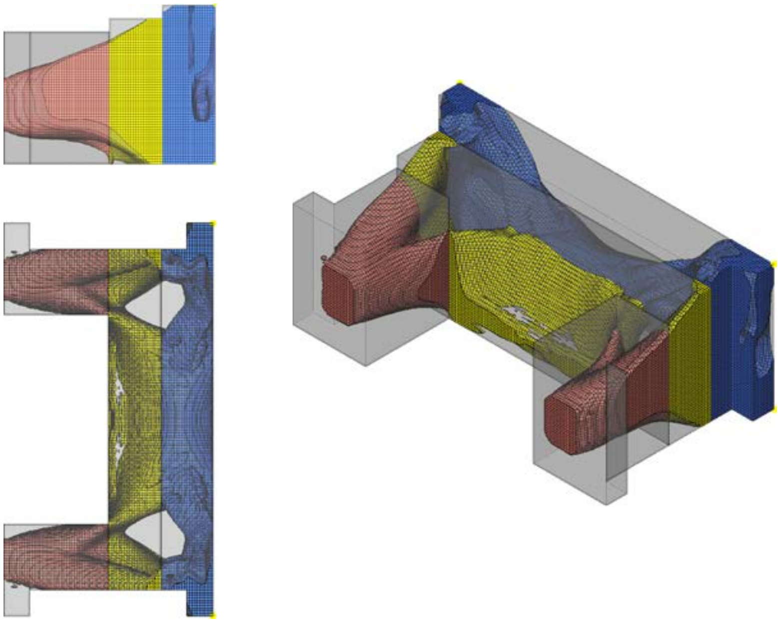

Fig.3: Optimal geometry for single replacement loading: side view (upper left), top view (lower left) and isometric view (right)

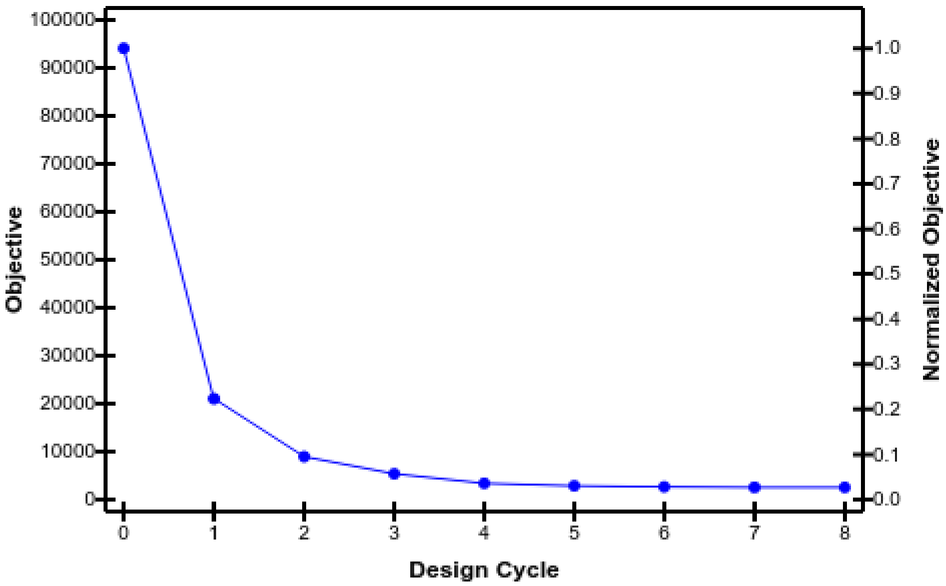

Fig.4: Convergence behavior for topology optimization with single replacement loading

This method is quick to implement and is reasonable to be used for transient nonlinear finite-element simulations to design those parts of the structure, which are expected to behave linearly or with only minuscule nonlinear behavior, i.e. light plasticity. This method is flexible with respect to the formulation of optimization problem including objectives and constraints. As this is a fully linear elastostatic method, it is not possible to use constraints in the time domain, including and related to acceleration. Inertia effects of heavy masses can be appropriately considered with finite-element analysis with inertia relief. When these assumptions are met, a single static replacement load can lead to expeditious feedback concerning optimal load paths.

3.1.2 *Multiple replacement loads*

A further development of the method described above was achieved by implementing several specific linear elastostatic replacement load cases over the time domain Volz [2011]. These load cases represent changes in the load magnitudes and direction over time, which were ignored above. Contact, such as that of the engine against a component, can be represented by the introduction of a further load step, though this contact must be known and implemented a priori.

