



# ***GENESIS***

## **ANALYSIS EXAMPLES MANUAL**

***VERSION 11.0***

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# CHAPTER 1

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## Introduction

- Overview
- Analysis Examples





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## 1.1 Overview

The purpose of this volume is to provide a variety of simple analysis examples to assist the user in becoming familiar with the *GENESIS* analysis capabilities. These examples are also provided with the *GENESIS* software so they may be executed for comparison.

It should be remembered that, because of different machine precisions, the results may differ slightly from those presented here.

---

## 1.2 Analysis Examples

To assist the user in understanding the *GENESIS* analysis capabilities, a variety of examples are presented in Chapter 2.

The following table lists the examples, together with the features demonstrated by each:

| SECTION | PROBLEM SOLVED                          | SPECIAL FEATURES                                   |
|---------|---|--|
| 2.1     | Classical Three Rod Truss               | Mass Element.                                      |
| 2.2     | Ten Rod Truss                           | None.  |
| 2.3     | Portal Frame                            | None.  |
| 2.4     | Circular Ring                           | Cylindrical Coordinate System                      |
| 2.5     | Wide Column/Wide Beam Frame             | Bar offsets  |
| 2.6     | Deformed Beam                           | Enforced displacements.                            |
| 2.7     | Two Bar Symmetric Frame                 | Multi-point constraints. Load combinations.        |
| 2.8     | Three Story Facade with a Panel - A     | Rigid elements.                                    |
| 2.9     | Three Story Facade with a Panel - B     | None.  |
| 2.10    | Vibration of a Skewed Beam              | Two coordinate systems.                            |
| 2.11    | Beam with Temperature Load              | Single point constraints.                          |
| 2.12    | Portal Frame with Flexible Nodes        | Multi-point constraints. Scalar springs.           |
| 2.13    | Beam Supported by Springs               | Scalar springs.                                    |
| 2.14    | Planar Grillage with Internal Pin       | Pin flags.   |
| 2.15    | Rod Modeled with Tetra Elements         | None.  |
| 2.16    | Rod Modeled with Penta Elements         | None.  |
| 2.17    | Anisotropic Plate with Thermal Load - A | Orthotropic material, MAT8.                        |
| 2.18    | Anisotropic Plate with Thermal Load - B | Anisotropic material, MAT2.                        |
| 2.19    | Straight Uniform Fin Analysis           | Convection using boundary elements LINE and POINT. |

| SECTION | PROBLEM SOLVED  | SPECIAL FEATURES  |
|---------|---|---|
| 2.20    | Square Plate With Volume Heat Generation and Enforced Temperature         | Enforced temperature.   |
| 2.21    | Conduction on an Elliptical Bar Heated by a Distant Source                | Utilization of boundary element ELCYL and vector flux loading.                        |
| 2.22    | Heat Conduction in an Anisotropic Plate                                   | Utilization of anisotropic material MAT5 with PSHELL.                                 |
| 2.23    | Heat Conduction in an Anisotropic Solid                                   | Utilization of anisotropic material MAT5 with PSOLID.                                 |
| 2.24    | Convection-Conduction Heat Transfer in the Straight Cantilever Problem    | Utilization of ambient temperature loading and use of boundary element AREA4.         |
| 2.25    | Dynamic Response of the Cantilevered Beam                                 | Frequency response analysis using direct dynamic analysis and modal dynamic analysis. |
| 2.26    | Dynamic Response of the Cantilevered Beam With Element Structural Damping | Use of element structural damping, GE value in MAT1.                                  |
| 2.27    | Dynamic Response of the Cantilevered Beam With and Without Modal Damping  | Use of modal structural damping, TABDMP1.   |
| 2.28    | Dynamic Response of a CELAS1/CDAMP1/CMASS1 Structure                      | Use of scalar elements and SPOINTs.   |
| 2.29    | Three Rod Truss - Analysis Using Multidisciplinary Load Cases             | Use of multiple load case types.  |
| 2.30    | Thin Axisymmetric Annulus   | None.   |
| 2.31    | Three Independent Beams   | PBEAM statement.  |
| 2.32    | Two Pairs of Beams with and without Warping Coefficients                  | Use warping coefficients in the PBEAM statement.                                      |
| 2.33    | Two Independant Beams, with and without Offset                            | Use offset in the CBEAM statement.  |
| 2.34    | Buckling Analysis of a Tripod   | Use of buckling analysis.   |
| 2.35    | Enforced Dynamic Displacement with the Large Mass Method                  | Large Mass Method for enforced dynamic displacement.                                  |
| 2.36    | Enforced Dynamic Displacement with the Lagrange Multiplier Method         | Use of constraint equations using DMIG.   |

|             |   |                        |
|-------------|---|------------------------|
| <b>2.37</b> | <b>Random Response of a Simple Spring/Mass/Damper Structure</b> | Use of Random response |
| <b>2.38</b> | <b>Random Response of a Cantilever Plate</b>                    | Use of Random response |
| <b>2.39</b> | <b>Rigid Body Modes of Doubly Curved Shells</b>                 | Use of weld element    |
| <b>2.40</b> | <b>Rigid Body Modes of Double Hat Structure</b>                 | Use of weld element    |





# CHAPTER 2

---

## Analysis Examples

- Classical Three Rod Truss
- Ten Rod Truss
- Portal Frame
- Circular Ring
- Wide Column/Wide Beam Frame
- Deformed Beam
- Two Bar Symmetric Frame
- Three Story Facade with a Panel - A
- Three Story Facade with a Panel - B
- Vibration of a Skewed Beam
- Beam with Temperature Load
- Portal Frame with Flexible Nodes
- Beam Supported by Springs
- Planar Grillage with Internal Pin
- Rod Modeled with Tetra Elements
- Rod Modeled with Penta Elements
- Anisotropic Plate with Thermal Load - A
- Anisotropic Plate with Thermal Load - B
- Straight Uniform Fin Analysis

- **Square Plate With Volume Heat Generation and Enforced Temperature**
- **Conduction on an Elliptical Bar Heated by a Distant Source**
- **Heat Conduction in an Anisotropic Plate**
- **Heat Conduction in an Anisotropic Solid**
- **Convection-Conduction Heat Transfer in the Straight Cantilever Problem**
- **Dynamic Response of the Cantilevered Beam**
- **Dynamic Response of the Cantilevered Beam With Element Structural Damping**
- **Dynamic Response of the Cantilevered Beam With and Without Modal Damping**
- **Dynamic Response of a CELAS1/CDAMP1/CMASS1 Structure**
- **Three Rod Truss - Analysis Using Multidisciplinary Load Cases**
- **Thin Axisymmetric Annulus**
- **Three Independent Beams**
- **Two Pairs of Beams with and without Warping Coefficients**
- **Two Independent Beams, with and without Offset**
- **Buckling Analysis of a Tripod**
- **Enforced Dynamic Displacement with the Large Mass Method**
- **Enforced Dynamic Displacement with the Lagrange Multiplier Method**
- **Random Response of a Simple Spring/Mass/Damper Structure**
- **Random Response of a Cantilever Plate**
- **Rigid Body Modes of Doubly Curved Shells**
- **Rigid Body Modes of Double Hat Structure**



---

## 2.1 Classical Three Rod Truss

### Example ID

A001

### Analysis Data Used

GRID, GRDSET, CONM2, CROD, PROD, MAT1, LOADCASE, LOAD, FORCE, METHOD, EIGR, ELFORCE, OLOAD, SPCFORCES, STRESS, DISP and SVECTOR.

### Special Features Used

Utilization of Mass element CONM2.

### Problem Statement

The 3 rod truss structure of **Figure 2-1** is loaded in two separate load cases. For the first load case find the nodal displacements, element forces, stresses, and reaction forces. For the second load case find the two natural frequencies and the corresponding two modes shapes with a lumped mass of 1000.0 at node 4.

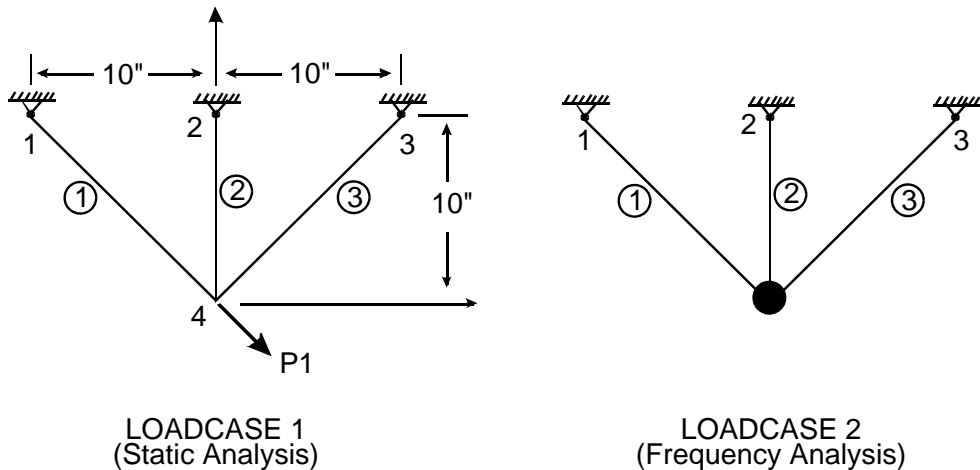
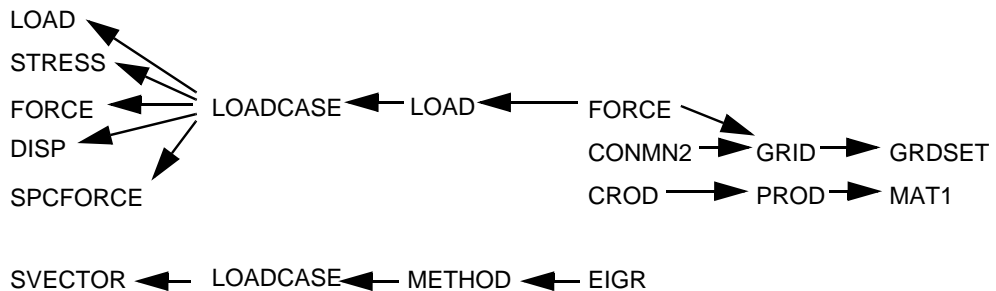


Figure 2-1

## Structural Analysis Model

1. Three CROD elements and one CONM2 mass element.
2. Section properties:  $A1=A3=1.0 \text{ in}^2$ .  $A2=2.0 \text{ in}^2$
3. Material:  $E=1.0E7 \text{ psi}$
4. Two load cases.
  1.  $P1=20000 \text{ lb}$ .
  2. Natural vibrations. Lumped mass:  $M=1000$ . Distributed mass:  $m=0.1$

## Analysis Data Relationships



## Special Modeling Techniques

NONE

## Reference Solutions

NONE

---

## 2.1.1 Input Data

```
ID A001
SOL COMPAT0
CEND
TITLE = THREE BAR TRUSS
ECHO=NONE
LINE=64,80
SPC = 100
LOADCASE 1
    LABEL = LOAD CONDITION 1
    LOAD = 300
    ELFORCE=ALL
    OLOAD=ALL
    SPCFORCE=ALL
    STRESS=ALL
    DISP=ALL
LOADCASE 2
    METHOD=2
    SVECTOR=ALL
BEGIN BULK
$ GRID DATA
GRID    1           -10.    0.0    0.0
GRID    2           0.0    0.0    0.0
GRID    3          10.0    0.0    0.0
GRID    4           0.0   -10.0    0.0
$ SUPPORT DATA
SPC1    100    123456  1      2      3
SPC1    100    3456   4
$ ELEMENT DATA
CROD    1      11      1      4
CROD    2      12      2      4
CROD    3      13      3      4
$ MASS ELEMENTS DATA
CONM2    4      4           1000.0  0.0    0.0    0.0
+      0.0    0.0    0.0    0.0    0.0    0.0
$ PROPERTY DATA
PROD    11      1      1.0
PROD    12      1      2.0
PROD    13      1      1.0
$      2      3      4      5      6      7      8      9     10
MAT1    1      1.0E+7    0.33    0.1
PARAM,WTMASS,.00259
$ EXTERNAL LOADS DATA
FORCE   300      4           20000.  0.8   -0.6
$
GRDSET                                3456
$
$ EIGENSOLVER DEFINITION
EIGR    2      SUB                2      MAX
$
ENDDATA
```

---

## 2.1.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A001  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
THREE BAR TRUSS

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 4
NUMBER OF CROD ELEMENTS: 3
NUMBER OF CONM2 ELEMENTS: 1
```

|                                     |   |
|-------------------------------------|---|
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 4 |
| NUMBER OF ELEMENT PROPERTIES:       | 3 |
| NUMBER OF MATERIALS:                | 1 |
| NUMBER OF DEGREES OF FREEDOM:       | 2 |

#### LOAD CASES SUMMARY

|                                      |   |
|--------------------------------------|---|
| NUMBER OF STATIC LOAD CASES:         | 1 |
| NUMBER OF USER FREQUENCY LOAD CASES: | 1 |
| TOTAL NUMBER OF LOAD CASES:          | 2 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
THREE BAR TRUSS

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS)\*  
\*\*\*\*\*

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THREE BAR TRUSS

#### M A S S / V O L U M E S U M M A R Y

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 2.602506E+00 |
| SYSTEM VOLUME      | 4.828427E+01 |
| SYSTEM MASS/VOLUME | 5.389966E-02 |
| WTMASS PARAMETER   | 2.590000E-03 |
| SYSTEM MASS/WTMASS | 1.004828E+03 |

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THREE BAR TRUSS

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 2.759199E-16 ; STRAIN ENERGY : 2.076160E+02  
1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
THREE BAR TRUSS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2          | T3         | R1         | R2         | R3         |
|----------|------------|-------------|------------|------------|------------|------------|
| 1        | 1.6000E+04 | -1.2000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 1.6000E+05 |

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THREE BAR TRUSS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

1 -1.6000E+04 1.2000E+04 0.0000E+00 0.0000E+00 0.0000E+00 -1.6000E+05  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 THREE BAR TRUSS

#### MAXIMUM APPLIED FORCE

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3         |
|----------|---------|------------|------------|-------------|------------|
| 1        | 4       | 2.0000E+04 | 1.6000E+04 | -1.2000E+04 | 0.0000E+00 |

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 THREE BAR TRUSS

#### MAXIMUM SPC FORCE

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3         |
|----------|---------|------------|-------------|------------|------------|
| 1        | 1       | 1.3530E+04 | -9.5672E+03 | 9.5672E+03 | 0.0000E+00 |

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 THREE BAR TRUSS

#### MAXIMUM DISPLACEMENT

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3         |
|----------|---------|------------|------------|-------------|------------|
| 1        | 4       | 2.3058E-02 | 2.2627E-02 | -4.4328E-03 | 0.0000E+00 |

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 THREE BAR TRUSS

LOAD CONDITION 1 LOADCASE 1

#### LOAD VECTOR

| ID | F1<br>M1                     | F2<br>M2                      | F3<br>M3                     |
|----|------------------------------|-------------------------------|------------------------------|
| 4  | 1.600000E+04<br>0.000000E+00 | -1.200000E+04<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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 THREE BAR TRUSS

LOAD CONDITION 1 LOADCASE 1

#### GRID DISPLACEMENTS

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

|   |                              |                               |                              |
|---|------------------------------|-------------------------------|------------------------------|
| 2 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 3 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 4 | 2.262742E-02<br>0.000000E+00 | -4.432777E-03<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 12  
THREE BAR TRUSS

LOAD CONDITION 1 LOADCASE 1

### REACTION FORCES

| ID | F1<br>M1                      | F2<br>M2                      | F3<br>M3                     |
|----|-------------------------------|-------------------------------|------------------------------|
| 1  | -9.567223E+03<br>0.000000E+00 | 9.567223E+03<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 2  | 0.000000E+00<br>0.000000E+00  | 8.865554E+03<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 3  | -6.432777E+03<br>0.000000E+00 | -6.432777E+03<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 13  
THREE BAR TRUSS

LOAD CONDITION 1 LOADCASE 1

### FORCES IN ROD ELEMENTS

| ROD ID | FORCE-A       | FORCE-B       |
|--------|---------------|---------------|
| 1      | 1.353010E+04  | 1.353010E+04  |
| 2      | 8.865554E+03  | 8.865554E+03  |
| 3      | -9.097320E+03 | -9.097320E+03 |

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THREE BAR TRUSS

LOAD CONDITION 1 LOADCASE 1

### STRESSES IN ROD ELEMENTS

| ROD ID | STRESS-A      | STRESS-B      |
|--------|---------------|---------------|
| 1      | 1.353010E+04  | 1.353010E+04  |
| 2      | 4.432777E+03  | 4.432777E+03  |
| 3      | -9.097320E+03 | -9.097320E+03 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 15  
THREE BAR TRUSS

## E I G E N V A L U E S

|                      |              |                                    |              | GENERALIZED  | GENERALIZED  |
|----------------------|--------------|------------------------------------|--------------|--------------|--------------|
| MODE                 | CYCLES       | EIGENVALUE                         | RADIANS      | MASS         | STIFFNESS    |
| 1                    | 8.309283E+01 | 2.725755E+05                       | 5.220876E+02 | 2.594169E+00 | 7.071068E+05 |
| 2                    | 1.625825E+02 | 1.043535E+06                       | 1.021536E+03 | 2.594169E+00 | 2.707107E+06 |
| 1GENESIS VERSION 7.2 |              | DATE 12-13-2002 TIME 12:49 PAGE 16 |              |              |              |
| THREE BAR TRUSS      |              |                                    |              |              |              |

## E I G E N V E C T O R

EIGENVECTOR NUMBER 1 FREQUENCY (HZ) = 8.309283E+01  
EIGENVALUE = 2.725755E+05

| GRID ID | T1<br>R1                     | T2<br>R2                      | T3<br>R3                     |
|---------|------------------------------|-------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 2       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 4       | 1.000000E+00<br>0.000000E+00 | -2.250613E-18<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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THREE BAR TRUSS

## E I G E N V E C T O R

EIGENVECTOR NUMBER 2 FREQUENCY (HZ) = 1.625825E+02  
EIGENVALUE = 1.043535E+06

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 4       | 2.250613E-18                 | 1.000000E+00                 | 0.000000E+00                 |



0.000000E+00      0.000000E+00      0.000000E+00

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.2 Ten Rod Truss

### Example ID

A002

### Analysis Data Used

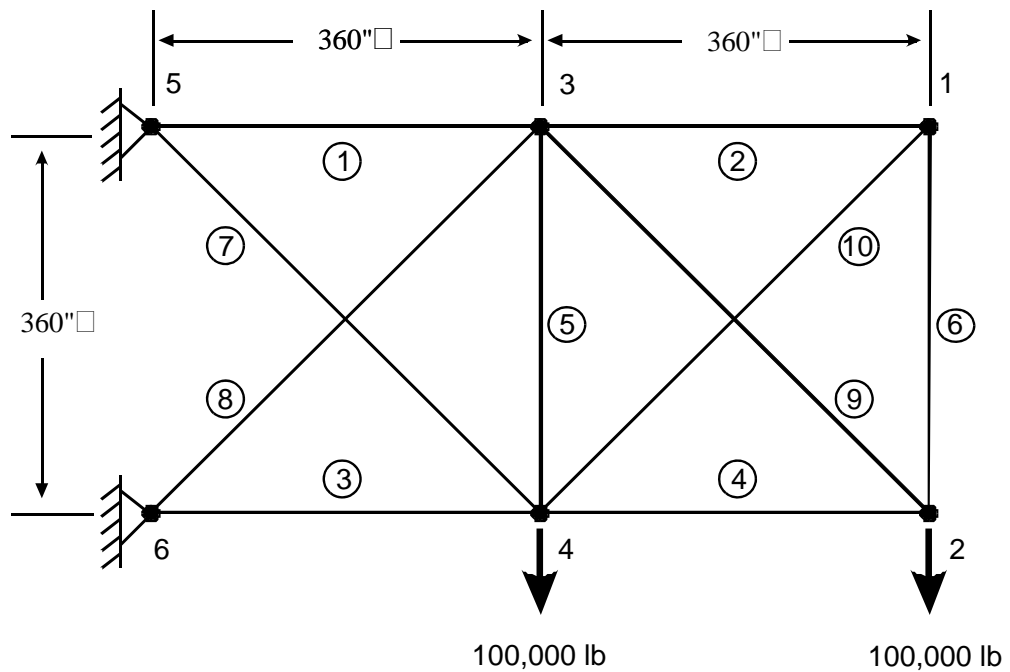
GRID, GRDSET, CROD, PROD, MAT1,  
LOADCASE, LOAD, FORCE, DISP, FORCE, STRESS

### Special Features Used

NONE

### Problem Statement

Find the nodal displacements, element forces and the stresses of the ten rod truss (see **Figure 2-2**).

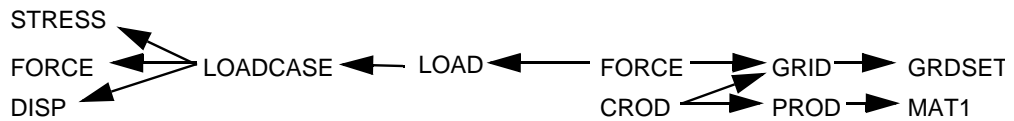


**Figure 2-2**

## Structural Analysis Model

1. Planar truss structure modeled with 10 CROD elements.
2. Section properties: Area=5.0in<sup>2</sup>
3. Material: E=1.0E7 psi.
4. One load case: Two 100,000 lb loads applied simultaneously to GRIDS 2 and 4.

## Analysis Data Relationships



## Special Modeling Techniques

NONE

## Reference Solutions

NONE

---

## 2.2.1 Input Data