Again, the geometry based on MUTE and Visio.M is used to demonstrate this method. Load cases were derived based on empirical knowledge of crash behavior of this vehicle. A series of loads have been used to represent different events within an offset front impact as by Euro NCAP. The first step is the activation of the left crash box resulting in a load of 50 kN. This is followed by a split of the load path into the lower and upper longitudinal members of 100 kN. The last load case represents the impact of the crushed members against the entire front block of the passenger cell, resulting in a distributed force of 150 kN.

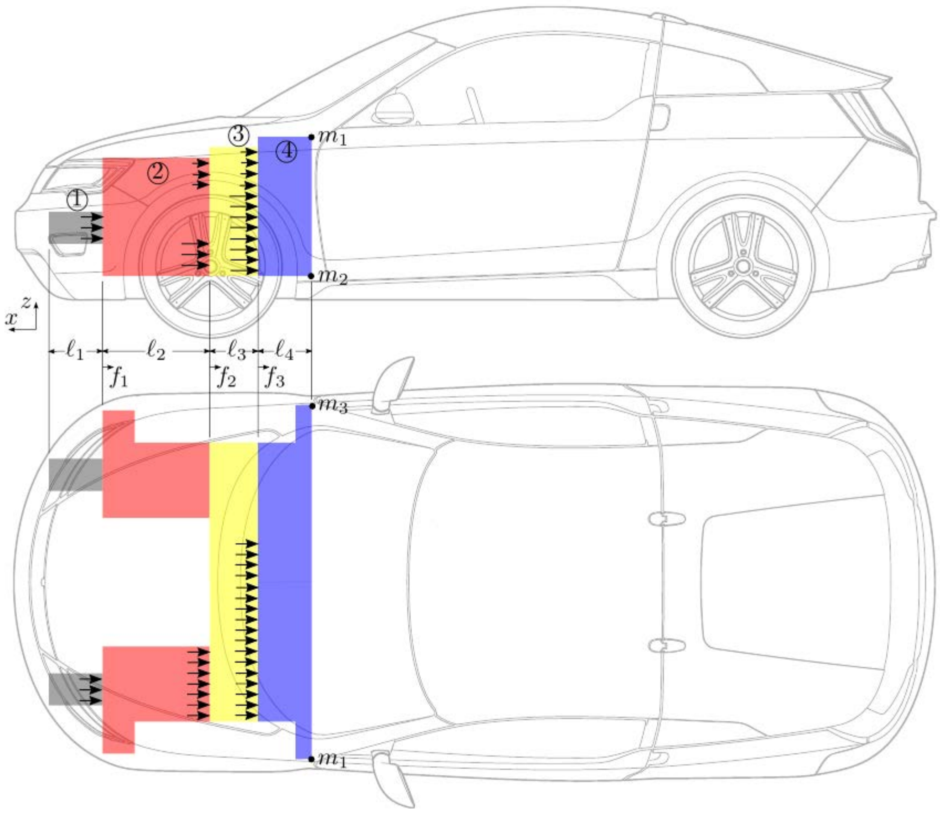

Fig.5: Load case for multiple replacement loading with four subdomains; 1. (grey): load-speed absorption zone, 2. (red) highly dynamic, 3. (yellow) dynamic, 4. (blue) elastic

These three loads cases are used in an equally weighted summed compliance problem, which is formulated mathematically as follows:

$$\min \sum_l \{u_l^T K u_l\}$$
where
$$x \in [0,1]$$
$$E_i = x_i E_{0,i}$$
$$\rho_i = x_i \rho_{0,i} \qquad (6)$$
such that
$$h(\rho) = m(\rho) - m_{\text{spec}}$$
governed by
$$K u_l = f_l^{ext}$$

Again for this problem, a quick convergence behavior was achieved using GENESIS coming to an optimum in 11 iterations (Fig. 7). The evaluation time was nearly double of that of the previous example due to the two further load cases. The optimal topology is seen in Fig. 6 and shows drastic differences in the yellow and blue regions due to the added load cases.

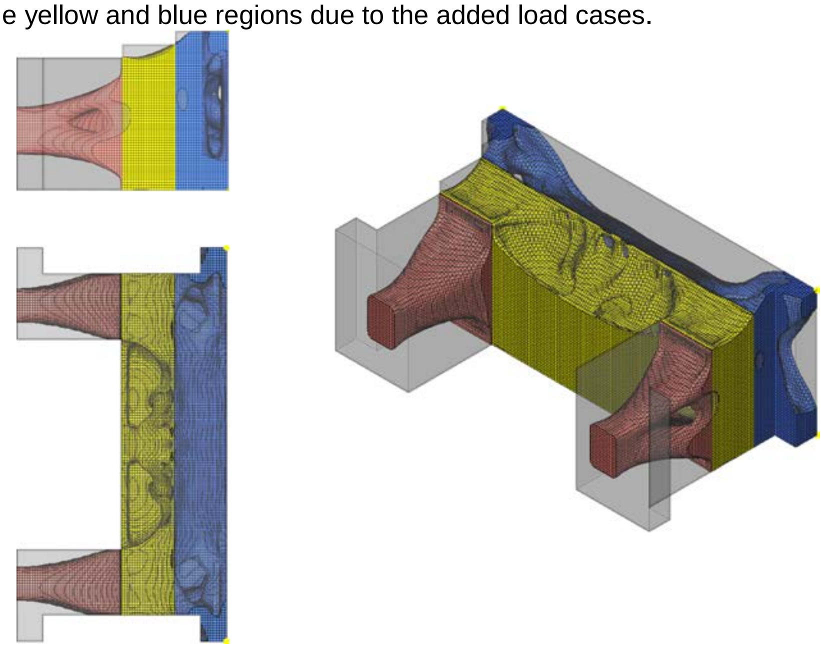

Fig.6: Optimal topology for multiple replacement loading: side view (upper left), top view (lower left) and isometric view (right)

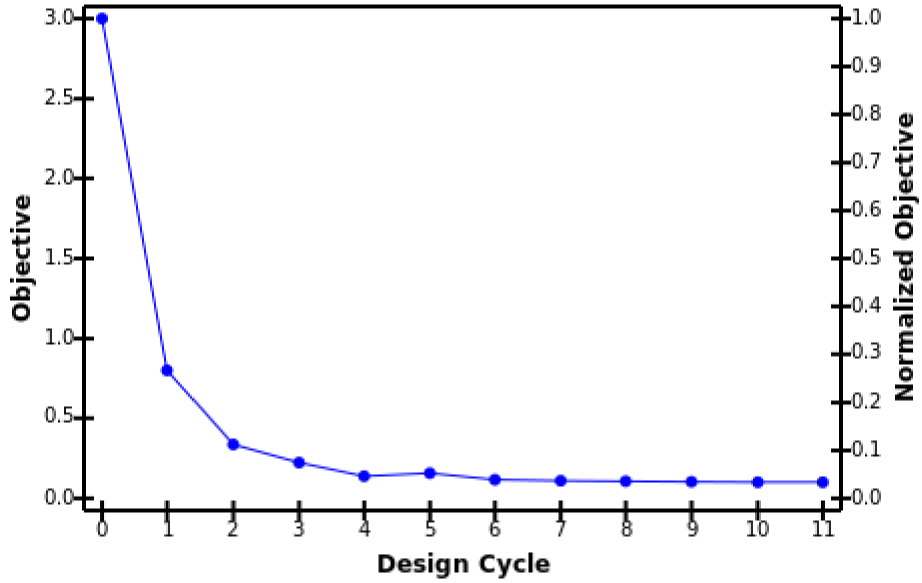

Fig.7: Convergence behavior of topology optimization with multiple replacement loading

The method of using multiple static replacement loads can be used, especially in early design phases, to identify optimal load paths. Here, though, much more a priori knowledge of the structural behavior is necessary. This knowledge is needed to be able to properly choose the location and magnitude of forces, which indeed plays a dominating role for the course in the load paths that will be identified. This method is similar to the above in the flexibility of the optimization formulation and its inability to consider acceleration aspects.

3.1.3 Multiple replacement loading with updates

A more refined multiple replacement loading methods was developed by Park [2011]. This method develops eequivalent static loads (ESL) for discrete time steps during the nonlinear process. It does this by first calculating the nonlinear analysis giving a the nodal displacements $u(t)$. Then using the linear stiffness matrix K, respective linear static loads f^{equiv} are found that give the calculated nonlinear nodal displacements at certain time steps, which are a subset of the nonlinear time steps. Like the previously mentioned methods, we can now utilize the analytical design sensitivities as linear elastostatic finite-element analysis is used. Park's method goes further and after convergence of the linear elastostatic design optimization, it recalculates the nonlinear finite-element analysis and updates the nonlinear displacements and, therefore, the equivalent static loads f^{equiv}. For topology optimization using this method, a minimum compliance design no longer makes sense as this would require a summation over all time steps.