```
ID AAA
SOL COMPAT1
CEND
TITLE=10 ROD TRUSS
SUBTITLE=STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS
ECHO = NONE
LINE = *,80
LOADCASE      1
  LABEL=
  LOAD   =      1
  DISP   =ALL
  FORCE   =ALL
  STRESS =ALL
$
BEGIN BULK
$
$   GRID POINTS
$
GRDSET                                0   3456
GRID      1      720.000 360.000   .000
GRID      2      720.000   .000   .000
GRID      3      360.000 360.000   .000
GRID      4      360.000   .000   .000
GRID      5          .000 360.000   .000   123456
GRID      6          .000   .000   .000   123456
$
$   ELEMENT DEFINITIONS
$
CROD      1      1      5      3
CROD      2      1      3      1
CROD      3      1      6      4
CROD      4      1      4      2
CROD      5      1      3      4
CROD      6      1      1      2
CROD      7      1      5      4
CROD      8      1      6      3
CROD      9      1      3      2
CROD     10      1      4      1
$
$   MATERIAL DEFINITIONS
$
MAT1     11      1.0+7      0.3
$
$   PROPERTY DEFINITIONS
$
PROD      1      11.500E+01      .000E+00      P1
$
$   LOAD DEFINITIONS
$
FORCE      1      2      0  1.0      .0 -100000.      .0
FORCE      1      4      0  1.0      .0 -100000.      .0
ENDDATA
```



---

## 2.2.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A002  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
10 ROD TRUSS  
STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS

### ANALYSIS PROBLEM SUMMARY

|                                     |    |
|-------------------------------------|----|
| NUMBER OF GRID POINTS:              | 6  |
| NUMBER OF CROD ELEMENTS:            | 10 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 10 |

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 8

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 10 ROD TRUSS  
 STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 10 ROD TRUSS  
 STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 2.098234E+04  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 10 ROD TRUSS  
 STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -1.844394E-15 ; STRAIN ENERGY : 5.741690E+05  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 10 ROD TRUSS  
 STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

|   |            |             |            |            |            |             |
|---|------------|-------------|------------|------------|------------|-------------|
| 1 | 0.0000E+00 | -2.0000E+05 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -1.0800E+08 |
|---|------------|-------------|------------|------------|------------|-------------|

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 10 ROD TRUSS  
 STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

|   |            |            |            |            |            |            |
|---|------------|------------|------------|------------|------------|------------|
| 1 | 0.0000E+00 | 2.0000E+05 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 1.0800E+08 |
|---|------------|------------|------------|------------|------------|------------|

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 10 ROD TRUSS  
 STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE  | GRID ID     | MAGNITUDE  | T1         | T2              | T3         |        |
|---|-------------|------------|------------|-----------------|------------|--------|
| 1   | 2           | 1.0000E+05 | 0.0000E+00 | -1.0000E+05     | 0.0000E+00 |        |
| 1GENESIS  | VERSION 7.2 |            |            | DATE 12-13-2002 | TIME 12:49 | PAGE 8 |
| 10 ROD TRUSS                                    |             |            |            |                 |            |        |
| STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS |             |            |            |                 |            |        |

# M A X I M U M   S P C   F O R C E

| LOADCASE  | GRID ID     | MAGNITUDE  | T1          | T2              | T3         |        |
|---|-------------|------------|-------------|-----------------|------------|--------|
| 1   | 5           | 3.1772E+05 | -3.0000E+05 | 1.0464E+05      | 0.0000E+00 |        |
| 1GENESIS  | VERSION 7.2 |            |             | DATE 12-13-2002 | TIME 12:49 | PAGE 9 |
| 10 ROD TRUSS                                    |             |            |             |                 |            |        |
| STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS |             |            |             |                 |            |        |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE  | GRID ID     | MAGNITUDE  | T1          | T2              | T3         |         |
|---|-------------|------------|-------------|-----------------|------------|---------|
| 1   | 2           | 8.1060E+00 | -1.9045E+00 | -7.8791E+00     | 0.0000E+00 |         |
| 1GENESIS  | VERSION 7.2 |            |             | DATE 12-13-2002 | TIME 12:49 | PAGE 10 |
| 10 ROD TRUSS                                    |             |            |             |                 |            |         |
| STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS |             |            |             |                 |            |         |

LOADCASE 1

# G R I D   D I S P L A C E M E N T S

| GRID ID   | T1<br>R1                      | T2<br>R2                      | T3<br>R3                     |                    |
|---|-------------------------------|-------------------------------|------------------------------|--------------------|
| 1   | 1.695525E+00<br>0.000000E+00  | -7.590253E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |                    |
| 2   | -1.904475E+00<br>0.000000E+00 | -7.879150E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |                    |
| 3   | 1.406628E+00<br>0.000000E+00  | -3.348705E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |                    |
| 4   | -1.473372E+00<br>0.000000E+00 | -3.604230E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |                    |
| 5   | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |                    |
| 6   | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |                    |
| 1GENESIS  | VERSION 7.2                   |                               | DATE 12-13-2002              | TIME 12:49 PAGE 11 |
| 10 ROD TRUSS                                    |                               |                               |                              |                    |
| STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS |                               |                               |                              |                    |



## F O R C E S I N R O D E L E M E N T S

| ROD ID | FORCE-A       | FORCE-B       |
|--------|---------------|---------------|
| 1      | 1.953650E+05  | 1.953650E+05  |
| 2      | 4.012463E+04  | 4.012463E+04  |
| 3      | -2.046350E+05 | -2.046350E+05 |
| 4      | -5.987537E+04 | -5.987537E+04 |
| 5      | 3.548962E+04  | 3.548962E+04  |
| 6      | 4.012463E+04  | 4.012463E+04  |
| 7      | 1.479763E+05  | 1.479763E+05  |
| 8      | -1.348665E+05 | -1.348665E+05 |
| 9      | 8.467656E+04  | 8.467656E+04  |
| 10     | -5.674480E+04 | -5.674480E+04 |

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10 ROD TRUSS

STRESSES, FORCES AND DISPLACEMENTS CALCULATIONS

## S T R E S S E S I N R O D E L E M E N T S

| ROD ID | STRESS-A      | STRESS-B      |
|--------|---------------|---------------|
| 1      | 3.907300E+04  | 3.907300E+04  |
| 2      | 8.024926E+03  | 8.024926E+03  |
| 3      | -4.092700E+04 | -4.092700E+04 |
| 4      | -1.197507E+04 | -1.197507E+04 |
| 5      | 7.097924E+03  | 7.097924E+03  |
| 6      | 8.024926E+03  | 8.024926E+03  |
| 7      | 2.959525E+04  | 2.959525E+04  |
| 8      | -2.697329E+04 | -2.697329E+04 |
| 9      | 1.693531E+04  | 1.693531E+04  |
| 10     | -1.134896E+04 | -1.134896E+04 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.3 Portal Frame

### Example ID

A003

### Analysis Data Used

GRID, GRDSET, CBAR, PBAR, MAT1, LOADCASE, LOAD, FORCE, MOMENT, SPC, SPC1, DISP, FORCE, STRESS.

### Special Features Used

NONE

### Problem Statement

Find the nodal displacements, element forces and stresses of the portal frame for each of the load cases indicated in **Figure 2-3**.

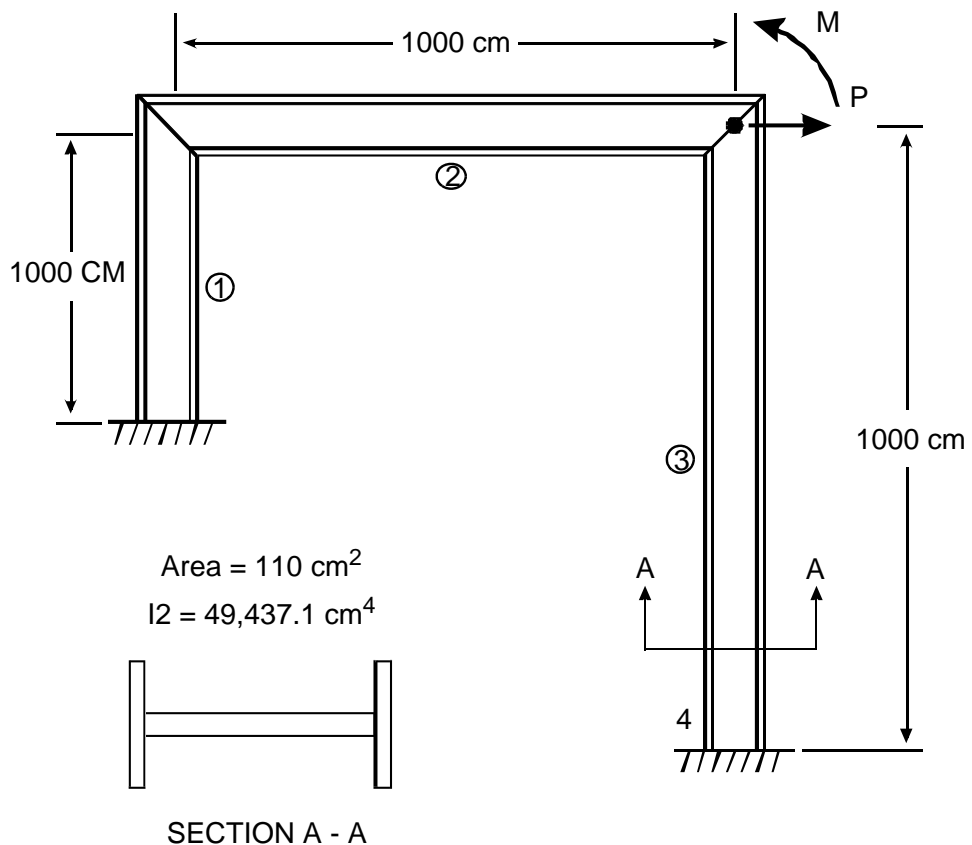
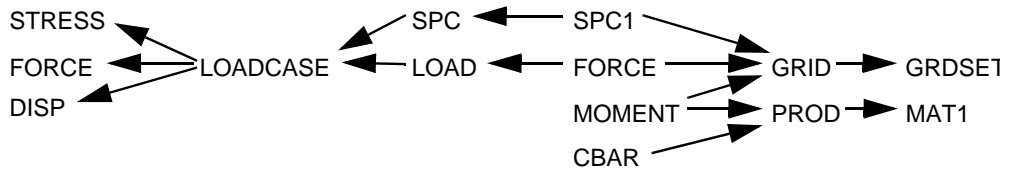


Figure 2-3

## Structural Analysis Model

1. Planar three bar frame assembled with 3 CBAR elements.
2. Section properties of I beams: Area=110.0cm<sup>2</sup>, I<sub>2</sub>=49437.1 cm<sup>4</sup>.
3. Material: Aluminum Alloy, E=7.06E+6 N/cm<sup>2</sup>,  $\nu$  = 0.33.
4. Two Load cases:
  1. P=50,000.0 N.
  2. M=20E+6 N-cm.

## Analysis Data Relationships



## Special Modeling Techniques

NONE

## Reference Solutions

NONE

---

## 2.3.1 Input Data

```
ID A003
SOL COMPAT1
CEND
TITLE = PORTAL FRAME
ECHO=NONE
LINE=64,80
DISP = ALL
STRESS = ALL
FORCE = ALL
SPC = 100
SUBCASE 1
    LABEL = A FORCE AT NODE 3
    LOAD = 300
SUBCASE 2
    LABEL = A MOMENT AT NODE 3
    LOAD = 310

$
BEGIN BULK
$
$    BOUNDARY CONDITIONS
$
SPC1    100    123456  1    4
$
$    GRID POINTS
$
GRDSET                                     345
GRID    1            0.0    500.    0.0
GRID    2            0.0    1000.    0.0
GRID    3            1000.    1000.    0.0
GRID    4            1000.    0.0    0.0
$
$    ELEMENT DEFINITIONS
$
CBAR    1            1            1            2            0.0    0.0    -1.0
CBAR    2            1            2            3            0.0    0.0    -1.0
CBAR    3            1            3            4            0.0    0.0    -1.0
$
$    PROPERTIY DEFINITIONS
$
PBAR    1            51            110.0    4504.0    49437.1
+P1      15.0    26.0    0.0    26.0    0.0    0.0    0.00    -26.0
$
$    MATERIAL DEFINITIONS
$
MAT1    51            7.06E6            0.33    0.1
$
$    LOAD DEFINITIONS
$
FORCE    300    3            50000.0    1.0    0.0    0.0
MOMENT    310    3            20.E6    0.0    0.0    -1.0
$
ENDDATA
```



---

## 2.3.2 Output

1

```
  GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N  E      S      I      S
G  GG  EEEE   N N N  EEEE   SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A003  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
PORTAL FRAME

### ANALYSIS PROBLEM SUMMARY

|                                     |   |
|-------------------------------------|---|
| NUMBER OF GRID POINTS:              | 4 |
| NUMBER OF CBAR ELEMENTS:            | 3 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 3 |

|                               |   |
|-------------------------------|---|
| NUMBER OF ELEMENT PROPERTIES: | 1 |
| NUMBER OF MATERIALS:          | 1 |
| NUMBER OF DEGREES OF FREEDOM: | 6 |

#### LOAD CASES SUMMARY

|                              |   |
|------------------------------|---|
| NUMBER OF STATIC LOAD CASES: | 2 |
| TOTAL NUMBER OF LOAD CASES:  | 2 |

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PORTAL FRAME

\*\*\*\*\*  
\* DESIGN CYCLE 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
PORTAL FRAME

#### MASS / VOLUME SUMMARY

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 2.750000E+04 |
| SYSTEM VOLUME      | 2.750000E+05 |
| SYSTEM MASS/VOLUME | 1.000000E-01 |

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PORTAL FRAME

#### SOLUTION RESIDUALS

|          |   |                          |                              |
|----------|---|--------------------------|------------------------------|
| LOADCASE | 1 | RESIDUAL : -2.089031E-14 | STRAIN ENERGY : 6.142640E+04 |
| LOADCASE | 2 | RESIDUAL : -2.171412E-16 | STRAIN ENERGY : 7.573223E+04 |

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PORTAL FRAME

#### LOAD RESULTANTS

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3          |
|----------|------------|------------|------------|------------|------------|-------------|
| 1        | 5.0000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -5.0000E+07 |
| 2        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -2.0000E+07 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
PORTAL FRAME

#### SPCFORCE RESULTANTS

| LOADCASE | T1          | T2         | T3         | R1         | R2         | R3         |
|----------|-------------|------------|------------|------------|------------|------------|
| 1        | -5.0000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 5.0000E+07 |
| 2        | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 2.0000E+07 |

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PORTAL FRAME

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        | 3       | 5.0000E+04 | 5.0000E+04 | 0.0000E+00 | 0.0000E+00 |
| 2        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 8  
PORTAL FRAME

# M A X I M U M   S P C   F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2          | T3         |
|----------|---------|------------|-------------|-------------|------------|
| 1        | 1       | 4.2634E+04 | -4.1139E+04 | -1.1193E+04 | 0.0000E+00 |
| 2        | 4       | 2.0446E+04 | 1.4716E+04  | 1.4195E+04  | 0.0000E+00 |

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PORTAL FRAME

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3         |
|----------|---------|------------|------------|-------------|------------|
| 1        | 3       | 2.4571E+00 | 2.4571E+00 | -1.4412E-02 | 0.0000E+00 |
| 2        | 3       | 2.7369E-01 | 2.7308E-01 | -1.8278E-02 | 0.0000E+00 |

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PORTAL FRAME

A FORCE AT NODE 3 LOADCASE 1

# G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                      | T3<br>R3                      |
|---------|------------------------------|-------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |
| 2       | 2.404083E+00<br>0.000000E+00 | 7.206126E-03<br>0.000000E+00  | 0.000000E+00<br>-4.705191E-03 |
| 3       | 2.457056E+00<br>0.000000E+00 | -1.441225E-02<br>0.000000E+00 | 0.000000E+00<br>-6.827082E-04 |
| 4       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |

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PORTAL FRAME



A MOMENT AT NODE 3

LOADCASE

2

## GRID DISPLACEMENTS

| GRID ID | T1<br>R1                     | T2<br>R2                      | T3<br>R3                      |
|---------|------------------------------|-------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |
| 2       | 2.541343E-01<br>0.000000E+00 | 9.138893E-03<br>0.000000E+00  | 0.000000E+00<br>7.402269E-04  |
| 3       | 2.730833E-01<br>0.000000E+00 | -1.827779E-02<br>0.000000E+00 | 0.000000E+00<br>-7.573223E-03 |
| 4       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |

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PORTAL FRAME

A FORCE AT NODE 3

LOADCASE

1

## FORCES IN BAR ELEMENTS

| BAR ID | END | BENDING MOMENT 1<br>SHEAR FORCE 1 | BENDING MOMENT 2<br>SHEAR FORCE 2 | AXIAL FORCE<br>TORQUE         |
|--------|-----|-----------------------------------|-----------------------------------|-------------------------------|
| 1      | A   | 0.000000E+00<br>0.000000E+00      | -1.356916E+07<br>-4.113878E+04    | 1.119255E+04<br>0.000000E+00  |
|        | B   | 0.000000E+00<br>0.000000E+00      | 7.000228E+06<br>-4.113878E+04     | 1.119255E+04<br>0.000000E+00  |
| 2      | A   | 0.000000E+00<br>0.000000E+00      | 7.000228E+06<br>1.119255E+04      | 4.113878E+04<br>0.000000E+00  |
|        | B   | 0.000000E+00<br>0.000000E+00      | -4.192326E+06<br>1.119255E+04     | 4.113878E+04<br>0.000000E+00  |
| 3      | A   | 0.000000E+00<br>0.000000E+00      | -4.192326E+06<br>-8.861219E+03    | -1.119255E+04<br>0.000000E+00 |
|        | B   | 0.000000E+00<br>0.000000E+00      | 4.668892E+06<br>-8.861219E+03     | -1.119255E+04<br>0.000000E+00 |

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PORTAL FRAME

A MOMENT AT NODE 3

LOADCASE

2

## FORCES IN BAR ELEMENTS

| BAR ID | END | BENDING MOMENT 1 | BENDING MOMENT 2 | AXIAL FORCE |
|--------|-----|------------------|------------------|-------------|
|--------|-----|------------------|------------------|-------------|

|   |   | SHEAR FORCE 1 | SHEAR FORCE 2 | TORQUE        |
|---|---|---------------|---------------|---------------|
| 1 | A | 0.000000E+00  | -3.162221E+06 | 1.419453E+04  |
|   |   | 0.000000E+00  | -1.471575E+04 | 0.000000E+00  |
|   | B | 0.000000E+00  | 4.195654E+06  | 1.419453E+04  |
|   |   | 0.000000E+00  | -1.471575E+04 | 0.000000E+00  |
| 2 | A | 0.000000E+00  | 4.195654E+06  | 1.471575E+04  |
|   |   | 0.000000E+00  | 1.419453E+04  | 0.000000E+00  |
|   | B | 0.000000E+00  | -9.998874E+06 | 1.471575E+04  |
|   |   | 0.000000E+00  | 1.419453E+04  | 0.000000E+00  |
| 3 | A | 0.000000E+00  | 1.000113E+07  | -1.419453E+04 |
|   |   | 0.000000E+00  | 1.471575E+04  | 0.000000E+00  |
|   | B | 0.000000E+00  | -4.714624E+06 | -1.419453E+04 |
|   |   | 0.000000E+00  | 1.471575E+04  | 0.000000E+00  |

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PORTAL FRAME

A FORCE AT NODE 3 LOADCASE 1

| S T R E S S E S I N B A R E L E M E N T S |     |               |               |               |               |
|---|-----|---------------|---------------|---------------|---------------|
| BAR ID                                    | END | STRESS-C      | STRESS-D      | STRESS-E      | STRESS-F      |
| 1   | A   | 7.238056E+03  | 7.238056E+03  | 1.017505E+02  | -7.034555E+03 |
|   | B   | -3.579815E+03 | -3.579815E+03 | 1.017505E+02  | 3.783316E+03  |
| 2   | A   | -3.307577E+03 | -3.307577E+03 | 3.739889E+02  | 4.055555E+03  |
|   | B   | 2.578821E+03  | 2.578821E+03  | 3.739889E+02  | -1.830843E+03 |
| 3   | A   | 2.103081E+03  | 2.103081E+03  | -1.017505E+02 | -2.306582E+03 |
|   | B   | -2.557218E+03 | -2.557218E+03 | -1.017505E+02 | 2.353717E+03  |

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PORTAL FRAME

A MOMENT AT NODE 3 LOADCASE 2

| S T R E S S E S I N B A R E L E M E N T S |     |               |               |               |               |
|---|-----|---------------|---------------|---------------|---------------|
| BAR ID                                    | END | STRESS-C      | STRESS-D      | STRESS-E      | STRESS-F      |
| 1   | A   | 1.792119E+03  | 1.792119E+03  | 1.290412E+02  | -1.534037E+03 |
|   | B   | -2.077541E+03 | -2.077541E+03 | 1.290412E+02  | 2.335623E+03  |
| 2   | A   | -2.072802E+03 | -2.072802E+03 | 1.337795E+02  | 2.340361E+03  |
|   | B   | 5.392395E+03  | 5.392395E+03  | 1.337795E+02  | -5.124836E+03 |
| 3   | A   | -5.388842E+03 | -5.388842E+03 | -1.290412E+02 | 5.130759E+03  |
|   | B   | 2.350478E+03  | 2.350478E+03  | -1.290412E+02 | -2.608560E+03 |

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.4 Circular Ring

### Example ID

A0046

### Analysis Data Used

GRID, CORD2C, CBAR, PBAR, MAT1, LOADCASE, PLOAD1, DISP

### Special Features Used

Utilization of cylindrical coordinate system for input grid coordinates and for the displacement output.

### Problem Statement

Find the radial and rotational displacements of a round, solid ring of radius  $R=25.0$  in. and circular cross section of  $D=4.0$  in. The ring is subjected to two separate load cases. The first load case corresponds to normal pressure while the second load case corresponds to constant torsion. Combinations of this two load cases are found in rings that are use to stiffen spherical domes.

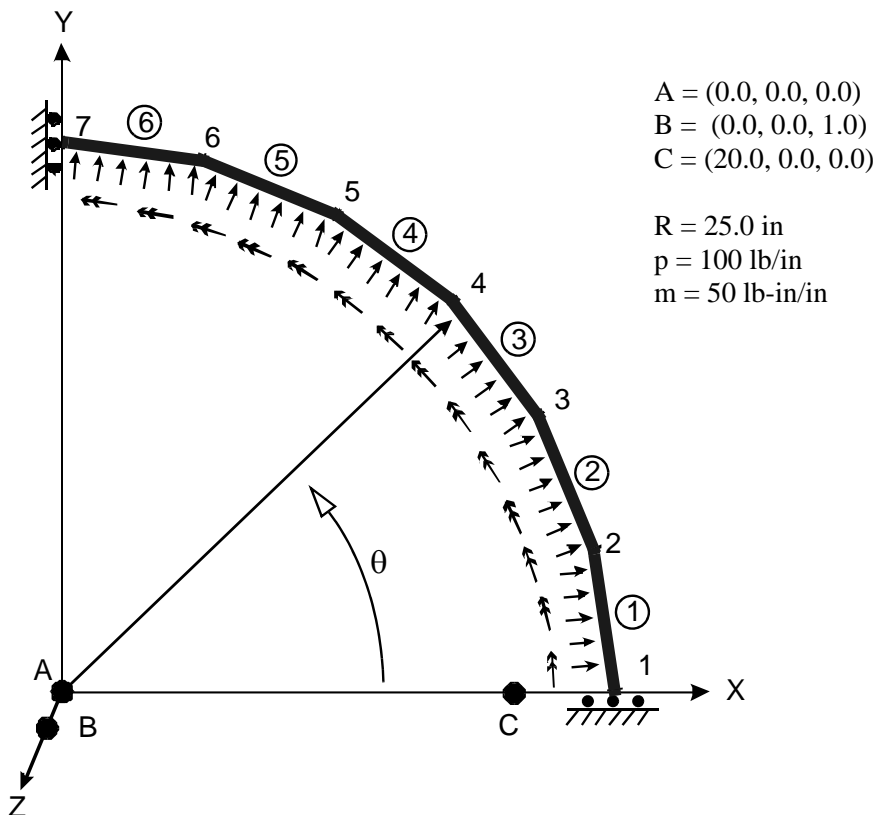
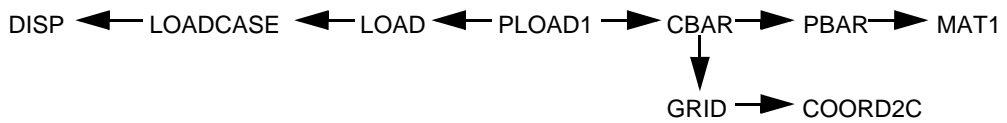


Figure 2-4

## Structural Analysis Model

1. Due to symmetry a quarter model with 6 round CBAR elements is used.
2. The section properties are: Area=12.57 m<sup>2</sup>, I=12.57 m<sup>4</sup>, J=25.14 m<sup>4</sup> and AS=11.16 m<sup>2</sup>
3. Material: E=1.0E7 psi.
4. Two load cases:
  1. Normal pressure p=100.0 lb/in.
  2. Constant torsion m= 50.0 lb-in/in.

## Analysis Data Relationships



## Special Modeling Techniques

NONE

## Reference Solutions

The analytical solutions assuming that inner radius is equal to the external radius are:

$$u_r = \frac{R^2 p}{EA} = 0.000497 \text{ in}$$

$$\theta = \frac{R^2 m}{EI} = 0.000246 \text{ rad}$$

where R is the radius, E the Young's modulus, A the cross sectional area, I the area moment of inertia, p is the normal pressure and m is the torsion per unit length.

## Calculated Solutions

$$u_r = 0.000493$$

$$\theta = 0.000246 \text{ rad}$$

## Comparison Between Calculated Solutions and Reference Solutions

The differences between analytical solutions and *GENESIS* solutions are:

$$u_r = -0.8\%$$

$$\theta = -0.8\%$$

---

## 2.4.1 Input Data

```
ID A004
SOL COMPAT1
CEND
TITLE= CIRCULAR RING
ECHO=NONE
LINE=64,80
LOADCASE 1
    LOAD=12
    DISP=ALL
LOADCASE 2
    LOAD=22
    DISP=ALL
BEGIN BULK
$
$   CYLINDRICAL COORDINATE SYSTEM
$
CORD2C  7          0.0    0.0    0.0    0.0    0.0    1.0
+      20.0    0.0    0.0
$
$   GRID DATA
$
GRID    1      7      25.0    00.0    0.0    7      246
GRID    2      7      25.0    15.0    0.0    7
GRID    3      7      25.0    30.0    0.0    7
GRID    4      7      25.0    45.0    0.0    7      3
GRID    5      7      25.0    60.0    0.0    7
GRID    6      7      25.0    75.0    0.0    7
GRID    7      7      25.0    90.0    0.0    7      246
$
$   ELEMENT DEFINITIONS
$
CBAR    1      1      1      2      -1.0    0.0    0.
CBAR    2      1      2      3      -1.0    0.0    0.
CBAR    3      1      3      4      -1.0    0.0    0.
CBAR    4      1      4      5      -1.0    0.0    0.
CBAR    5      1      5      6      -1.0    0.0    0.
CBAR    6      1      6      7      -1.0    0.0    0.
$
$   PROPERTY DATA: CIRCULAR BEAM,  D = 4.0 in.
$
PBAR    1      1      12.57    12.57    12.57    25.13
+      0.0      2.0      2.0      0.0      -2.0      0.0      0.0      -2.0
+      10.888    0.888    0.0
$
$   MATERIAL DEFINITION
$
MAT1    1      10.0+6      0.3      0.0      0.0      25.0
$
$   LOADS: NORMAL PRESSURE LOAD.
$
PLOAD1  12      1      FYE      -100.0      -100.0
PLOAD1  12      2      FYE      -100.0      -100.0
```

|  |    |   |     |        |        |
|--|----|---|-----|--------|--------|
| PLOAD1                                 | 12 | 3 | FYE | -100.0 | -100.0 |
| PLOAD1                                 | 12 | 4 | FYE | -100.0 | -100.0 |
| PLOAD1                                 | 12 | 5 | FYE | -100.0 | -100.0 |
| PLOAD1                                 | 12 | 6 | FYE | -100.0 | -100.0 |
| \$                                     |    |   |     |        |        |
| \$ LOADS: DISTRIBUTED TORCIONAL MOMENT |    |   |     |        |        |
| \$                                     |    |   |     |        |        |
| PLOAD1                                 | 22 | 1 | MXE | 50.0   | 50.0   |
| PLOAD1                                 | 22 | 2 | MXE | 50.0   | 50.0   |
| PLOAD1                                 | 22 | 3 | MXE | 50.0   | 50.0   |
| PLOAD1                                 | 22 | 4 | MXE | 50.0   | 50.0   |
| PLOAD1                                 | 22 | 5 | MXE | 50.0   | 50.0   |
| PLOAD1                                 | 22 | 6 | MXE | 50.0   | 50.0   |
| \$                                     |    |   |     |        |        |
| ENDDATA                                |    |   |     |        |        |



---

## 2.4.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A004

RUN STARTED: Dec 13, 2002 12:49

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

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DATE 12-13-2002 TIME 12:49 PAGE 1

CIRCULAR RING

### ANALYSIS PROBLEM SUMMARY

|                                     |   |
|-------------------------------------|---|
| NUMBER OF GRID POINTS:              | 7 |
| NUMBER OF LOCAL COORDINATE SYSTEMS: | 1 |
| NUMBER OF CBAR ELEMENTS:            | 6 |

TOTAL NUMBER OF NON RIGID ELEMENTS: 6  
 NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 35

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 2  
 TOTAL NUMBER OF LOAD CASES: 2

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 CIRCULAR RING

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

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 CIRCULAR RING

#### MASS / VOLUME SUMMARY

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 4.922143E+02  
 SYSTEM MASS/VOLUME 0.000000E+00

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 CIRCULAR RING

#### SOLUTION RESIDUALS

LOADCASE 1 ; RESIDUAL : -1.581308E-15 ; STRAIN ENERGY : 9.569094E-01  
 LOADCASE 2 ; RESIDUAL : 2.049844E-16 ; STRAIN ENERGY : 2.392273E-01  
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 CIRCULAR RING

#### LOAD RESULTANTS

| LOADCASE | T1         | T2         | T3         | R1          | R2         | R3         |
|----------|------------|------------|------------|-------------|------------|------------|
| 1        | 2.5000E+03 | 2.5000E+03 | 0.0000E+00 | 0.0000E+00  | 0.0000E+00 | 2.7285E-12 |
| 2        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -1.2500E+03 | 1.2500E+03 | 0.0000E+00 |

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 CIRCULAR RING

#### SPCFORCE RESULTANTS

| LOADCASE | T1          | T2          | T3         | R1         | R2         | R3          |
|----------|-------------|-------------|------------|------------|------------|-------------|
| 1        | -2.5000E+03 | -2.5000E+03 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -2.9104E-11 |

2 0.0000E+00 0.0000E+00 0.0000E+00 1.2500E+03 -1.2500E+03 0.0000E+00  
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CIRCULAR RING

#### MAXIMUM APPLIED FORCE

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        | 2       | 6.4705E+02 | 6.4705E+02 | 4.2633E-14 | 0.0000E+00 |
| 2        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

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CIRCULAR RING

#### MAXIMUM SPC FORCE

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3          |
|----------|---------|------------|------------|-------------|-------------|
| 1        | 1       | 2.5000E+03 | 2.2737E-13 | -2.5000E+03 | 0.0000E+00  |
| 2        | 4       | 5.6843E-14 | 0.0000E+00 | 0.0000E+00  | -5.6843E-14 |

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CIRCULAR RING

#### MAXIMUM DISPLACEMENT

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3          |
|----------|---------|------------|------------|------------|-------------|
| 1        | 7       | 4.9296E-04 | 4.9296E-04 | 0.0000E+00 | 0.0000E+00  |
| 2        | 1       | 2.5479E-18 | 0.0000E+00 | 0.0000E+00 | -2.5479E-18 |

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CIRCULAR RING

LOADCASE 1

#### GRID DISPLACEMENTS

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                      |
|---------|------------------------------|------------------------------|-------------------------------|
| 1       | 4.929618E-04<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  |
| 2       | 4.929618E-04<br>0.000000E+00 | 1.057097E-18<br>0.000000E+00 | 0.000000E+00<br>-2.303930E-19 |
| 3       | 4.929618E-04<br>0.000000E+00 | 1.680513E-18<br>0.000000E+00 | 0.000000E+00<br>-2.944887E-19 |
| 4       | 4.929618E-04<br>0.000000E+00 | 1.783794E-18<br>0.000000E+00 | 0.000000E+00<br>-1.848631E-19 |
| 5       | 4.929618E-04<br>0.000000E+00 | 1.460556E-18<br>0.000000E+00 | 0.000000E+00<br>-1.151965E-19 |

|   |                              |                              |                              |
|---|------------------------------|------------------------------|------------------------------|
| 6 | 4.929618E-04<br>0.000000E+00 | 8.233160E-19<br>0.000000E+00 | 0.000000E+00<br>2.232235E-23 |
| 7 | 4.929618E-04<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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CIRCULAR RING

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LOADCASE 2

# G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                      |
|---------|------------------------------|------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.464809E-04 | -2.547875E-18<br>0.000000E+00 |
| 2       | 0.000000E+00<br>1.236668E-19 | 0.000000E+00<br>2.464809E-04 | -1.882099E-18<br>0.000000E+00 |
| 3       | 0.000000E+00<br>1.846532E-19 | 0.000000E+00<br>2.464809E-04 | -8.030259E-19<br>0.000000E+00 |
| 4       | 0.000000E+00<br>1.777602E-19 | 0.000000E+00<br>2.464809E-04 | 0.000000E+00<br>0.000000E+00  |
| 5       | 0.000000E+00<br>1.389134E-19 | 0.000000E+00<br>2.464809E-04 | 1.192622E-18<br>0.000000E+00  |
| 6       | 0.000000E+00<br>5.844527E-20 | 0.000000E+00<br>2.464809E-04 | 2.250570E-18<br>0.000000E+00  |
| 7       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.464809E-04 | 2.263272E-18<br>0.000000E+00  |

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.5 Wide Column/Wide Beam Frame

### Example ID

A005

### Analysis Data Used

GRID, GRDSET, CBAR, PBAR, MAT1, LOAD, FORCE, SPC, SPC1, DISP, FORCE.

### Special Features Used

Utilization of offset capabilities of bar elements to model rigid connections of wide columns with wide beams.

### Problem Statement

Find the nodal displacements and the element forces for the wide column/wide beam frame shown in **Figure 2-5**.

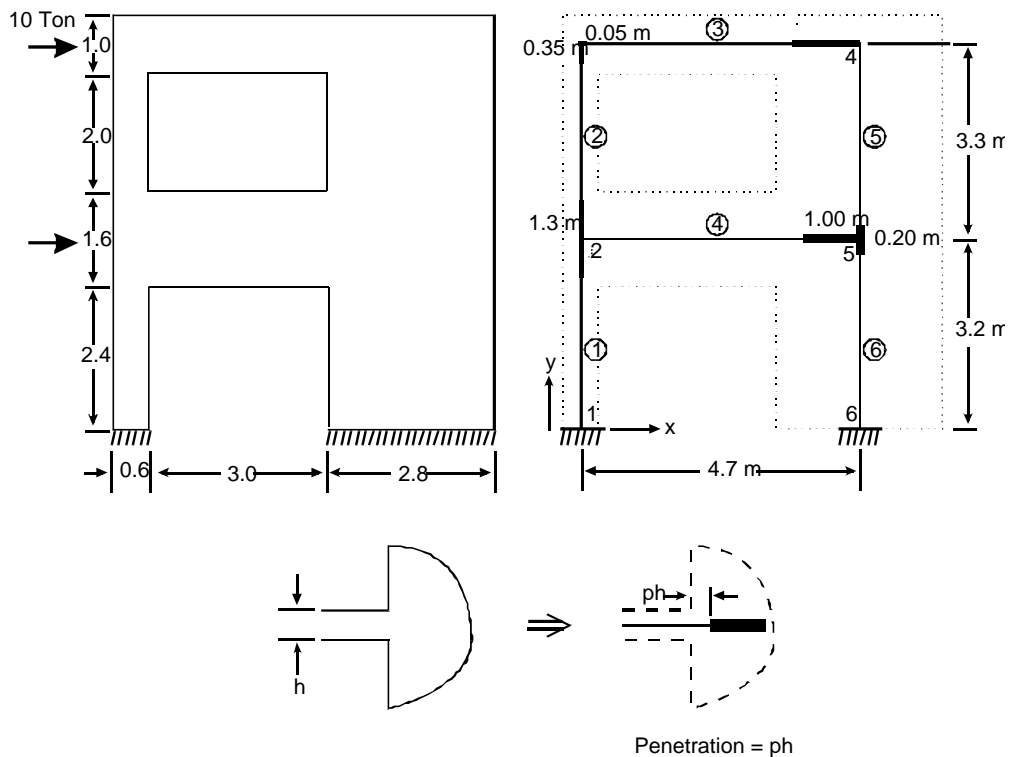


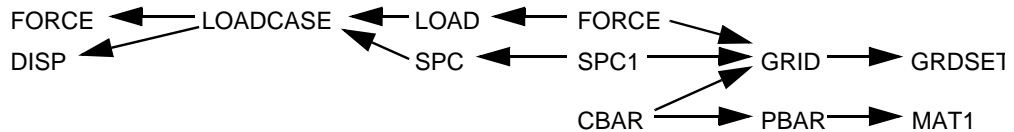
Figure 2-5

### Structural Analysis Model

1. Six CBAR elements with offsets.

2. Dimensions: thickness=0.2m
3. Materials: Concrete  $E=3.0E+6 \text{ T/m}^2$
4. One load case: Two nodal forces.  $P1=10 \text{ T}$  and  $P2=5 \text{ T}$ .