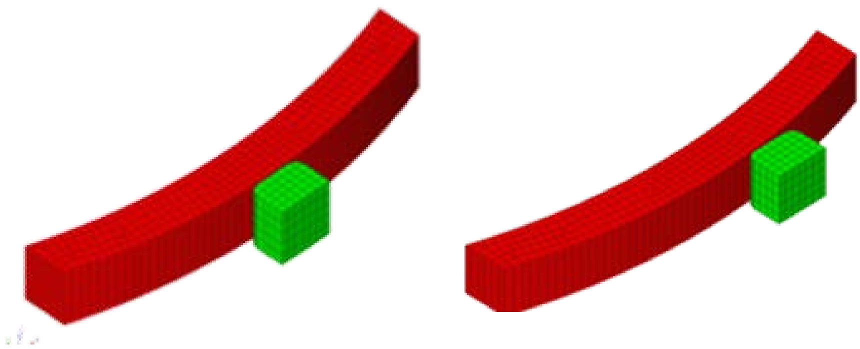

Fig.8: Mechanical analysis and starting topology for the center (left) and offset (right) load cases

Here ESLDYNA[5] is used which combines the linear elastostatic solver and optimizer GENESIS with the transient nonlinear solver LS-DYNA[6]. To illustrate this method, a standard example from ESLDYNA is used [N.N. 2012] designing the topology of an automotive bumper. In this case the optimal topology is to be found for a specified mass that minimizes the intrusion (displacement) for the two load cases shown in Fig. 8. The mathematical formulation is somewhat more abstract concerning the governing equations and this is shown here as

$$\begin{aligned}
&\min\{u\} \\
&\text{where} \\
&x \in [0,1] \\
&E_i = x_i E_{0,i} \\
&\rho_i = x_i \rho_{0,i} \\
&\text{such that} \\
&h(\rho) = m(\rho) - m_{\text{spec}} \\
&\text{governed by} \\
&K u_l^{lin} = f_l^{equiv} \\
&\text{where} \\
&f_l^{\mathbf{equiv}} = Ku \\
&\text{and} \\
&M\ddot{u} + D\dot{u} + f^{int} = f^{ext}
\end{aligned} \tag{6}$$

Further in Eq. 6, l represents the discretization for the equivalent static steps which is a subset of the time discretization of the nonlinear finite-element analysis.

The convergence history for this method is remarkable as it converges four times (Fig. 10). Convergence in each linear step is followed by a jump, which is caused by the update in loads via the new nonlinear analysis. The nonlinear analyses are, thus, used to improve the equivalent static loads but kept a minimum to keep computational effort at a manageable level. Here four nonlinear analyses are required and 35 linear. The converged topology can be seen in Fig. 9.

5 www.vrand.com/ESLDyna.html
6 www.lstc.com

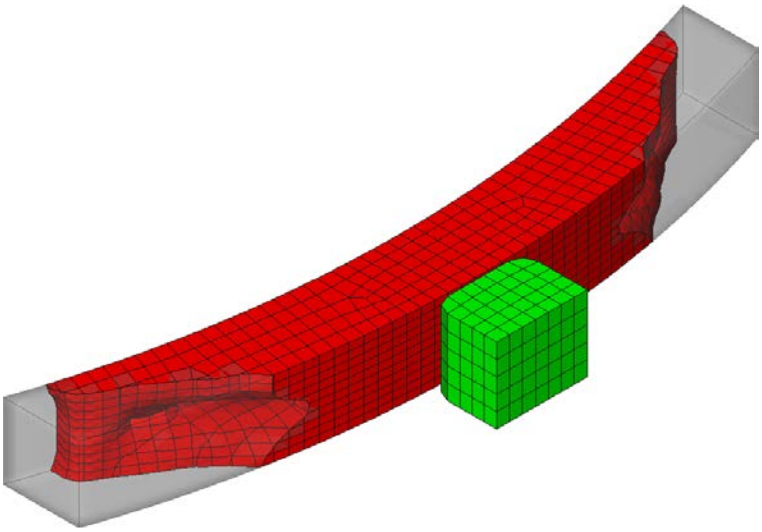

Fig.9: Optimum topology using multiple replacement loading with updates

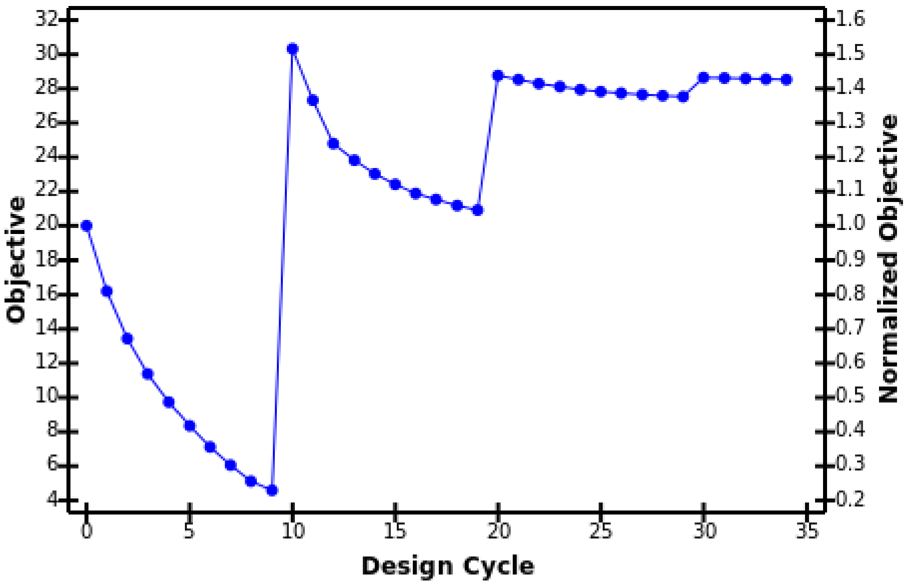

Fig.10: Convergence behavior of topology optimization with multiple replacement loading with updates

In contrast to the methods above, no a priori knowledge is necessary for the derivation of the linear replacement loads; these are derived directly from the nonlinear finite-element analysis. This results in increased computational effort: multiple convergences of the optimization in linear domain followed by updating of the static replacement loads. The formulation of the optimization problem remains flexible, though standard topology optimization via minimum compliance design is not feasible. Constraints related to acceleration may be possible via finite differencing schemes over the larger steps of the linear temporal subdomain; this though has not been implemented in the commercial software used here.

Although it was outside the scope of this study to investigate this method for use on structures having progressive failure behavior, e.g. crash boxes, this is planned to be analyzed in the immediate future.

3.2 *Methods based on nonlinear simulations*

In this section, the second family of methods of using full transient nonlinear structural-mechanical analysis in concert with very efficient, gradient-free optimization methods to find optimal topologies will be discussed. Although other, related methods do indeed exist, this section will concentrate on hybrid cellular automata (HCA). Further methods include the poorly named, as it has nothing to do with evolutionary strategies, *evolutionary structural optimization* (ESO) from Xie and Steven [1997] and its extended relative *bidirectional evolutionary structural optimization* (BESO) from Querin et al. [1998].

Cellular automata were first used in topological design of mechanical structures by Inou et al. [1994]. Building on the work of Tovar [2004], Patel [2007] expanded this for use in concert with crashworthiness considerations via in nonlinear finite-element analyses. The method was implemented in the commercial software LS-TaSC[7,8]. Hunkeler [2013] went further specifically adapting HCA to work with thin-walled structures in crashworthiness. The results from two versions of hybrid cellular automata will be shown below.

Hybrid cellular automata strive to homogenize a system state of each cell and in use in automotive crashworthiness design this is chosen to be internal energy U, which is then homogenized to some reference U^*. The optimization problem for HCA for crashworthiness can be generally defined as

$$\begin{aligned}
&\sum_i \left\{U_i - U_i^*\right\} \\
&\text{where} \\
&x \in [0,1] \\
&\rho_i = x_i \rho_{0,i} \\
&E_i = x_i E_{0,i} \\
&E_{h,i} = x_i E_{h,0,i} \\
&\sigma_{y,i} = x_i \sigma_{y,0,i} \\
&\text{such that} \\
&h(\rho) = m(\rho) - m_{\text{spec}} \\
&\text{governed by} \\
&M\ddot{u} + D\{\dot{u} + f^{int} = f^{ext}
\end{aligned} \tag{7}$$

with E_h being the strain-hardening modulus and σ_y the yield stress.

As such is the possibility of different objectives limited and multiple only abstractly, if at all. Constraints can be implemented, but only in such a way that it is known a priori if more or less material via increase of mass forces the design into the feasible domain. Violation of displacement constraints result in increased mass and violation force constraints (i.e. peak force) results in removal of mass. Because of this, general nonlinear constraints are not possible.