## Analysis Data Relationships



## Special Modeling Techniques

The analysis of the facade of the figure below is done through a simplified model of a 6 bar frame. The model includes shear deformation and rigid segments. The rigid segments, which are generated with the offsets of the beam, are used to model the intersections of elements in massive nodes. The lengths of the rigid segments are obtained by subtracting the distance of the model axes and the penetration of each element in the massive node. The penetration of each element can be calculated by the product of Muto's penetration factor ( $p=0.25$ ) and the elements' height (see the figure).

Note that the offsets are specified in the general coordinate system (displacement and constraint output coordinate system); not in the basic coordinate system. When no specification of displacement coordinate system is supplied (in field 7 of GRID entries), the general coordinate system is the basic coordinate system by default. In this example, the general coordinate system is the basic coordinate system.

## Reference Solution

NONE

---

## 2.5.1 Input Data

```
ID A005
SOL COMPAT1
CEND
TITLE= ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME
SUBTITLE=SIX BAR MODEL WITH RIGID OFFSETS
ECHO=NONE
LINE=64,80
    LOAD=100
    SPC =100
    DISP=ALL
    FORCE=ALL
BEGIN BULK
$
$  GRID DATA
$
GRDSET                                     345
GRID   1          0.0      0.0      0.0
GRID   2          0.0      3.2      0.0
GRID   3          0.0      6.5      0.0
GRID   4          4.7      6.5      0.0
GRID   5          4.7      3.2      0.0
GRID   6          4.7      0.0      0.0
$
$  BOUNDARY CONDITIONS
$
SPC1    100      123456  1      6
$
$  ELEMENT DATA
$
CBAR    1      12      1      2      0.0      0.0      -1.0      0.0
+      0.0      0.0      0.0      0.0      -0.65      0.0
CBAR    2      12      2      3      0.0      0.0      -1.0      0.0
+      0.0      0.65      0.0      0.0      -0.35      0.0
CBAR    3      3      3      4      0.0      0.0      -1.0      0.0
+      0.05      0.0      0.0      -1.15      0.00      0.0
CBAR    4      4      2      5      0.0      0.0      -1.0      0.0
+      0.0      0.0      0.0      -1.0      0.00      0.0
CBAR    5      56      5      4      0.0      0.0      -1.0      0.0
+      0.0      0.1      0.0      0.0      0.00      0.0
CBAR    6      56      6      5      0.0      0.0      -1.0      0.0
+      0.0      0.0      0.0      0.0      -0.10      0.0
$
$  PROPERTY DEFINITIONS
$
PBAR    12      1      .12      .0004      .0036
+      0.1      0.3      0.1      -0.3      -0.1      -0.3      -0.1      0.3
+      0.8333330.8333330.0
PBAR    3      1      .20      .00067      .01667
+      0.1      0.5      0.1      -0.5      -0.1      -0.5      -0.1      0.5
+      0.8333330.8333330.0
PBAR    4      1      .32      .00107      .06827
+      0.1      0.8      0.1      -0.8      -0.1      -0.8      -0.1      0.8
```

```

+      0.8333330.8333330.0
PBAR   56      1      .56      .00187   .3659
+      0.1      1.4      0.1      -1.4     -0.1     -1.4     -0.1     1.4
+      0.8333330.8333330.0
$
$   MATERIAL DATA
$
MAT1    1      3.0+6   1.2+6
$
$   LOAD DATA
$
FORCE   100     3      10.0     1.0     0.0     0.0
FORCE   100     2      5.0      1.0     0.0     0.0
$
ENDDATA

```



---

## 2.5.2 Output

1

```
  GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N  E      S      I      S
G  GG  EEEE   N N N  EEEE   SSSS   I      SSSS
G  G   E      N  NN  E      S      I      S
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A005  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME  
SIX BAR MODEL WITH RIGID OFFSETS

### ANALYSIS PROBLEM SUMMARY

|                                     |   |
|-------------------------------------|---|
| NUMBER OF GRID POINTS:              | 6 |
| NUMBER OF CBAR ELEMENTS:            | 6 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 6 |

NUMBER OF ELEMENT PROPERTIES: 4  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 12

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME  
 SIX BAR MODEL WITH RIGID OFFSETS

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME  
 SIX BAR MODEL WITH RIGID OFFSETS

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 5.994000E+00  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME  
 SIX BAR MODEL WITH RIGID OFFSETS

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 7.085516E-16 ; STRAIN ENERGY : 3.451449E-03  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME  
 SIX BAR MODEL WITH RIGID OFFSETS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

|   |            |            |            |            |            |             |
|---|------------|------------|------------|------------|------------|-------------|
| 1 | 1.5000E+01 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -8.1000E+01 |
|---|------------|------------|------------|------------|------------|-------------|

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME  
 SIX BAR MODEL WITH RIGID OFFSETS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

|   |             |            |            |            |            |            |
|---|-------------|------------|------------|------------|------------|------------|
| 1 | -1.5000E+01 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 8.1000E+01 |
|---|-------------|------------|------------|------------|------------|------------|

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME  
 SIX BAR MODEL WITH RIGID OFFSETS

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE                                       | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|--|-------------|------------|-----------------|------------|------------|---|
| 1  | 3           | 1.0000E+01 | 1.0000E+01      | 0.0000E+00 | 0.0000E+00 |   |
| 1GENESIS                                       | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 8 |
| ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME |             |            |                 |            |            |   |
| SIX BAR MODEL WITH RIGID OFFSETS               |             |            |                 |            |            |   |

# M A X I M U M   S P C   F O R C E

| LOADCASE                                       | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|--|-------------|------------|-----------------|------------|------------|---|
| 1  | 6           | 1.5470E+01 | -1.3640E+01     | 7.2974E+00 | 0.0000E+00 |   |
| 1GENESIS                                       | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 9 |
| ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME |             |            |                 |            |            |   |
| SIX BAR MODEL WITH RIGID OFFSETS               |             |            |                 |            |            |   |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE                                       | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |    |
|--|-------------|------------|-----------------|------------|------------|----|
| 1  | 3           | 5.7427E-04 | 5.7043E-04      | 6.6263E-05 | 0.0000E+00 |    |
| 1GENESIS                                       | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 10 |
| ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME |             |            |                 |            |            |    |
| SIX BAR MODEL WITH RIGID OFFSETS               |             |            |                 |            |            |    |

LOADCASE 1

# G R I D   D I S P L A C E M E N T S

| GRID ID  | T1<br>R1                     | T2<br>R2                      | T3<br>R3                      |                    |
|--|------------------------------|-------------------------------|-------------------------------|--------------------|
| 1  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |                    |
| 2  | 2.397111E-04<br>0.000000E+00 | 5.169027E-05<br>0.000000E+00  | 0.000000E+00<br>-1.915512E-05 |                    |
| 3  | 5.704342E-04<br>0.000000E+00 | 6.626258E-05<br>0.000000E+00  | 0.000000E+00<br>-2.782742E-05 |                    |
| 4  | 5.252333E-04<br>0.000000E+00 | -1.781007E-05<br>0.000000E+00 | 0.000000E+00<br>-8.167393E-05 |                    |
| 5  | 2.170035E-04<br>0.000000E+00 | -1.346553E-05<br>0.000000E+00 | 0.000000E+00<br>-6.705721E-05 |                    |
| 6  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |                    |
| 1GENESIS                                       | VERSION 7.2                  |                               | DATE 12-13-2002               | TIME 12:49 PAGE 11 |
| ONE BAY, TWO STORY WIDE COLUMN/WIDE BEAM FRAME |                              |                               |                               |                    |
| SIX BAR MODEL WITH RIGID OFFSETS               |                              |                               |                               |                    |

## F O R C E S   I N   B A R   E L E M E N T S

| BAR ID | END | BENDING MOMENT 1<br>SHEAR FORCE 1 | BENDING MOMENT 2<br>SHEAR FORCE 2 | AXIAL FORCE<br>TORQUE |
|--------|-----|-----------------------------------|-----------------------------------|-----------------------|
| 1      | A   | 0.000000E+00                      | -1.814576E+00                     | 7.297449E+00          |
|        |     | 0.000000E+00                      | -1.359567E+00                     | 0.000000E+00          |
|        | B   | 0.000000E+00                      | 1.652321E+00                      | 7.297449E+00          |
|        |     | 0.000000E+00                      | -1.359567E+00                     | 0.000000E+00          |
| 2      | A   | 0.000000E+00                      | -2.629698E+00                     | 2.280884E+00          |
|        |     | 0.000000E+00                      | -2.251283E+00                     | 0.000000E+00          |
|        | B   | 0.000000E+00                      | 2.548253E+00                      | 2.280884E+00          |
|        |     | 0.000000E+00                      | -2.251283E+00                     | 0.000000E+00          |
| 3      | A   | 0.000000E+00                      | 3.222158E+00                      | -7.748717E+00         |
|        |     | 0.000000E+00                      | 2.280884E+00                      | 0.000000E+00          |
|        | B   | 0.000000E+00                      | -4.760937E+00                     | -7.748717E+00         |
|        |     | 0.000000E+00                      | 2.280884E+00                      | 0.000000E+00          |
| 4      | A   | 0.000000E+00                      | 6.629071E+00                      | -5.891716E+00         |
|        |     | 0.000000E+00                      | 5.016565E+00                      | 0.000000E+00          |
|        | B   | 0.000000E+00                      | -1.193222E+01                     | -5.891716E+00         |
|        |     | 0.000000E+00                      | 5.016565E+00                      | 0.000000E+00          |
| 5      | A   | 0.000000E+00                      | -1.741194E+01                     | -2.280884E+00         |
|        |     | 0.000000E+00                      | -7.748717E+00                     | 0.000000E+00          |
|        | B   | 0.000000E+00                      | 7.383955E+00                      | -2.280884E+00         |
|        |     | 0.000000E+00                      | -7.748717E+00                     | 0.000000E+00          |
| 6      | A   | 0.000000E+00                      | -4.488741E+01                     | -7.297449E+00         |
|        |     | 0.000000E+00                      | -1.364043E+01                     | 0.000000E+00          |
|        | B   | 0.000000E+00                      | -2.602070E+00                     | -7.297449E+00         |
|        |     | 0.000000E+00                      | -1.364043E+01                     | 0.000000E+00          |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.6 Deformed Beam

### Example ID

A006

### Analysis Data Used

GRID, CBAR, PBAR, MAT1, LOADCASE, LOAD, SPC, SPCD, SPC1, STRESS.

### Special Features Used

This problem shows the capabilities of *GENESIS* to analyze structures with enforced displacements.

### Problem Statement

The beam in **Figure 2-6** has an enforced displacement of 1 inch in the z direction. Neglecting shear deformation find the tensile stresses at the corners of the beam.

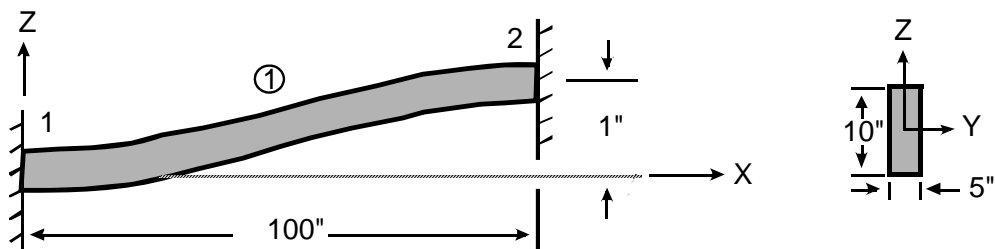


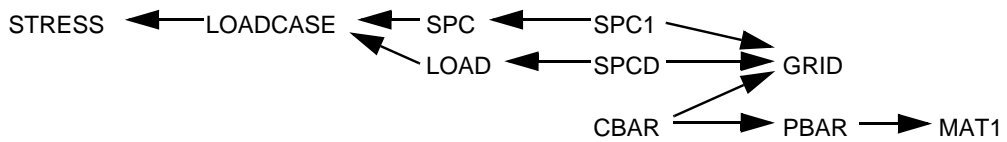
Figure 2-6

### Structural Analysis Model

1. One CBAR element in the x-z plane.
2. The relevant section properties are: Area=50 in<sup>2</sup>, I<sub>2</sub>=416.7 in<sup>4</sup>
3. Material E=1.0E7 psi.
4. One displacement load case  $u_z = 1.0$  in.

The enforced displacement is specified with a SPCD data entry. In addition, it is necessary to constrain the degree of freedom. Degrees of freedom are constrained with SPC1 data entries.

## Analysis Data Relationships



## Special Modeling Techniques

NONE

## Reference Solution

The analytical solution for the stresses in the corner of the beam neglecting shear deformation are:

$$\sigma = \pm \frac{Mh}{2I}$$

where M is the moment, I the area moment of inertia and h the total height. The moment can be calculated using the following relationship:

$$M = \frac{6EI}{L^2}$$

where E is the Young's modulus and L is the length of the beam.

Therefore

$$\sigma = \mp 30,000.0 \text{ psi}$$

## Calculated Solution

$$\sigma = \mp 30,000 \text{ psi}$$

## Comparison Between Calculated Solution and Reference Solution

There are no differences between analytical solutions and the *GENESIS* solution.

---

## 2.6.1 Input Data

```
ID A006
SOL COMPAT1
CEND
TITLE= DEFORMEND BEAM
ECHO=NONE
LINE=64,80
LOADCASE 1
    LOAD=71
    SPC=81
    STRESS=ALL
BEGIN BULK
$
$  GRID DATA
$
GRID    1          0.0    0.0    0.0          123456
GRID    2          100.0    0.    0.          12456
$
$  ELEMENT DEFINITIONS
$
CBAR    1          1          1          2          0.    1.0    0.
+          0.    .0    0.0    0.0    0.0    0.0
$
$  PROPERTY DEFINITIONS
$
PBAR    1          1          50.0    104.2    416.7    288.5    0.0
+          2.5    5.0    2.5    -5.0    -2.5    -5.0    -2.5    5.0
$
$  MATERIAL DATA
$
MAT1    1          10.0+6          0.3    0.0    0.0    25.0
$
$  ENFORCED DISPLACEMENT DATA
$
SPCD    71          2          3          1.0
SPC1    81          3          2
$
ENDDATA
```

---

## 2.6.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A006

RUN STARTED: Dec 13, 2002 12:49

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
DEFORMEND BEAM

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 2
NUMBER OF CBAR ELEMENTS: 1
TOTAL NUMBER OF NON RIGID ELEMENTS: 1
```



NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 1

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 DEFORMEND BEAM

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 DEFORMEND BEAM

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 5.000000E+03  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 DEFORMEND BEAM

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 0.000000E+00 ; STRAIN ENERGY : 2.500200E+04  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 DEFORMEND BEAM

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 DEFORMEND BEAM

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 DEFORMEND BEAM

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE       | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|----------------|-------------|------------|-----------------|------------|------------|---|
| 1              |             | 0.0000E+00 | 0.0000E+00      | 0.0000E+00 | 0.0000E+00 |   |
| 1GENESIS       | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 8 |
| DEFORMEND BEAM |             |            |                 |            |            |   |

# M A X I M U M   S P C   F O R C E

| LOADCASE       | GRID ID     | MAGNITUDE  | T1              | T2         | T3          |   |
|----------------|-------------|------------|-----------------|------------|-------------|---|
| 1              | 1           | 5.0004E+04 | 0.0000E+00      | 0.0000E+00 | -5.0004E+04 |   |
| 1GENESIS       | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE        | 9 |
| DEFORMEND BEAM |             |            |                 |            |             |   |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE       | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |    |
|----------------|-------------|------------|-----------------|------------|------------|----|
| 1              | 2           | 1.0000E+00 | 0.0000E+00      | 0.0000E+00 | 1.0000E+00 |    |
| 1GENESIS       | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 10 |
| DEFORMEND BEAM |             |            |                 |            |            |    |

LOADCASE                      1

# S T R E S S E S   I N   B A R   E L E M E N T S

| BAR ID | END | STRESS-C      | STRESS-D      | STRESS-E      | STRESS-F      |
|--------|-----|---------------|---------------|---------------|---------------|
| 1      | A   | -3.000000E+04 | 3.000000E+04  | 3.000000E+04  | -3.000000E+04 |
|        | B   | 3.000000E+04  | -3.000000E+04 | -3.000000E+04 | 3.000000E+04  |

1                      \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.7 Two Bar Symmetric Frame

### Example ID

A007

### Analysis Data Used

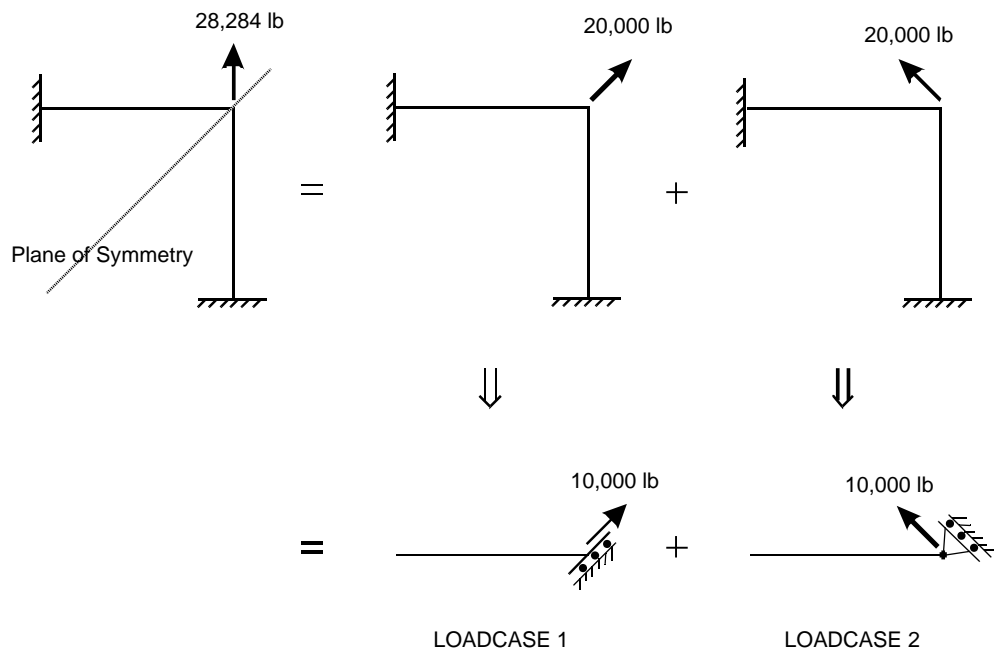
GRID, CBAR, PBAR, MAT1, LOADCASE, LOADCOM, LOADSEQ, LOAD, FORCE, SPC, SPC1, DISPLACEMENT, STRESS

### Special Features Used

Utilization of multi-point constraints (MPC) to break a symmetric structure into a half model under two load conditions.

### Problem Statement

Find nodal displacements of the structure shown in **Figure 2-7**.



**Figure 2-7**

## Structural Analysis Model

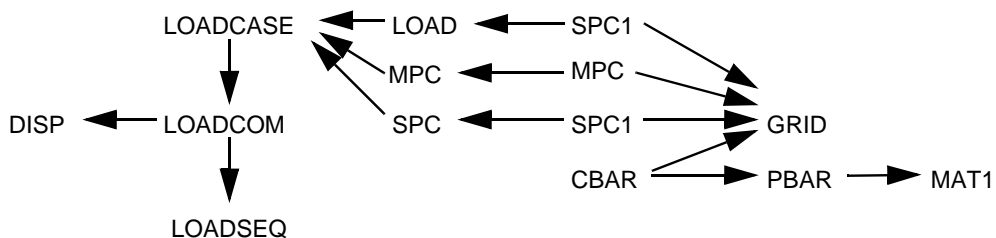
1. One beam structure in the X-Z plane (half model, CBAR element)
2. Section properties: Area=50 in<sup>2</sup>, I2=416.7 in<sup>4</sup>
3. Material E=1.0E7 psi.
4. Two load cases: each of them has different boundary conditions

B.C. 1:  $U_x - U_z = 0$  (MPC=1),  $U_y = 0$ ,  $\theta_x = \theta_y = \theta_z = 0$ .

B.C. 2:  $U_x + U_z = 0$  (MPC=2),  $U_y = 0$ ,  $\theta_x = \theta_z = 0$ .

5. One load combination that represent the solution of the left part of the structure.

## Analysis Data Relationships



## Special Modeling Techniques

**Utilization of Symmetry:** First the load is divided in the sum of its symmetric and antisymmetric components. Then, each of the forces is applied to the structure as a separate load case. Third, the structure in each load case is broken in half by considering suitable boundary conditions and load. Finally, the solution of the left part of the structure is recovered by adding the solutions of the symmetric and antisymmetric load case.

## Reference Solution

The reference solution was constructed using the full model.

## Calculated Solution

The *GENESIS* solution is presented in the output.

## Comparison Between Calculated Solution and Reference

No differences between the full model and half model were found.

---

## 2.7.1 Input Data

```
ID A007
SOL COMPAT1
CEND
TITLE= TWO BAR SYMMETRIC FRAME
ECHO=NONE
LINE=64,80
LOADCASE 1
    LOAD=1
    MPC=1
    SPC=1
LOADCASE 2
    LOAD=2
    MPC=2
    SPC=2
LOADCOM 3
LOADSEQ 1.0,1.0
    DISPLACEMENT=ALL
BEGIN BULK
$
$  GRID DATA
$
GRID    1           0.0    0.0    0.0           123456
GRID    2          100.0    0.    0.           246
$
$  ELEMENT DEFINITIONS
$
CBAR    1          1      1      2      0.    1.0    0.
+              0.    .0    0.0    0.0    0.0    0.0
$
$  PROPERTY DEFINITIONS
$
PBAR    1          1      50.0    104.2    416.7    288.5    0.0
+          2.5      5.0      2.5     -5.0     -2.5     -5.0     -2.5      5.0
+              0.0
$
$  MATERIAL DATA
$
MAT1    1          10.0+6           0.3      0.0      0.0      25.0
$
$  LOAD DATA
$
FORCE   1          2           1.0+4    .707      0.0      0.707
FORCE   2          2           1.0+4    -.707      0.0      0.707
$
$  BOUNDARY CONDITIONS
$
MPC     1          2      1      1.0      2      3      -1.0
MPC     2          2      1      1.0      2      3      1.0
SPC1    1          2456      2
SPC1    2          246      2
$
ENDDATA
```



---

## 2.7.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A007

RUN STARTED: Dec 13, 2002 12:49

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
TWO BAR SYMMETRIC FRAME

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 2
NUMBER OF CBAR ELEMENTS: 1
TOTAL NUMBER OF NON RIGID ELEMENTS: 1
```

|                               |   |
|-------------------------------|---|
| NUMBER OF ELEMENT PROPERTIES: | 1 |
| NUMBER OF MATERIALS:          | 1 |
| NUMBER OF DEGREES OF FREEDOM: | 3 |

#### LOAD CASES SUMMARY

|                                      |   |
|--------------------------------------|---|
| NUMBER OF STATIC LOAD CASES:         | 2 |
| NUMBER OF STATIC LOADCOM LOAD CASES: | 1 |
| TOTAL NUMBER OF LOAD CASES:          | 3 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
TWO BAR SYMMETRIC FRAME

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
TWO BAR SYMMETRIC FRAME

#### M A S S / V O L U M E S U M M A R Y

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 0.000000E+00 |
| SYSTEM VOLUME      | 5.000000E+03 |
| SYSTEM MASS/VOLUME | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
TWO BAR SYMMETRIC FRAME

#### S O L U T I O N R E S I D U A L S

|          |     |                           |                              |
|----------|-----|---------------------------|------------------------------|
| LOADCASE | 1 ; | RESIDUAL : 2.572828E-16 ; | STRAIN ENERGY : 1.979598E+01 |
| LOADCASE | 2 ; | RESIDUAL : 1.286414E-16 ; | STRAIN ENERGY : 1.994410E+01 |
| LOADCOM  | 3 ; |                           | STRAIN ENERGY : 3.974008E+01 |

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TWO BAR SYMMETRIC FRAME

#### O L O A D R E S U L T A N T S

| LOADCASE | T1          | T2         | T3         | R1         | R2          | R3         |
|----------|-------------|------------|------------|------------|-------------|------------|
| 1        | 7.0700E+03  | 0.0000E+00 | 7.0700E+03 | 0.0000E+00 | -7.0700E+05 | 0.0000E+00 |
| 2        | -7.0700E+03 | 0.0000E+00 | 7.0700E+03 | 0.0000E+00 | -7.0700E+05 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
TWO BAR SYMMETRIC FRAME

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|



```

      1 -1.4000E+04  0.0000E+00 -1.4001E+02  0.0000E+00  1.4001E+04  0.0000E+00
      2  1.4105E+04  0.0000E+00 -3.5265E+01  0.0000E+00  3.5265E+03  0.0000E+00
1GENESIS  VERSION  7.2                      DATE 12-13-2002  TIME 12:49  PAGE    7
      TWO BAR SYMMETRIC FRAME

```

#### MAXIMUM APPLIED FORCE

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3         |
|----------|---------|------------|-------------|------------|------------|
| 1        | 2       | 9.9985E+03 | 7.0700E+03  | 0.0000E+00 | 7.0700E+03 |
| 2        | 2       | 9.9985E+03 | -7.0700E+03 | 0.0000E+00 | 7.0700E+03 |

```

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      TWO BAR SYMMETRIC FRAME

```

#### MAXIMUM SPC FORCE

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3          |
|----------|---------|------------|-------------|------------|-------------|
| 1        | 1       | 1.4001E+04 | -1.4000E+04 | 0.0000E+00 | -1.4001E+02 |
| 2        | 1       | 1.4105E+04 | 1.4105E+04  | 0.0000E+00 | -3.5265E+01 |

```

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      TWO BAR SYMMETRIC FRAME

```

#### MAXIMUM DISPLACEMENT

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3         |
|----------|---------|------------|-------------|------------|------------|
| 1        | 2       | 3.9598E-03 | 2.8000E-03  | 0.0000E+00 | 2.8000E-03 |
| 2        | 2       | 3.9894E-03 | -2.8209E-03 | 0.0000E+00 | 2.8209E-03 |

```

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      TWO BAR SYMMETRIC FRAME

```

LOADCASE 3

#### GRID DISPLACEMENTS

| GRID ID | T1<br>R1                      | T2<br>R2                      | T3<br>R3                     |
|---------|-------------------------------|-------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 2       | -2.094929E-05<br>0.000000E+00 | 0.000000E+00<br>-4.231421E-05 | 5.620945E-03<br>0.000000E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.8 Three Story Facade with a Panel - A

### Example ID

A008

### Analysis Data Used

GRID, GRDSET, RBE2, CBAR, PBAR, MAT1, LOAD, FORCE, SPC, SPC1, DISP.

### Special Features Used

Utilization of rigid body elements (RBE2) to model a panel zone in a frame.

### Problem Statement

Considering shear deformation find the nodal displacements of the facade of **Figure 2-8**.

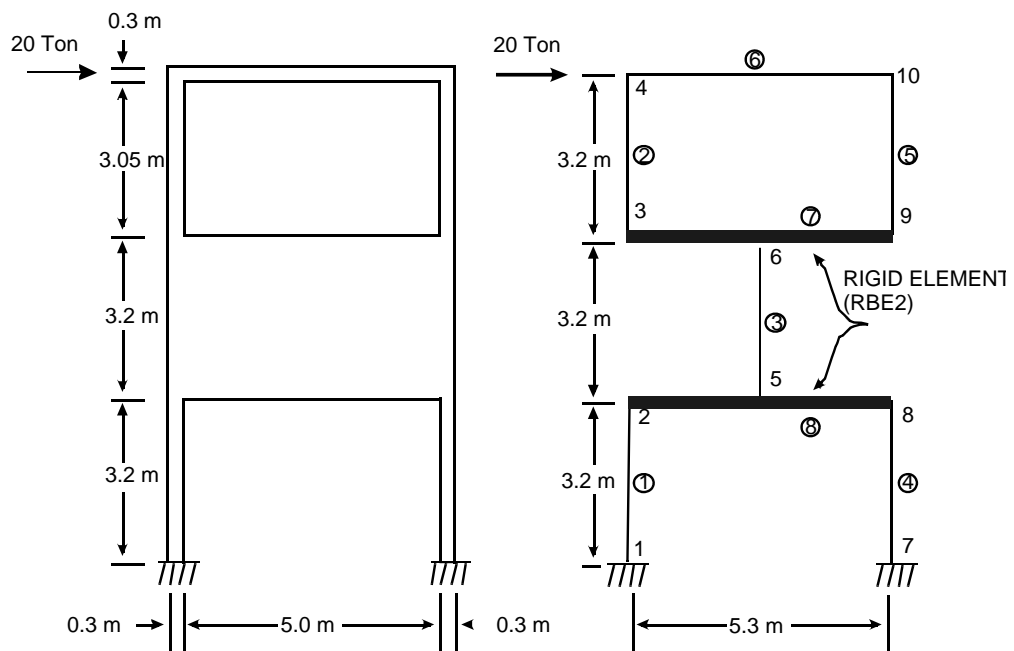
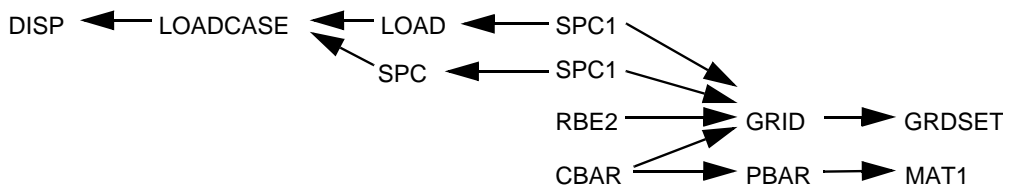


Figure 2-8

## Structural Analysis Model

1. Analysis of a two dimensional structure with a simplified model of 6 bar elements (CBAR) and 2 rigid body elements (RBE2).
2. Section properties of flexible beams: Area=0.06 m<sup>2</sup>, I2=0.0045 m<sup>4</sup>, AS2=0.05 m<sup>2</sup>
3. Section properties of modeled beam: Area=1.6 m<sup>2</sup>, I2=3.2614 m<sup>4</sup>, AS2=1.248 m<sup>2</sup>
4. Material: E=3.0+6 T/m<sup>2</sup>, G=1.2+6 T/m<sup>2</sup>.
5. One load case. Loading: 20.0 T.

## Analysis Data Relationships



## Special Modeling Techniques

The analysis of the facade is performed using a stick model. Panel zones, as in this problem, can be modeled using two rigid elements and one beam element with modified properties. In the figure below, a detail of the panel model is given. These rigid elements, that are parallel to the horizontal force, cause that planes that they are located in to remain plane after deformation. Shear deformation is included in all beam elements.

The properties of the equivalent column is:

$$A = \frac{at}{1-\nu}$$

$$I = \frac{a^3 t}{12(1-\nu^2)} \left[ 1 + \frac{1-\nu}{2} \left( \frac{b}{a} \right)^2 \right]$$

$$AS = \frac{A}{(1-\nu) \left[ 1 + \frac{1-\nu}{2} \left( \frac{b}{a} \right)^2 \right]}$$

where  $t$  is the thickness of the panel.

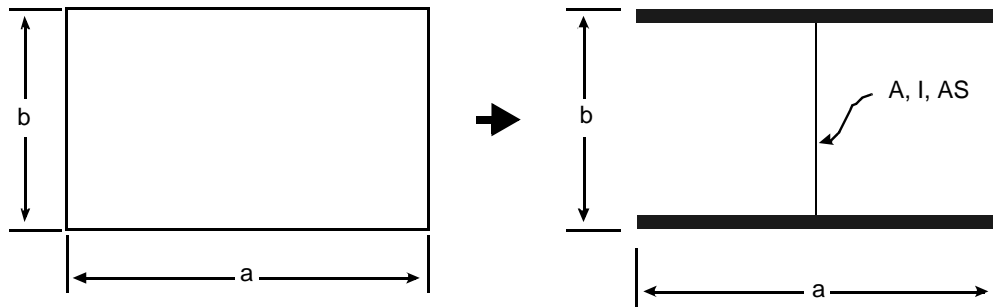


Figure 2-9

## Reference Solutions

See Example [A009](#). The horizontal displacements of the facades at nodes 2,3 and 4 are:

$$U2 = 0.02076 \text{ m}$$

$$U3 = 0.02110 \text{ m}$$

$$U4 = 0.05596 \text{ m}$$

## Calculated Solutions

The horizontal displacements of the facades at nodes 2,3 and 4 are:

$$U2 = 0.02108 \text{ m}$$

$$U3 = 0.02183 \text{ m}$$

$$U4 = 0.05675 \text{ m}$$

## Comparison Between Calculated Solutions and Reference Solutions

The differences between reference solution and *GENESIS* solutions are:

$$U2: -2\%$$

$$U3: -3\%$$

$$U4: -1\%$$

---

## 2.8.1 Input Data

```
ID A008
SOL COMPAT1
CEND
TITLE=THREE STORY FACADE WITH A PANEL - I
SUBTITLE=SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS
ECHO=NONE
LINE=64,80
    LOAD=100
    SPC =100
    DISP=ALL
BEGIN BULK
$
$  GRID DATA
$
GRDSET                                                    246
$
GRID      1           0.0      0.0      0.0
GRID      2           0.0      0.0      3.2
GRID      3           0.0      0.0      6.4
GRID      4           0.0      0.0      9.6
GRID      5           2.8      0.0      3.2
GRID      6           2.8      0.0      6.4
GRID      7           5.3      0.0      0.0
GRID      8           5.3      0.0      3.2
GRID      9           5.3      0.0      6.4
GRID     10           5.3      0.0      9.6
$
$  BOUNDARY CONDITIONS
$
SPC1      100      123456  1      7
$
$  ELEMENT DEFINITIONS
$
BAROR      1           0.0      1.0      0.0
CBAR      1      1      1      2      0.0      1.0      0.0
CBAR      2      1      3      4      0.0      1.0      0.0
CBAR      4      1      7      8      0.0      1.0      0.0
CBAR      5      1      9     10      0.0      1.0      0.0
CBAR      6      1      4     10      0.0      1.0      0.0
$
$  PANEL MODELED AS A BEAM DATA
$
CBAR      3      100      5      6      0.0      1.0      0.0
$
$  RIGID ELEMENTS DATA
$
RBE2      7      6     135      3      9
RBE2      8      5     135      2      8
$
$  PROPERTY DEFINITIONS
$
PBAR      1      1      0.06      0.0002      0.00045
```

```

+      0.1      0.15      0.1      -0.15  -0.1      -0.15  -0.1      0.15
+      0.8333330.8333330.00
PBAR   100      1        1.6      0.0037  3.2614
+      0.1      2.8      0.1      -2.8     -0.1      -2.8     -0.1      2.8
+      0.8333330.8333330.0
$
$   MATERIAL DATA
$
MAT1    1        3.0+6    1.2+6
$
$   LOAD DATA
$
FORCE   100      4        20.0     1.0      0.0      0.0
$
ENDDATA

```

---

## 2.8.2 Output

1

```
      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A008  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
THREE STORY FACADE WITH A PANEL - I  
SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

### ANALYSIS PROBLEM SUMMARY

|                                     |    |
|-------------------------------------|----|
| NUMBER OF GRID POINTS:              | 10 |
| NUMBER OF CBAR ELEMENTS:            | 6  |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 6  |

NUMBER OF RBE2 ELEMENTS: 2  
 NUMBER OF ELEMENT PROPERTIES: 2  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 24

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 THREE STORY FACADE WITH A PANEL - I  
 SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 THREE STORY FACADE WITH A PANEL - I  
 SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 6.206000E+00  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 THREE STORY FACADE WITH A PANEL - I  
 SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 1.684547E-13 ; STRAIN ENERGY : 5.674781E-01  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 THREE STORY FACADE WITH A PANEL - I  
 SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 2.0000E+01 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 1.9200E+02 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 THREE STORY FACADE WITH A PANEL - I  
 SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1          | T2         | T3         | R1         | R2          | R3         |
|----------|-------------|------------|------------|------------|-------------|------------|
| 1        | -2.0000E+01 | 0.0000E+00 | 8.8818E-16 | 0.0000E+00 | -1.9200E+02 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 THREE STORY FACADE WITH A PANEL - I



# SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

## M A X I M U M   A P P L I E D   F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        | 4       | 2.0000E+01 | 2.0000E+01 | 0.0000E+00 | 0.0000E+00 |

1GENESIS    VERSION 7.2                      DATE 12-13-2002    TIME 12:49    PAGE    8

THREE STORY FACADE WITH A PANEL - I

SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

## M A X I M U M   S P C   F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3          |
|----------|---------|------------|-------------|------------|-------------|
| 1        | 1       | 3.1771E+01 | -1.0000E+01 | 0.0000E+00 | -3.0156E+01 |

1GENESIS    VERSION 7.2                      DATE 12-13-2002    TIME 12:49    PAGE    9

THREE STORY FACADE WITH A PANEL - I

SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

## M A X I M U M   D I S P L A C E M E N T

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        | 4       | 5.6752E-02 | 5.6748E-02 | 0.0000E+00 | 7.0790E-04 |

1GENESIS    VERSION 7.2                      DATE 12-13-2002    TIME 12:49    PAGE    10

THREE STORY FACADE WITH A PANEL - I

SIX BAR MODEL WITH TWO RIGID BODY ELEMENTS

LOADCASE                      1

## G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                      |
|---------|------------------------------|------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  |
| 2       | 2.108419E-02<br>0.000000E+00 | 0.000000E+00<br>2.023076E-04 | 5.361151E-04<br>0.000000E+00  |
| 3       | 2.182739E-02<br>0.000000E+00 | 0.000000E+00<br>2.337051E-04 | 6.240282E-04<br>0.000000E+00  |
| 4       | 5.674781E-02<br>0.000000E+00 | 0.000000E+00<br>8.576776E-03 | 7.079012E-04<br>0.000000E+00  |
| 5       | 2.108419E-02<br>0.000000E+00 | 0.000000E+00<br>2.023076E-04 | -3.034614E-05<br>0.000000E+00 |
| 6       | 2.182739E-02<br>0.000000E+00 | 0.000000E+00<br>2.337051E-04 | -3.034614E-05<br>0.000000E+00 |
| 7       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  |

|    |                              |                              |                               |
|----|------------------------------|------------------------------|-------------------------------|
| 8  | 2.108419E-02<br>0.000000E+00 | 0.000000E+00<br>2.023076E-04 | -5.361151E-04<br>0.000000E+00 |
| 9  | 2.182739E-02<br>0.000000E+00 | 0.000000E+00<br>2.337051E-04 | -6.146090E-04<br>0.000000E+00 |
| 10 | 5.645426E-02<br>0.000000E+00 | 0.000000E+00<br>8.472251E-03 | -6.984819E-04<br>0.000000E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.9 Three Story Facade with a Panel - B

### Example ID

A009

### Analysis Data Used

GRID, GRDSET, CQUAD4, CBAR, PBAR, PSHELL, MAT1, LOAD, FORCE, SPC, SPC1, DISP

### Special Features Used

NONE

### Problem Statement

Considering shear deformation find the nodal displacements of the facade of **Figure 2-10**.