The full transient nonlinear finite-element method is used for the system evaluations and this allows for the full spectrum of nonlinearities to be considered. Due to the nature of HCA, the number of design variables plays no or little role in the convergence time. HCA utilizes the differences between neighboring cells as basis for the design of the next iteration and, therefore, these differences can be seen analogous to gradients in standard structural design optimization.

3.2.1 *Hybrid cellular automaton in LS-TaSC*

In HCA of LS-TaSC every finite element is considered a cell to be homogenized. This leads to a great deal of design variables, which can still be handled very efficiently. Two verification cases are used here with LS-TaSC: the transversely loaded beam, representing an automotive side sill undergoing a

7 ww.lstc.com/products/ls-tasc
8 Version 3.0

pole impact; and the axially loaded column, representing a crash box being activated to absorb energy in a frontal impact. Each of these structures is modeled as extruded aluminum sections with an internal grid of thin-walled reinforcements. The wall thicknesses (going down to not existing) of this grid are the design variables used for the optimization.

Transversely loaded beam (side sill in pole crash)

In the first problem shown, a extruded beam is transversely impacted from the side (Fig. 11). This is a simplified example for the side sill of an automobile loaded in the case of pole impact. HCA is used to homogenize the internal energies of all elements in the structure.

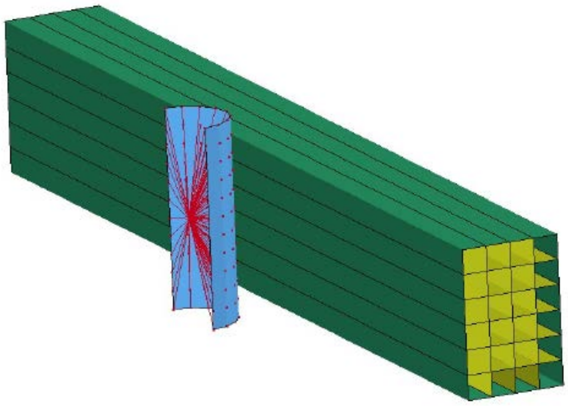

Fig.11: Configuration of transversely loaded beam

The optimization with HCA converges in spite of the high number of design variables to an optimal design within 50 iterations. As we see that the redistribution of mass never reaches a level under ca. 0.015, a hard convergence is not achieved. It can be seen, though, that this algorithm rapidly reduces intrusion in the structure.

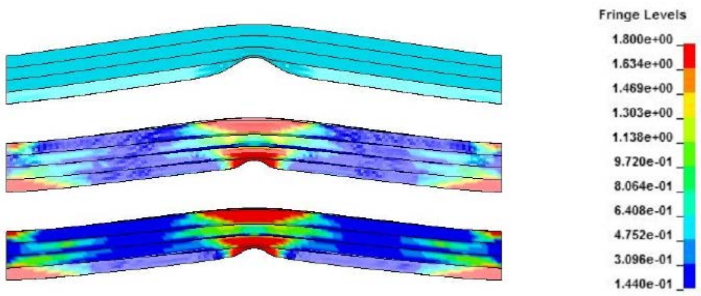

Fig.12: Convergence of designs of the transversely loaded beam with LS-TaSC showing optimal wall thicknesses in mm (from top to bottom: 1^{st}, 25^{th} and 50^{th} iteration)

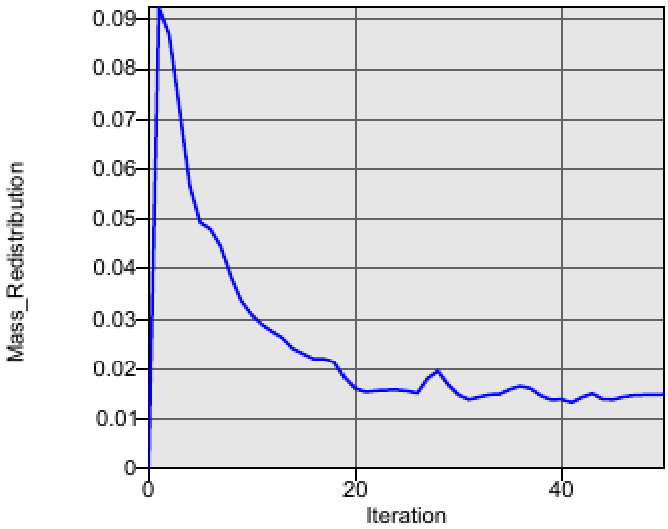

Fig.13: Convergence behavior of the transversely loaded beam with LS-TaSC

Axially loaded column

A much more challenging structure to optimize is the axially loaded column, an abstracted crash box (Fig. 14). Again, the objective of the algorithm is to homogenize the internal energy of all elements. In this case though, this leads to a cyclic behavior of removing from the bottom and then the top, showing no convergence. To avoid this problem a manufacturing constraint was added, which states that all elements in extrusion direction must have the same wall thickness.

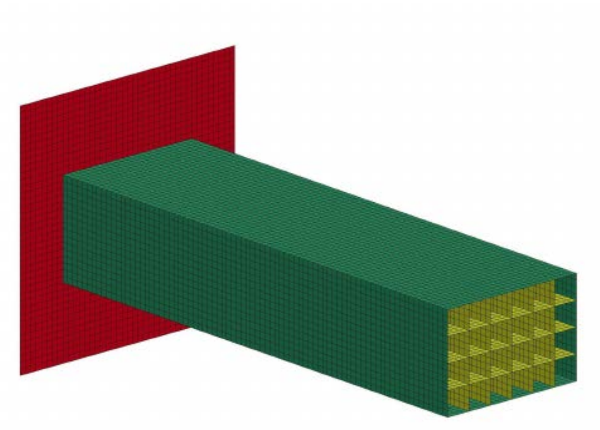

Fig.14: Configuration of axially loaded column

The algorithm shows again good convergence, though never reaching hard convergence. It is to be observed though that the crushing begins at different parts of the extruded section. It is further to be noted that the internal parts of the grid become very thin; the quality of this optimization may be increased by increasing the wall thickness in which elements are deleted.

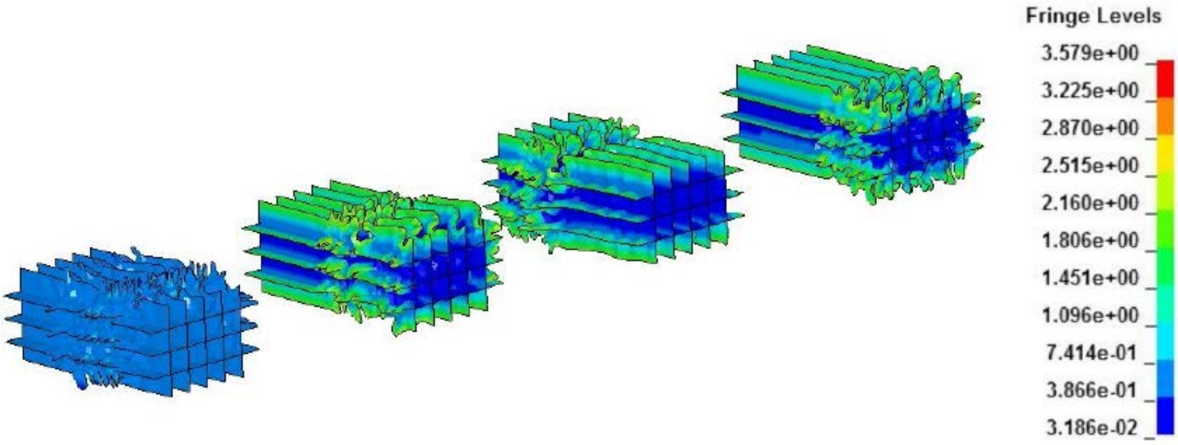

Fig.15: Convergence of designs of the axially loaded column with LS-TaSC showing optimal wall thicknesses in mm (from left to right: 1st, 17th, 33rd, and 50th iteration)

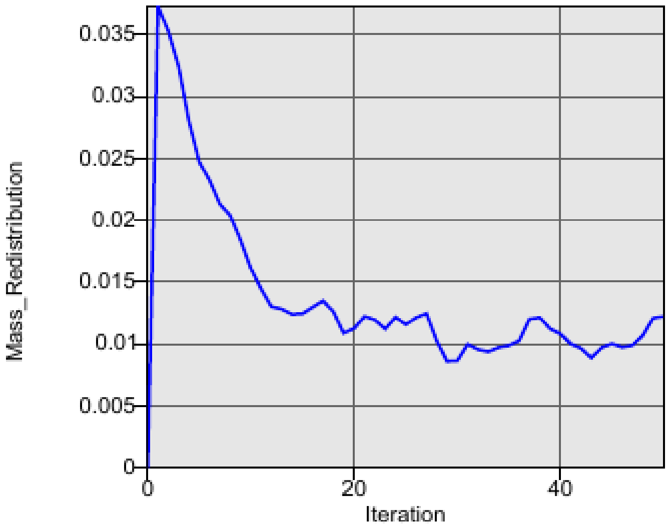

Fig.16: Convergence behavior of the axially loaded column with LS-TaSC

Summary of HCA

With LS-TaSC, topology optimization problems were easily implemented and the number of evaluations was relatively small leading to fast results. Both the optimization formulation along with constraint possibilities is somewhat limited in the current version of LS-TaSC. Neither of the cases used reached a hard convergence and this will be investigated in the near future. Further, though, the axial case did show improvement over the starting design, this did not show the preform as desired for use in the topology optimization of crash boxes.