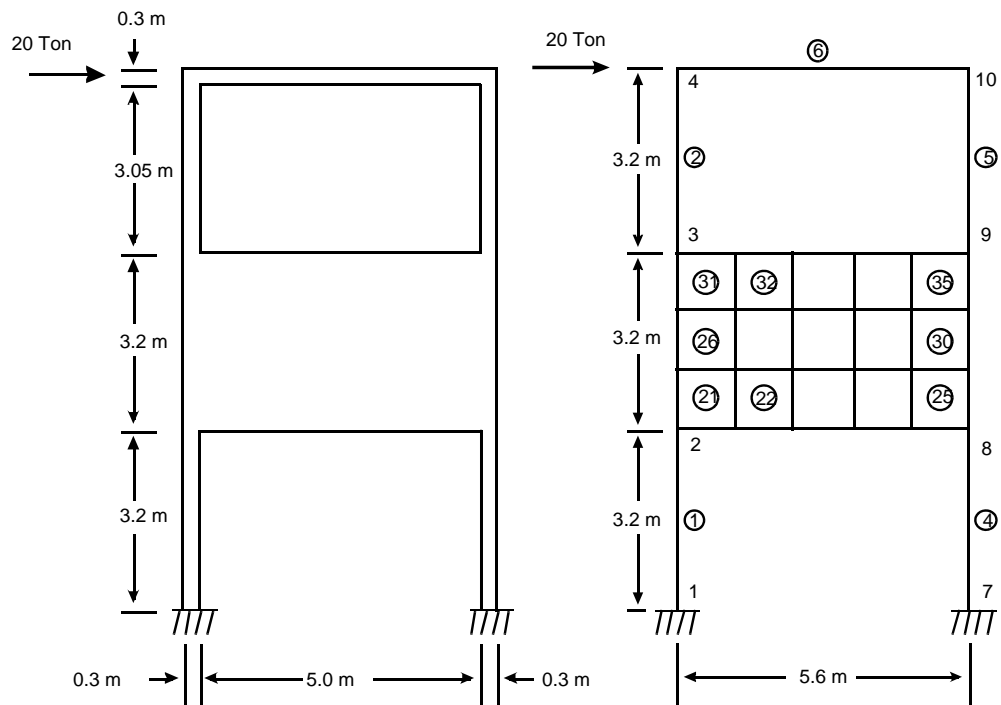


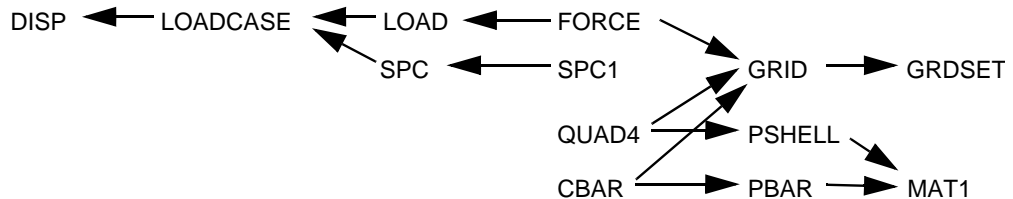
Figure 2-10

## Structural Analysis Model

1. Five beam elements (CBAR) and 15 shell elements (CQUAD4), thickness = 0.2 m.
2. Section properties: Area=0.06m<sup>2</sup>, I2=0.00045 m<sup>4</sup>, AS2=0.05 m<sup>2</sup>.
3. Material: E=3.0+6 T/m<sup>2</sup>, G=1.2+6 T/m<sup>2</sup>.
4. One load case. Loading: P= 20.0 T.

Quad elements and beam elements are incompatible elements. To overcome this incompatibility in this example nodes 2, 8, 3 and 9 were restricted to not rotate in plane. The last restriction is considered an adequate assumption because the points in the panel can not rotate much. The points of the panel do not rotate much because the columns do not deform much in the axial direction and the panel is very rigid, compared to the columns.

## Analysis Data Relationships



## Special Modeling Techniques

NONE

## Reference Solutions

NONE

---

## 2.9.1 Input Data

```
ID A009
SOL COMPAT1
CEND
TITLE=THREE STORY FACADE WITH A PANEL-II
ECHO=NONE
LINE=64,80
    LOAD=100
    SPC =200
    DISP=ALL
BEGIN BULK
$
$   GRID DATA
$
$
GRDSET                                246
GRID    1                0.0        0.0        0.0
GRID    2                0.0        0.0        3.2
GRID    3                0.0        0.0        6.4
GRID    4                0.0        0.0        9.6
GRID    7                5.6        0.0        0.0
GRID    8                5.6        0.0        3.2
GRID    9                5.6        0.0        6.4
GRID   10                5.6        0.0        9.6
$
GRID   20                1.1        0.0        3.2        2456
GRID   21                2.2        0.0        3.2        2456
GRID   22                3.4        0.0        3.2        2456
GRID   23                4.5        0.0        3.2        2456
$
GRID   24                0.0        0.0        4.3        2456
GRID   25                1.1        0.0        4.3        2456
GRID   26                2.2        0.0        4.3        2456
GRID   27                3.4        0.0        4.3        2456
GRID   28                4.5        0.0        4.3        2456
GRID   29                5.6        0.0        4.3        2456
$
GRID   30                0.0        0.0        5.3        2456
GRID   31                1.1        0.0        5.3        2456
GRID   32                2.2        0.0        5.3        2456
GRID   33                3.4        0.0        5.3        2456
GRID   34                4.5        0.0        5.3        2456
GRID   35                5.6        0.0        5.3        2456
$
GRID   36                1.1        0.0        6.4        2456
GRID   37                2.2        0.0        6.4        2456
GRID   38                3.4        0.0        6.4        2456
GRID   39                4.5        0.0        6.4        2456
$
$   BOUNDARY CONDITIONS
$
SPC1    200      123456  1      7
SPC1    200      2456   2      3      8      9
```

```

$
$   ELEMENT DATA
$
BAROR      1      0.0    1.0    0.0
CBAR    1      1      1      2      0.0    1.0    0.0
CBAR    2      1      3      4      0.0    1.0    0.0
CBAR    4      1      7      8      0.0    1.0    0.0
CBAR    5      1      9     10      0.0    1.0    0.0
CBAR    6      1      4     10      0.0    1.0    0.0
$
CQUAD4 21      2      2     20     25     24
CQUAD4 22      2     20     21     26     25
CQUAD4 23      2     21     22     27     26
CQUAD4 24      2     22     23     28     27
CQUAD4 25      2     23      8     29     28
$
CQUAD4 26      2     24     25     31     30
CQUAD4 27      2     25     26     32     31
CQUAD4 28      2     26     27     33     32
CQUAD4 29      2     27     28     34     33
CQUAD4 30      2     28     29     35     34
$
CQUAD4 31      2     30     31     36      3
CQUAD4 32      2     31     32     37     36
CQUAD4 33      2     32     33     38     37
CQUAD4 34      2     33     34     39     38
CQUAD4 35      2     34     35      9     39
$
$   PROPERTY DEFINITIONS
$
PBAR    1      1      0.06    0.0002  0.00045  0.1
+      0.1      0.15      0.1    -0.15    -0.1    -0.15    -0.1    0.15
+      0.83333330.83333330.00
PSHELL  2      1      .2
$
$   MATERIAL DATA
$
MAT1    1      3.0+6    1.2+6
$
$   LOAD DATA
$
FORCE   100      4      20.0    1.0    0.0    0.0
$
ENDDATA

```

---

## 2.9.2 Output

1

```
  GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N  E      S      I      S
G  GG  EEEE   N N N  EEEE   SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A009

RUN STARTED: Dec 13, 2002 12:49

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
THREE STORY FACADE WITH A PANEL-II

### ANALYSIS PROBLEM SUMMARY

|                                    |    |
|------------------------------------|----|
| NUMBER OF GRID POINTS:             | 28 |
| NUMBER OF CBAR ELEMENTS:           | 5  |
| NUMBER OF CQUAD4->PSHELL ELEMENTS: | 15 |

TOTAL NUMBER OF NON RIGID ELEMENTS: 20  
 NUMBER OF ELEMENT PROPERTIES: 2  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 54

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 THREE STORY FACADE WITH A PANEL-II

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

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 THREE STORY FACADE WITH A PANEL-II

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 4.688000E+00  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 THREE STORY FACADE WITH A PANEL-II

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 1.601635E-13 ; STRAIN ENERGY : 5.595812E-01  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 THREE STORY FACADE WITH A PANEL-II

#### O L O A D R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

|   |            |            |            |            |            |            |
|---|------------|------------|------------|------------|------------|------------|
| 1 | 2.0000E+01 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 1.9200E+02 | 0.0000E+00 |
|---|------------|------------|------------|------------|------------|------------|

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 THREE STORY FACADE WITH A PANEL-II

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

|   |             |            |             |            |             |            |
|---|-------------|------------|-------------|------------|-------------|------------|
| 1 | -2.0000E+01 | 0.0000E+00 | -1.9371E-12 | 0.0000E+00 | -1.9200E+02 | 0.0000E+00 |
|---|-------------|------------|-------------|------------|-------------|------------|

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 THREE STORY FACADE WITH A PANEL-II



# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE                           | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|------------------------------------|-------------|------------|-----------------|------------|------------|---|
| 1                                  | 4           | 2.0000E+01 | 2.0000E+01      | 0.0000E+00 | 0.0000E+00 |   |
| 1GENESIS                           | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 8 |
| THREE STORY FACADE WITH A PANEL-II |             |            |                 |            |            |   |

# M A X I M U M   S P C   F O R C E

| LOADCASE                           | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|------------------------------------|-------------|------------|-----------------|------------|------------|---|
| 1                                  | 7           | 1.3885E+01 | -1.0000E+01     | 0.0000E+00 | 9.6332E+00 |   |
| 1GENESIS                           | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 9 |
| THREE STORY FACADE WITH A PANEL-II |             |            |                 |            |            |   |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE                           | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |    |
|------------------------------------|-------------|------------|-----------------|------------|------------|----|
| 1                                  | 4           | 5.5959E-02 | 5.5958E-02      | 0.0000E+00 | 3.2806E-04 |    |
| 1GENESIS                           | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 10 |
| THREE STORY FACADE WITH A PANEL-II |             |            |                 |            |            |    |

LOADCASE                      1

# G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                      |
|---------|------------------------------|------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  |
| 2       | 2.076046E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.712565E-04<br>0.000000E+00  |
| 3       | 2.110456E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.498061E-04<br>0.000000E+00  |
| 4       | 5.595812E-02<br>0.000000E+00 | 0.000000E+00<br>8.767236E-03 | 3.280593E-04<br>0.000000E+00  |
| 7       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  |
| 8       | 2.076053E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.712565E-04<br>0.000000E+00 |
| 9       | 2.110408E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -2.496603E-04<br>0.000000E+00 |

|    |                              |                              |                               |
|----|------------------------------|------------------------------|-------------------------------|
| 10 | 5.564799E-02<br>0.000000E+00 | 0.000000E+00<br>8.655570E-03 | -3.279135E-04<br>0.000000E+00 |
| 20 | 2.078759E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.193432E-04<br>0.000000E+00  |
| 21 | 2.079381E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 4.170112E-05<br>0.000000E+00  |
| 22 | 2.079383E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -4.183910E-05<br>0.000000E+00 |
| 23 | 2.078765E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.194170E-04<br>0.000000E+00 |
| 24 | 2.087139E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.013616E-04<br>0.000000E+00  |
| 25 | 2.086604E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.153716E-04<br>0.000000E+00  |
| 26 | 2.087417E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 4.069888E-05<br>0.000000E+00  |
| 27 | 2.087417E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -4.084917E-05<br>0.000000E+00 |
| 28 | 2.086604E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.154484E-04<br>0.000000E+00 |
| 29 | 2.087139E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -2.013474E-04<br>0.000000E+00 |

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THREE STORY FACADE WITH A PANEL-II

LOADCASE 1

# G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                      |
|---------|------------------------------|------------------------------|-------------------------------|
| 30      | 2.095337E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.204534E-04<br>0.000000E+00  |
| 31      | 2.095794E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.180255E-04<br>0.000000E+00  |
| 32      | 2.095691E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.814009E-05<br>0.000000E+00  |
| 33      | 2.095688E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.829158E-05<br>0.000000E+00 |

|    |                              |                              |                               |
|----|------------------------------|------------------------------|-------------------------------|
| 34 | 2.095786E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.181204E-04<br>0.000000E+00 |
| 35 | 2.095330E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -2.203859E-04<br>0.000000E+00 |
| 36 | 2.105951E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.179869E-04<br>0.000000E+00  |
| 37 | 2.104040E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 4.000493E-05<br>0.000000E+00  |
| 38 | 2.104033E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -4.013590E-05<br>0.000000E+00 |
| 39 | 2.105930E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.180740E-04<br>0.000000E+00 |

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.10 Vibration of a Skewed Beam

### Example ID

A010

### Analysis Data Used

GRID, CORD1R, CBAR, PBAR, MAT1, LOADCASE, METHOD, EIGR, SVECTOR

### Special Features Used

Utilization of two different coordinate systems. The first one is the basic coordinate system and is used to define the grid coordinates. The second is a rotated coordinate system with its x-axis parallel to the cantilever beam. This coordinate system is used to describe the mode shapes of the beam.

### Problem Statement

Find the first four natural frequencies and mode shapes of the cantilever beam shown in **Figure 2-11**. Neglect shear deformation and axial motion. Consider only the motion in the x-z plane.

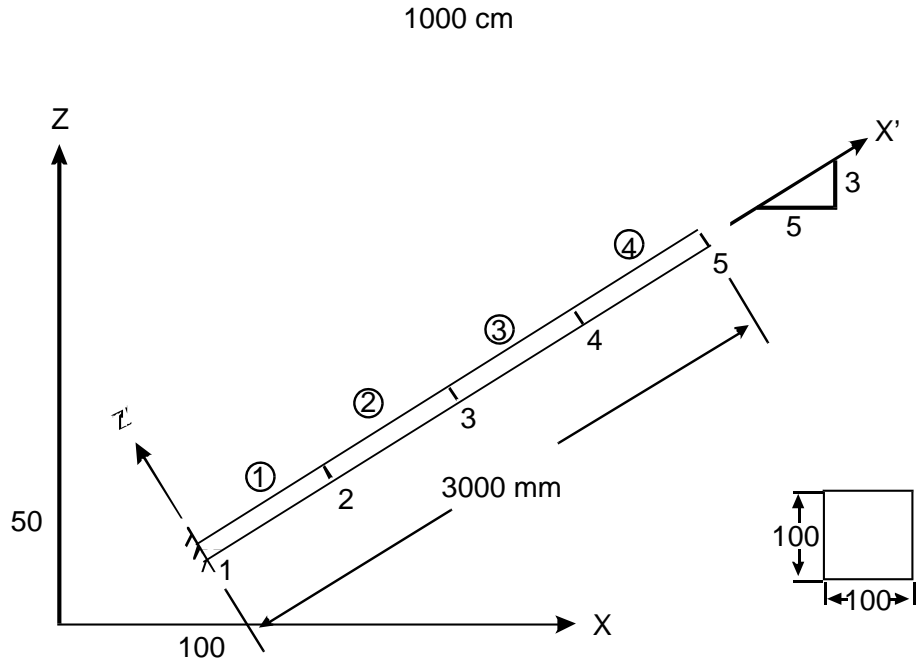
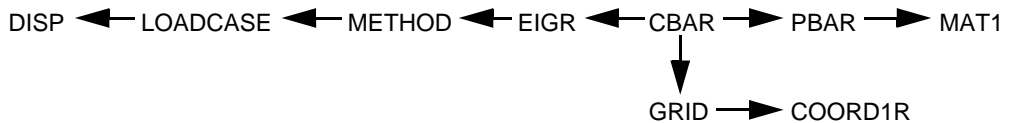


Figure 2-11

## Structural Analysis Model

1. Four CBAR elements.
2. The section properties are: Area=10000 mm<sup>2</sup>, I=8.3+6 mm<sup>4</sup>.
3. Material: E=2.07+5 MPa, ρ=8.0-9 N-sec<sup>2</sup>/mm<sup>4</sup>.
4. One eigenvalue load case.

## Analysis Data Relationships



## Special Modeling Techniques

NONE

## Reference Solutions

The analytical solutions for Bernulli-Euler beams for the first four modes are:

$$f1 = 0.5596 \sqrt{\frac{EI}{\rho AL^2}} = 9.13 \text{ hz}$$

$$f2 = 3.5069 \sqrt{\frac{EI}{\rho AL^2}} = 57.21 \text{ hz}$$

$$f3 = 9.8194 \sqrt{\frac{EI}{\rho AL^2}} = 160.2 \text{ hz}$$

$$f4 = 19.257 \sqrt{\frac{EI}{\rho AL^2}} = 314.19 \text{ hz}$$

where E the Young's modulus, A the cross section area, I the area moment of inertia, ρ the mass density and L the length.

## Calculated Solutions

$$f1 = 9.128 \text{ hz}$$

$$f2 = 57.20 \text{ hz}$$

$$f3 = 160.87 \text{ hz}$$

$$f4 = 316.39 \text{ hz}$$

## Comparison Between Calculated Solutions and Reference Solutions

The differences between analytical solutions and the *GENESIS* solutions are:

f1: 0.1%

f2: 0.1%

f3: 0.5%

f4: 1%

---

## 2.10.1 Input Data

```
ID A010
SOL COMPAT1
CEND
TITLE=VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM
LINE=64,80
$
LOADCASE 4
METHOD=4
SVECTOR=ALL
$
BEGIN BULK
$
$   COORDINATE SYSTEM
$
GRID    101           10.0   0.0    5.0    2    123456
GRID    102           7.0    0.0   10.0    2    123456
GRID    103          15.0    0.0    8.0    2    123456
CORD1R  2           101    102    103
$
$   GRID DATA
$
GRID     1           100.00  0.0    50.0    2    123456
GRID     2           743.11  0.    435.87  2    1246
GRID     3           1386.24  0.    821.74  2    1246
GRID     4           2029.36  0.    1207.62  2    1246
GRID     5           2672.48  0.    1593.48  2    1246
$
$   ELEMENT DEFINITIONS
$
CBAR     1           1       1       2       0.       1.0       0.
CBAR     2           1       2       3       0.       1.0       0.
CBAR     3           1       3       4       0.       1.0       0.
CBAR     4           1       4       5       0.       1.0       0.
$
$   PROPERTY DEFINITIONS
$
PBAR     1           1       1.0+4    8.333+6  8.333+6  0.0
$
$   MATERIAL DATA
$
MAT1     1           2.07+5          0.3      8.0-9
$
$   EIGENVALUES DATA
$
EIGR     4           SUB                      4      MAX
$
ENDDATA
```

---

## 2.10.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A010  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 8
NUMBER OF LOCAL COORDINATE SYSTEMS: 1
NUMBER OF CBAR ELEMENTS: 4
```



TOTAL NUMBER OF NON RIGID ELEMENTS: 4  
 NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 8

#### LOAD CASES SUMMARY

NUMBER OF USER FREQUENCY LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 2.399998E-01  
 SYSTEM VOLUME 2.999997E+07  
 SYSTEM MASS/VOLUME 8.000000E-09

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM

LOADCASE 4

#### E I G E N V A L U E S

| MODE | CYCLES       | EIGENVALUE   | RADIANS      | GENERALIZED MASS | GENERALIZED STIFFNESS |
|------|--------------|--------------|--------------|------------------|-----------------------|
| 1    | 9.128321E+00 | 3.289588E+03 | 5.735493E+01 | 6.001553E-02     | 1.974264E+02          |
| 2    | 5.719729E+01 | 1.291548E+05 | 3.593812E+02 | 5.994689E-02     | 7.742430E+03          |
| 3    | 1.608667E+02 | 1.021626E+06 | 1.010755E+03 | 5.897686E-02     | 6.025233E+04          |
| 4    | 3.163879E+02 | 3.951841E+06 | 1.987924E+03 | 6.188160E-02     | 2.445463E+05          |

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 VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM

LOADCASE 4

#### E I G E N V E C T O R

EIGENVECTOR NUMBER 1 FREQUENCY (HZ) = 9.128321E+00  
 EIGENVALUE = 3.289588E+03

| GRID ID | T1           | T2           | T3           |
|---------|--------------|--------------|--------------|
|         | R1           | R2           | R3           |
| 1       | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

|     |                              |                               |                              |
|-----|------------------------------|-------------------------------|------------------------------|
|     | 0.000000E+00                 | 0.000000E+00                  | 0.000000E+00                 |
| 2   | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-2.426782E-04 | 9.727488E-02<br>0.000000E+00 |
| 3   | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-3.876783E-04 | 3.395039E-01<br>0.000000E+00 |
| 4   | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-4.490414E-04 | 6.577331E-01<br>0.000000E+00 |
| 5   | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-4.588620E-04 | 1.000000E+00<br>0.000000E+00 |
| 101 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 102 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 103 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM

|                       |                              |                               | LOADCASE                      | 4 |
|-----------------------|------------------------------|-------------------------------|-------------------------------|---|
| E I G E N V E C T O R |                              |                               |                               |   |
| EIGENVECTOR NUMBER    | 2                            | FREQUENCY (HZ) =              | 5.719729E+01                  |   |
|                       |                              | EIGENVALUE =                  | 1.291548E+05                  |   |
| GRID ID               | T1<br>R1                     | T2<br>R2                      | T3<br>R3                      |   |
| 1                     | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |   |
| 2                     | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>7.624584E-04  | -4.172720E-01<br>0.000000E+00 |   |
| 3                     | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-1.502159E-04 | -7.141837E-01<br>0.000000E+00 |   |
| 4                     | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-1.297497E-03 | -1.358161E-01<br>0.000000E+00 |   |
| 5                     | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-1.595286E-03 | 1.000000E+00<br>0.000000E+00  |   |
| 101                   | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |   |
| 102                   | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |   |

|     |              |              |              |
|-----|--------------|--------------|--------------|
| 103 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|     | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM

LOADCASE 4

E I G E N V E C T O R

EIGENVECTOR NUMBER 3 FREQUENCY (HZ) = 1.608667E+02  
 EIGENVALUE = 1.021626E+06

| GRID ID | T1<br>R1                     | T2<br>R2                      | T3<br>R3                      |
|---------|------------------------------|-------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |
| 2       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-5.340301E-04 | 7.276161E-01<br>0.000000E+00  |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>1.853566E-03  | 2.429590E-02<br>0.000000E+00  |
| 4       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-8.777811E-04 | -5.853796E-01<br>0.000000E+00 |
| 5       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-2.631730E-03 | 1.000000E+00<br>0.000000E+00  |
| 101     | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |
| 102     | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |
| 103     | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00  |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 8  
 VIBRATIONS OF CANTILEVER BEAM IN A SKEWED COORDINATE SYSTEM

LOADCASE 4

E I G E N V E C T O R

EIGENVECTOR NUMBER 4 FREQUENCY (HZ) = 3.163879E+02  
 EIGENVALUE = 3.951841E+06

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

|     |              |               |               |
|-----|--------------|---------------|---------------|
| 2   | 0.000000E+00 | 0.000000E+00  | -7.221294E-01 |
|     | 0.000000E+00 | -1.095162E-03 | 0.000000E+00  |
| 3   | 0.000000E+00 | 0.000000E+00  | 7.433052E-01  |
|     | 0.000000E+00 | -7.855713E-05 | 0.000000E+00  |
| 4   | 0.000000E+00 | 0.000000E+00  | -6.382017E-01 |
|     | 0.000000E+00 | 9.580850E-04  | 0.000000E+00  |
| 5   | 0.000000E+00 | 0.000000E+00  | 1.000000E+00  |
|     | 0.000000E+00 | -3.748253E-03 | 0.000000E+00  |
| 101 | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
|     | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
| 102 | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
|     | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
| 103 | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
|     | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.11 Beam with Temperature Load

### Example ID:

A011

### Analysis Feature Used:

GRID, CBAR, PBAR, MAT1, LINE, LOADCASE, LABEL, SPC, SPC1, TEMPD, TEMP, STRESS, DISP

### Special Features Used:

Single point constraint.

### Problem Statement:

The beam of **Figure 2-12** was installed free of stress with a temperature of 5 degrees. The temperature is then increased to 25 degrees.

For  $T=25$  degrees:

1. Calculate the axial displacement of the beam at node 2 when node 2 is free.
2. Calculate the stresses that would develop in the beam the axial displacement of node 2 is constrained.

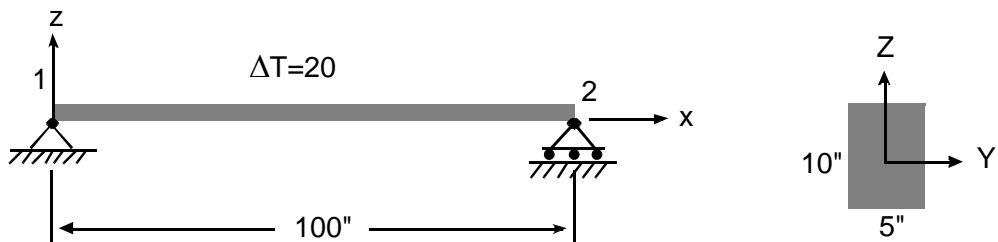


Figure 2-12

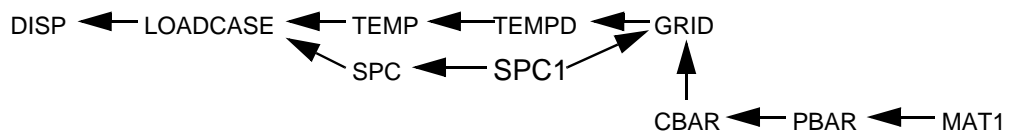
## Structural Analysis Model:

1. One CBAR elements.
2. The section properties are: Area=50 in<sup>2</sup>.  $\alpha = 10\text{E-}6$  in/degree.
3. Material: E=1.0E7 psi.

Two load cases:

1. Change of temperature.
1. Change of temperature and lateral constraint (SPC).

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE

## Reference Solutions:

The analytical solutions for a beam under change of temperature are:

$$U_x = \alpha L(T - T_0) = 0.02 \text{ in. (free to move)}$$

$$\sigma = -E\alpha(T - T_0) = -2000.0 \text{ psi (constrained to move)}$$

where E is Young's modulus,  $\alpha$  is the thermal expansion coefficient, L is the length and  $(T - T_0)$  is the difference between the final temperature and initial temperature.

CALCULATED SOLUTIONS:

$$U_x = 0.02 \text{ in.}$$

$$\sigma = -2000.0 \text{ psi}$$

## Comparison Between Calculated Solutions and Reference Solutions:

There are no differences between analytical solutions and the *GENESIS* solutions.

---

## 2.11.1 Input Data

```
ID A011
SOL COMPAT1
CEND
TITLE= BEAM UNDER CHANGES OF TEMPERATURE
LINE=64,80
LOADCASE 1
LABEL= LOAD CASE 1: FREE RIGHT DISPLACEMENT
    TEMP=1
    DISPLACEMENT=ALL
LOADCASE 2
LABEL= LOAD CASE 2: CONSTRAIN RIGHT DISPLACEMENT
    TEMP=1
    SPC=1
    STRESS=ALL
BEGIN BULK
$
$   BOUNDARY CONDITIONS FOR LOAD CASE 2
$
SPC1    1      1      2
$
$   GRID DATA
$
GRID    1      0.0    0.0    0.0      12346
GRID    2     100.0    0.0    0.0      2346
$
$   ELEMENT DEFINITIONS
$
CBAR    1      1      1      2      0.    1.0    0.
+        0.    .0    0.0    0.0    0.0    0.0
$
$   PROPERTY DEFINITIONS
$
PBAR    1      1      50.0    104.2    416.7    288.5    0.0
+        2.5    5.0    2.5    -5.0    -2.5    -5.0    -2.5    5.0
$
$   MATERIAL DATA
$
MAT1    1      10.0+6      0.3    0.0    10.-6    5.0
$
$   TEMPERATURE DATA
$
TEMPD   1      25.0
$
ENDDATA
```

---

## 2.11.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A011

RUN STARTED: Dec 13, 2002 12:49

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
BEAM UNDER CHANGES OF TEMPERATURE

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 2
NUMBER OF CBAR ELEMENTS: 1
TOTAL NUMBER OF NON RIGID ELEMENTS: 1
```



|                               |   |
|-------------------------------|---|
| NUMBER OF ELEMENT PROPERTIES: | 1 |
| NUMBER OF MATERIALS:          | 1 |
| NUMBER OF DEGREES OF FREEDOM: | 3 |

#### LOAD CASES SUMMARY

|                              |   |
|------------------------------|---|
| NUMBER OF STATIC LOAD CASES: | 2 |
| TOTAL NUMBER OF LOAD CASES:  | 2 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
BEAM UNDER CHANGES OF TEMPERATURE

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
BEAM UNDER CHANGES OF TEMPERATURE

#### MASS / VOLUME SUMMARY

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 0.000000E+00 |
| SYSTEM VOLUME      | 5.000000E+03 |
| SYSTEM MASS/VOLUME | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
BEAM UNDER CHANGES OF TEMPERATURE

#### SOLUTION RESIDUALS

|          |   |            |              |                 |              |
|----------|---|------------|--------------|-----------------|--------------|
| LOADCASE | 1 | RESIDUAL : | 0.000000E+00 | STRAIN ENERGY : | 1.000000E+03 |
| LOADCASE | 2 | RESIDUAL : | 0.000000E+00 | STRAIN ENERGY : | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
BEAM UNDER CHANGES OF TEMPERATURE

#### LOAD RESULTS

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 2        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
BEAM UNDER CHANGES OF TEMPERATURE

#### SPCFORCE RESULTS

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 2        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

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BEAM UNDER CHANGES OF TEMPERATURE

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        | 2       | 1.0000E+05 | 1.0000E+05 | 0.0000E+00 | 0.0000E+00 |
| 2        | 2       | 1.0000E+05 | 1.0000E+05 | 0.0000E+00 | 0.0000E+00 |

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BEAM UNDER CHANGES OF TEMPERATURE

# M A X I M U M   S P C   F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 2        | 1       | 1.0000E+05 | 1.0000E+05 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 9  
BEAM UNDER CHANGES OF TEMPERATURE

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        | 2       | 2.0000E-02 | 2.0000E-02 | 0.0000E+00 | 0.0000E+00 |
| 2        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

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BEAM UNDER CHANGES OF TEMPERATURE

LOAD CASE 1: FREE RIGHT DISPLACEMENT LOADCASE 1

# G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 2.000000E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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BEAM UNDER CHANGES OF TEMPERATURE

LOAD CASE 2: CONSTRAIN RIGHT DISPLACEMENT LOADCASE 2

# S T R E S S E S   I N   B A R   E L E M E N T S

| BAR ID | END | STRESS-C | STRESS-D | STRESS-E | STRESS-F |
|--------|-----|----------|----------|----------|----------|
|--------|-----|----------|----------|----------|----------|

```
1      A  -2.000000E+03  -2.000000E+03  -2.000000E+03  -2.000000E+03
      B  -2.000000E+03  -2.000000E+03  -2.000000E+03  -2.000000E+03
```

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.12 Portal Frame with Flexible Nodes

### Example ID:

A012

### Analysis Data Used:

GRID, GRDSET, CBAR, CELAS1, PBAR, PELAS, MAT1, LOAD, FORCE, PLOADA, SPC, SPC1, MPC, MPC, FORCE, SPCFORCES

### Special Features Used:

Utilization of multi-point constraints (MPC) and scalar springs (CELAS) to model a flexible node.

### Problem Statement:

Find the internal forces and reaction forces of the portal frame with angular flexible nodes. Consider that the angular flexibility of the nodes is  $f = L/(12EI)$ .

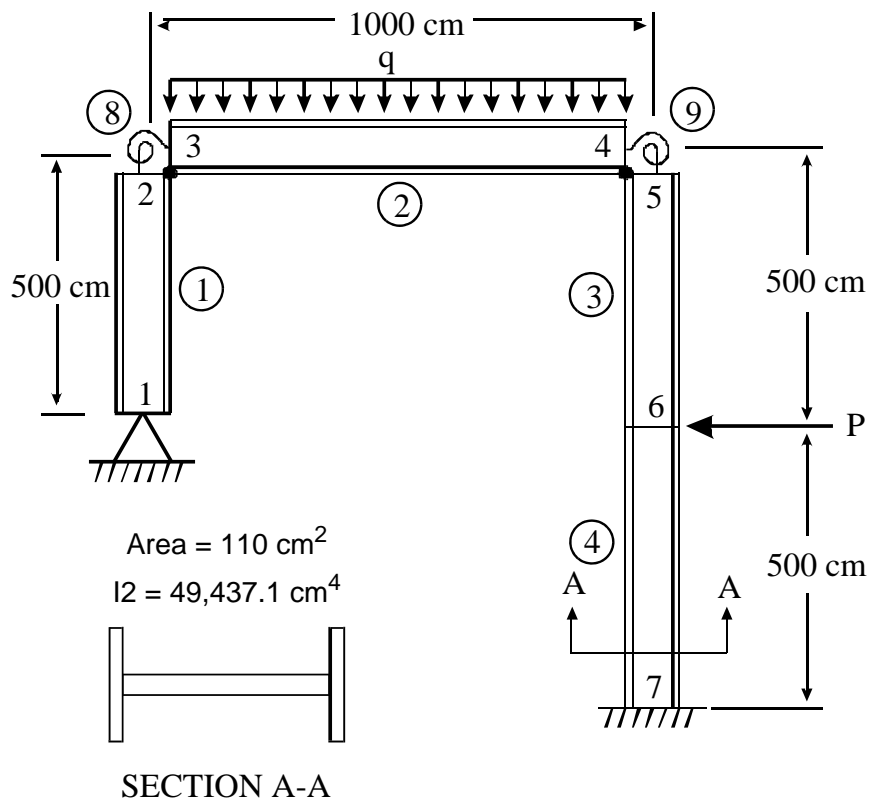
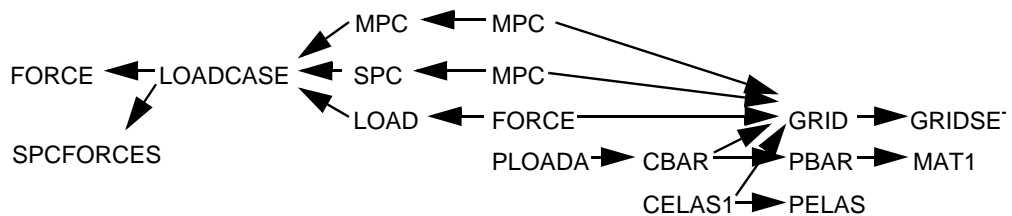


Figure 2-13

## Structural Analysis Model:

1. Planar frame assembled with 4 CBAR elements and two CELAS elements.
2. Section properties of RAIL: Area=110.0cm<sup>2</sup>, I2=49437.1 cm<sup>4</sup>.  
of CELAS:  $k = \frac{12EI}{L} = 4.188\text{E}+9\text{N-cm}$ .
3. Material: Aluminum Alloy, E=7.06E+6 N/cm<sup>2</sup>, ν=0.33.
4. One load case: P=10,000.0 N and q=10 N/cm.
5. Compatibility: U<sub>x</sub>(GRID 2)-U<sub>x</sub>(GRID 3)=0, U<sub>y</sub>(GRID 2)-U<sub>y</sub>(GRID 3)=0,  
U<sub>x</sub>(GRID 4)-U<sub>x</sub>(GRID 5)=0, U<sub>y</sub>(GRID 4)-U<sub>y</sub>(GRID 5)=0.

## Analysis Data Relationships:



## Special Modeling Techniques:

The angular flexibility of the upper left node was modeled by creating two grids for that node (GRID 2 and GRID 3). Then bars 1 and 2 were connected to the GRIDS 2 and 3 respectively. Then the CELAS element (EID=8) was connected to grid 2 and 3. Finally, to achieve displacement compatibilities, two MPC equations were written. The angular flexibility of the upper right node was modeled in a similar way.

## Reference Solutions:

A solution of this problem was obtained using the manual force method.

$$M1a = 0.0$$

$$M1b = -\frac{LX_1}{2} = -1408935 \text{ N-cm}$$

$$M2a = -\frac{LX_1}{2} = -1408935 \text{ N-cm}$$

$$M2b = -\frac{LX_1}{2} + LX_2 - \frac{qL^2}{2} = -240550 \text{ N-cm}$$

$$M3a = -\frac{LX_1}{2} + LX_2 - \frac{qL^2}{2} = -240550 \text{ N-cm}$$

$$M4b = -\frac{LX_1}{2} + LX_2 - \frac{qL^2}{2} = -240550 \text{ N-cm}$$

$$X1 = 2817.87 \text{ N}$$

$$X2 = 6168.38 \text{ N}$$

To get this solution, P was written as  $P=qL$ .  $X1$  and  $X2$  were selected as the unknowns forces. These unknown forces correspond to the reaction forces at grid number 1.

## Calculated Solutions:

$$M1a = 0.0 \text{ N-cm}$$

$$M1b = -1407252 \text{ N-cm}$$

$$M2a = -1407252 \text{ N-cm}$$

$$M2b = -2393214 \text{ N-cm}$$

$$M3a = -2393214 \text{ N-cm}$$

$$M4b = -2424818 \text{ N-cm}$$

$$X1 = 2814.50 \text{ N}$$

$$X2 = 6167.93 \text{ N}$$

## Comparison Between Calculated Solutions and Reference Solutions:

The differences between analytical solutions and the *GENESIS* solutions are:

M1a: 0%

M1b: 0.1%

M2a: 0.1%

M2b: 0.5%

M3a: 0.5%

M4b: 0.1%

X1: 0.1%

X2: 0.01%

---

## 2.12.1 Input Data

```
ID A012
SOL COMPAT1
CEND
TITLE = PORTAL FRAME WITH FLEXIBLE NODES
$ECHO=NONE
LINE=64,80
SET 1000 =1,7
SUBCASE 1
    LOAD = 300
    SPC = 100
    MPC = 100
    LABEL = A FORCE AT NODE 3
    FORCE = ALL
    SPCFORCES = 1000
$
BEGIN BULK
$
$    BOUNDARY CONDITIONS
$
SPC1    100    12345    1
SPC1    100    123456    7
MPC     100    2        1        1.0    3        1        -1.0
MPC     100    2        2        1.0    3        2        -1.0
$
MPC     100    4        1        1.0    5        1        -1.0
MPC     100    4        2        1.0    5        2        -1.0
$
$    GRID POINTS
$
GRDSET                                     345
GRID    1                0.0    500.    0.0
GRID    2                0.0    1000.    0.0
GRID    3                0.0    1000.    0.0
GRID    4                1000.    1000.    0.0
GRID    5                1000.    1000.    0.0
GRID    6                1000.    500.    0.0
GRID    7                1000.    0.0    0.0
$
$    ELEMENT DEFINITIONS
$
CBAR    1        1        1        2        0.0    0.0    -1.0
CBAR    2        1        3        4        0.0    0.0    -1.0
CBAR    3        1        5        6        0.0    0.0    -1.0
CBAR    4        1        6        7        0.0    0.0    -1.0
$
$    SCALAR SPRING
$
CELAS1  8        2        2        6        3        6
CELAS1  9        3        4        6        5        6
$
$    PROPERTY DEFINITIONS
$
```