3.2.2 *Hybrid cellular automaton for thin-walled structures*

Hunkeler [2013] further extended this method to properly handle thin-walled extrusions especially under axial loadings as is common in automotive crash boxes, naming this hybrid cellular automaton for thin-walled structures (HCA-TWS). This method differentiates itself from that implemented in LS-TaSC in that its cells are not every finite element. Instead the cells are defined as domains of numerous finite elements. The HCA algorithm then tries to homogenize the behavior of each domain and not each finite element.

Using the same configuration introduced above, this was optimized using HCA-TWS in 100 evaluations. The cells are defined by the grid, including all cells in crushing direction (Fig. 17). No hard convergence was reached and the best design was taken. The optimal design (Fig. 18), though, shows drastic improvement over both starting configuration and other standard configurations.

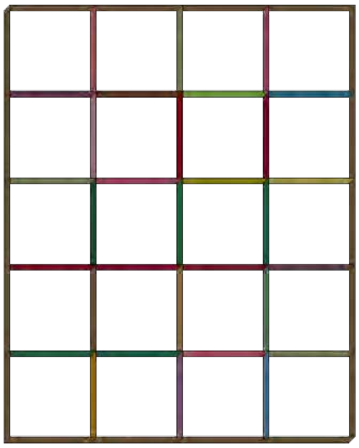

Fig.17: Starting configuration of the axially loaded column for HCA-TWA along with cell definitions (every color a cell) [Hunkeler 2013]

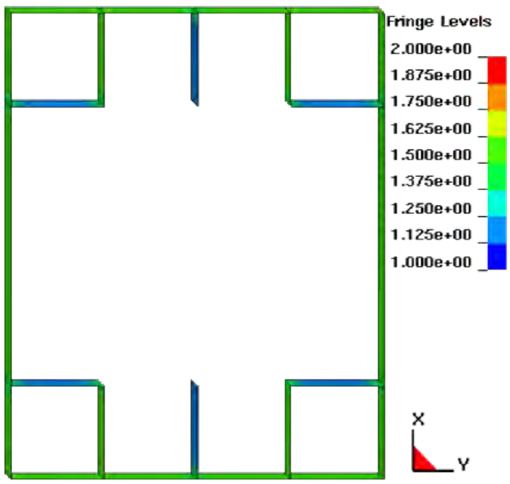

Fig.18: Optimal topology of the axially loaded column with HCA-TWA [Hunkeler 2013]

This method, through its inherent nature of averaging several cells and not homogenizing locally (finite elements) is much better conditioned to be used in conditions of energy absorption such as crushing. Though like LS-TaSC, further strides need to be taken to verify convergence behavior.

4 Review and comparison of methods

After studying the use of the methods for topology optimization of transient nonlinear structures illustrated above, a comparison is proposed here. As this study was not exhaustive in all regards of all methods, further investigations are needed to validate and substantiate these findings. From the experience of these optimization results amongst others, Table 1 has been created to ascertain the performance of the methods for different design criteria and structural-mechanical behavior.

Table 1: *Comparison of methods for topology optimization of transient nonlinear structures. Criteria being very appropriate ++, appropriate: +, neutral: 0, somewhat inappropriate : -, very inappropriate: - -, no rating, further investigation must be completed: **

Category	Method	Number of design variables	Multiple objectives	Constraint flexibility	as constraint	Highly nonlinear behavior	Light plasticity	Bending	Progressive collapse
Replacement loading	Single replacement	+	++	++	- -	- -	0	-	--
	Multiple replacement	+	++	++	- -	- -	+	0	-
	Multiple replacement with update	+	++	++	-	-	++	+	*
Efficient optimization algorithms	HCA in LS-TaSC	++	- -	- -	++	++	++	+ +	-
	HCA-TWS	++	-	-	+	++	++	+ +	+

Future studies are planned and running to verify the statements of Table 1. Further benchmark example will be necessary to further quantify the results as well as the advantages and disadvantages of these methods.

5 Literature

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