```

PBAR      1      51      110.0    4504.0  49437.1
+P1       15.0    26.0     0.0     26.0    0.0     0.0     0.00    -26.0
PELAS     2      4.188+9          1.0
PELAS     3      4.188+9          1.0
$PELAS    3      2.792+9          1.0
$
$      MATERIAL DEFINITIONS
$
MAT1      51      7.06E6          0.33
$
$      LOAD DEFINITIONS
$
FORCE     300     6          -10000.0  1.0     0.0     0.0
PLOADA    300     2          0.0      -1.0     0.0    10.0    10.0
$
ENDDATA

```

---

## 2.12.2 Output

1

```
  GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N  E      S      I      S
G  GG  EEEE   N N N  EEEE   SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A012  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
PORTAL FRAME WITH FLEXIBLE NODES

### ANALYSIS PROBLEM SUMMARY

|                              |   |
|------------------------------|---|
| NUMBER OF GRID POINTS:       | 7 |
| NUMBER OF CBAR ELEMENTS:     | 4 |
| NUMBER OF CELAS1/2 ELEMENTS: | 2 |

TOTAL NUMBER OF NON RIGID ELEMENTS: 6  
 NUMBER OF ELEMENT PROPERTIES: 3  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 16

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 PORTAL FRAME WITH FLEXIBLE NODES

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 PORTAL FRAME WITH FLEXIBLE NODES

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 2.750000E+05  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 PORTAL FRAME WITH FLEXIBLE NODES

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 5.403643E-14 ; STRAIN ENERGY : 2.231866E+03  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 PORTAL FRAME WITH FLEXIBLE NODES

#### O L O A D R E S U L T A N T S

| LOADCASE | T1          | T2          | T3         | R1         | R2         | R3         |
|----------|-------------|-------------|------------|------------|------------|------------|
| 1        | -1.0000E+04 | -1.0000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 PORTAL FRAME WITH FLEXIBLE NODES

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3          |
|----------|------------|------------|------------|------------|------------|-------------|
| 1        | 1.0000E+04 | 1.0000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -9.3132E-10 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 PORTAL FRAME WITH FLEXIBLE NODES

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE                         | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|----------------------------------|-------------|------------|-----------------|------------|------------|---|
| 1                                | 6           | 1.0000E+04 | -1.0000E+04     | 0.0000E+00 | 0.0000E+00 |   |
| 1GENESIS                         | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 8 |
| PORTAL FRAME WITH FLEXIBLE NODES |             |            |                 |            |            |   |

# M A X I M U M   S P C   F O R C E

| LOADCASE                         | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|----------------------------------|-------------|------------|-----------------|------------|------------|---|
| 1                                | 7           | 8.1435E+03 | 7.1855E+03      | 3.8321E+03 | 0.0000E+00 |   |
| 1GENESIS                         | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 9 |
| PORTAL FRAME WITH FLEXIBLE NODES |             |            |                 |            |            |   |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE                         | GRID ID     | MAGNITUDE  | T1              | T2          | T3         |    |
|----------------------------------|-------------|------------|-----------------|-------------|------------|----|
| 1                                | 4           | 6.3940E-01 | -6.3938E-01     | -4.9344E-03 | 0.0000E+00 |    |
| 1GENESIS                         | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49  | PAGE       | 10 |
| PORTAL FRAME WITH FLEXIBLE NODES |             |            |                 |             |            |    |

A FORCE AT NODE 3 LOADCASE 1

# R E A C T I O N   F O R C E S

| ID                               | F1<br>M1                     | F2<br>M2                     | F3<br>M3                      |         |
|----------------------------------|------------------------------|------------------------------|-------------------------------|---------|
| 1                                | 2.814503E+03<br>0.000000E+00 | 6.167930E+03<br>0.000000E+00 | 0.000000E+00<br>-1.746230E-09 |         |
| 7                                | 7.185497E+03<br>0.000000E+00 | 3.832070E+03<br>0.000000E+00 | 0.000000E+00<br>-2.424818E+06 |         |
| 1GENESIS                         | VERSION 7.2                  | DATE 12-13-2002              | TIME 12:49                    | PAGE 11 |
| PORTAL FRAME WITH FLEXIBLE NODES |                              |                              |                               |         |

A FORCE AT NODE 3 LOADCASE 1

# F O R C E S   I N   E L A S 1 / 2   E L E M E N T S

| ELAS1 ID                         | FORCE        |  |
|----------------------------------|--------------|--|
| 8                                | 1.407252E+06 |  |
| 9                                | 2.393214E+05 |  |
| 1GENESIS                         | VERSION 7.2  | DATE 12-13-2002   TIME 12:49   PAGE 12 |
| PORTAL FRAME WITH FLEXIBLE NODES |              |  |

## F O R C E S   I N   B A R   E L E M E N T S

| BAR ID | END | BENDING MOMENT 1<br>SHEAR FORCE 1 | BENDING MOMENT 2<br>SHEAR FORCE 2 | AXIAL FORCE<br>TORQUE |
|--------|-----|-----------------------------------|-----------------------------------|-----------------------|
| 1      | A   | 0.000000E+00                      | 1.746230E-09                      | -6.167930E+03         |
|        |     | 0.000000E+00                      | 2.814503E+03                      | 0.000000E+00          |
|        | B   | 0.000000E+00                      | -1.407252E+06                     | -6.167930E+03         |
|        |     | 0.000000E+00                      | 2.814503E+03                      | 0.000000E+00          |
| 2      | A   | 0.000000E+00                      | -1.407252E+06                     | -2.814503E+03         |
|        |     | 0.000000E+00                      | -6.167930E+03                     | 0.000000E+00          |
|        | B   | 0.000000E+00                      | -2.393214E+05                     | -2.814503E+03         |
|        |     | 0.000000E+00                      | 3.832070E+03                      | 0.000000E+00          |
| 3      | A   | 0.000000E+00                      | -2.393214E+05                     | -3.832070E+03         |
|        |     | 0.000000E+00                      | -2.814503E+03                     | 0.000000E+00          |
|        | B   | 0.000000E+00                      | 1.167930E+06                      | -3.832070E+03         |
|        |     | 0.000000E+00                      | -2.814503E+03                     | 0.000000E+00          |
| 4      | A   | 0.000000E+00                      | 1.167930E+06                      | -3.832070E+03         |
|        |     | 0.000000E+00                      | 7.185497E+03                      | 0.000000E+00          |
|        | B   | 0.000000E+00                      | -2.424818E+06                     | -3.832070E+03         |
|        |     | 0.000000E+00                      | 7.185497E+03                      | 0.000000E+00          |

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.13 Beam Supported by Springs

### Example ID:

A013

### Analysis Data Used:

GRID, GRDSET, CBAR, CELAS1, PBAR, PELAS, MAT1, LOADCASE, LOAD, FORCE, FORCE (SOLUTION CONTROL)

### Special Features Used:

Utilization of scalar springs (CELAS).

### Problem Statement:

Find the moment at the center of the beam of **Figure 2-14**. Consider the rigidity of the external springs as  $k=1.0E7$  N/cm.

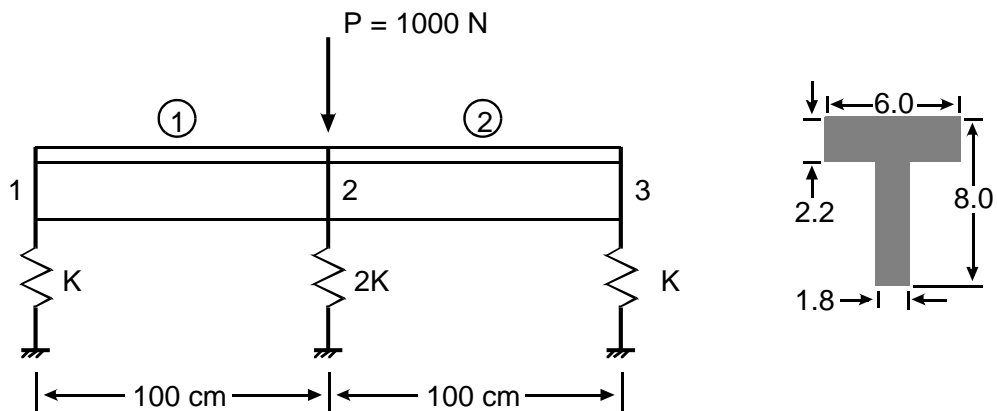
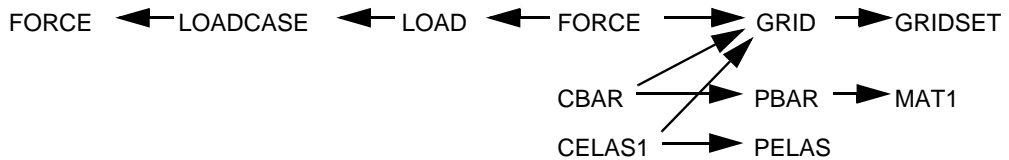


Figure 2-14

### Structural Analysis Model:

1. Planar structure assembled with 2 CBAR elements and three CELAS1 elements.
2. Section properties of T BEAM:  $\text{Area}=23.64 \text{ cm}^2$ ,  $I_2=42.41 \text{ cm}^4$ .
3. Stiffness of CELAS:  $k=1.0E+7$  N/cm.
4. Material: Aluminum Alloy,  $E=7.06E+6$  N/cm<sup>2</sup>,  $\nu=0.33$ .
5. One load case:  $P=1,000.0$  N.

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE

## Reference Solutions:

The analytical solution for this problem is:

$$M = \frac{P}{2\left(6EI + \frac{KL^3}{8}\right)} = 4.5084 \text{ N-cm}$$

where P is the applied load, E is the Young's modulus, I the area moment of inertia, k is the spring stiffness and L is the total length of the beam.

## Calculated Solutions:

$$M = 4.5084 \text{ N-cm}$$

## Comparison Between Calculated Solutions and Reference Solution:

There are no differences between the analytical solution and the *GENESIS* solution.

---

## 2.13.1 Input Data

```
ID A013
SOL COMPAT1
CEND
TITLE = BEAM SUPPORTED BY SPRINGS
LINE=64,80
LOADCASE 1
      LOAD = 300
      FORCE = ALL

$
BEGIN BULK
$
$   GRID POINTS
$
GRDSET                                     345
$
GRID   1           0.0    0.    0.0
GRID   2           100.0  0.    0.0      1345
GRID   3           200.0  0.    0.0
GRID   4           0.0   -1.    0.0      123456
GRID   5           100.0  -1.    0.0      123456
GRID   6           200.0  -1.    0.0      123456
$
$   ELEMENT DEFINITIONS
CBAR   1           1      1      2      0.0    0.0    -1.0
CBAR   2           1      2      3      0.0    0.0    -1.0
$
$   SCALAR SPRING
$
CELAS1  3           2      4      2      1      2
CELAS1  4           3      5      2      2      2
CELAS1  5           2      6      2      3      2
$
$   PROPERTY DEFINITIONS
$
PBAR   1           51      23.64  10.44  127.74
PELAS  2           3.+7      1.0
PELAS  3           6.+7      1.0
$
$   MATERIAL DEFINITIONS
$
MAT1   51           7.06E6      0.33
$
$   LOAD DEFINITIONS
$
FORCE  300         2           -1000.0  0.0    1.0    0.0
$
ENDDATA
```



---

## 2.13.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A013  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
BEAM SUPPORTED BY SPRINGS

### ANALYSIS PROBLEM SUMMARY

|                              |   |
|------------------------------|---|
| NUMBER OF GRID POINTS:       | 6 |
| NUMBER OF CBAR ELEMENTS:     | 2 |
| NUMBER OF CELAS1/2 ELEMENTS: | 3 |

|                                     |   |
|-------------------------------------|---|
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 5 |
| NUMBER OF ELEMENT PROPERTIES:       | 3 |
| NUMBER OF MATERIALS:                | 1 |
| NUMBER OF DEGREES OF FREEDOM:       | 8 |

#### LOAD CASES SUMMARY

|                              |   |
|------------------------------|---|
| NUMBER OF STATIC LOAD CASES: | 1 |
| TOTAL NUMBER OF LOAD CASES:  | 1 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
BEAM SUPPORTED BY SPRINGS

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
BEAM SUPPORTED BY SPRINGS

#### M A S S / V O L U M E S U M M A R Y

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 0.000000E+00 |
| SYSTEM VOLUME      | 4.728000E+03 |
| SYSTEM MASS/VOLUME | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
BEAM SUPPORTED BY SPRINGS

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -2.273204E-16 ; STRAIN ENERGY : 8.332582E-03  
1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
BEAM SUPPORTED BY SPRINGS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2          | T3         | R1         | R2         | R3          |
|----------|------------|-------------|------------|------------|------------|-------------|
| 1        | 0.0000E+00 | -1.0000E+03 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -1.0000E+05 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
BEAM SUPPORTED BY SPRINGS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 0.0000E+00 | 1.0000E+03 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 1.0000E+05 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
BEAM SUPPORTED BY SPRINGS

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE                  | GRID ID     | MAGNITUDE  | T1              | T2          | T3         |   |
|---------------------------|-------------|------------|-----------------|-------------|------------|---|
| 1                         | 2           | 1.0000E+03 | 0.0000E+00      | -1.0000E+03 | 0.0000E+00 |   |
| 1GENESIS                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49  | PAGE       | 8 |
| BEAM SUPPORTED BY SPRINGS |             |            |                 |             |            |   |

# M A X I M U M   S P C   F O R C E

| LOADCASE                  | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|---------------------------|-------------|------------|-----------------|------------|------------|---|
| 1                         | 5           | 9.9991E+02 | 0.0000E+00      | 9.9991E+02 | 0.0000E+00 |   |
| 1GENESIS                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 9 |
| BEAM SUPPORTED BY SPRINGS |             |            |                 |            |            |   |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE                  | GRID ID     | MAGNITUDE  | T1              | T2          | T3         |    |
|---------------------------|-------------|------------|-----------------|-------------|------------|----|
| 1                         | 2           | 1.6665E-05 | 0.0000E+00      | -1.6665E-05 | 0.0000E+00 |    |
| 1GENESIS                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49  | PAGE       | 10 |
| BEAM SUPPORTED BY SPRINGS |             |            |                 |             |            |    |

LOADCASE 1

# F O R C E S   I N   E L A S 1/2   E L E M E N T S

| ELAS1 ID                  | FORCE        |  |
|---------------------------|--------------|--|
| 3                         | 4.508409E-02 |  |
| 4                         | 9.999098E+02 |  |
| 5                         | 4.508409E-02 |  |
| 1GENESIS                  | VERSION 7.2  | DATE 12-13-2002   TIME 12:49   PAGE 11 |
| BEAM SUPPORTED BY SPRINGS |              |  |

LOADCASE 1

# F O R C E S   I N   B A R   E L E M E N T S

| BAR ID | END | BENDING MOMENT 1<br>SHEAR FORCE 1 | BENDING MOMENT 2<br>SHEAR FORCE 2 | AXIAL FORCE<br>TORQUE |
|--------|-----|-----------------------------------|-----------------------------------|-----------------------|
| 1      | A   | 0.000000E+00                      | 0.000000E+00                      | 0.000000E+00          |
|        |     | 0.000000E+00                      | -4.508409E-02                     | 0.000000E+00          |
|        | B   | 0.000000E+00                      | 4.508409E+00                      | 0.000000E+00          |
|        |     | 0.000000E+00                      | -4.508409E-02                     | 0.000000E+00          |
| 2      | A   | 0.000000E+00                      | 4.508409E+00                      | 0.000000E+00          |

|   |              |              |              |
|---|--------------|--------------|--------------|
|   | 0.000000E+00 | 4.508409E-02 | 0.000000E+00 |
| B | 0.000000E+00 | 1.776357E-15 | 0.000000E+00 |
|   | 0.000000E+00 | 4.508409E-02 | 0.000000E+00 |

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.14 Planar Grillage with Internal Pin

### Example ID:

A014

### Analysis Data Used:

GRID, CBAR, PBAR, MAT1, LOADCASE, LOAD, FORCE, MOMENT, DISP

### Special Features Used:

Utilization of PIN FLAGS of CBAR elements to represent an internal pin in the planar grillage.

### Problem Statement:

Find the angular displacement and internal moments of the elements of the planar grillage shown in **Figure 2-15**.

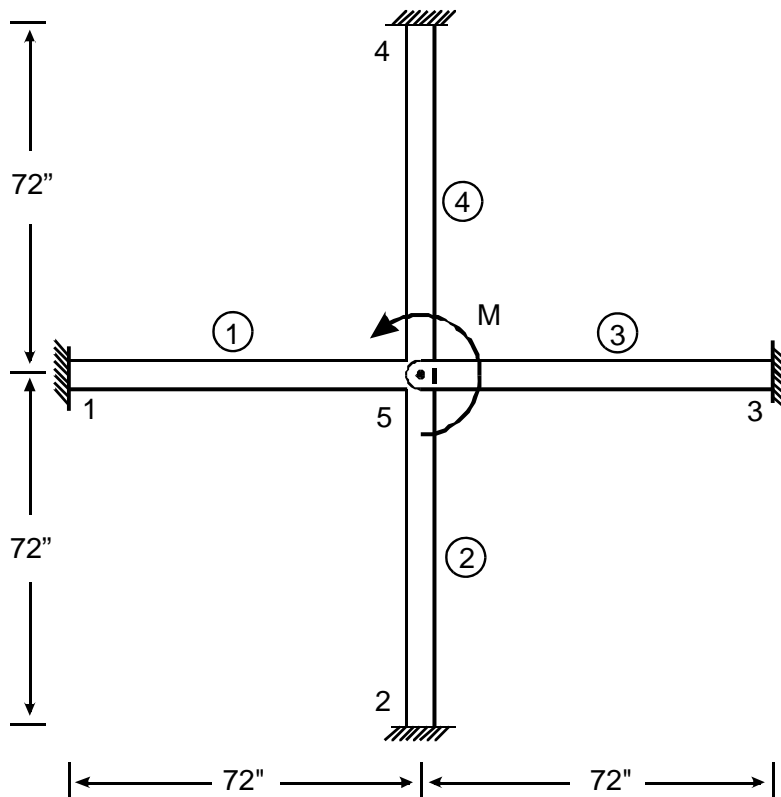
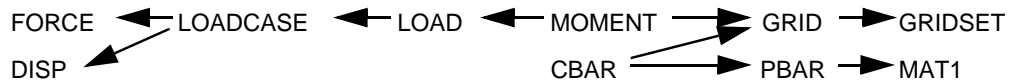


Figure 2-15

## Structural Analysis Model:

1. Planar grillage assembled with 4 CBAR elements.
2. Section properties of BEAMS: Area=1.0 in<sup>2</sup>, I2=0.0833 in<sup>4</sup>.
3. Material: E=6.902E+6 psi.
4. One Load cases: M=10,000.0 in-lb.

## Analysis Data Relationships:



## Special Modeling Techniques:

The internal pin of the grillage is represented in the CBAR (EID=3) data as PA=5. Notice that the degree of freedom being condensed is in the element coordinate system (not in the general coordinate system, nor in the basic coordinate system).

## Reference Solutions:

The analytical solutions for this problem are:

$$\theta = \frac{ML}{12EI} = 0.1043 \text{ Rad}$$

$$M1a = M2a = M4a = -\frac{4EI\theta}{L} = -3333.3 \text{ in-lb}$$

$$M1b = M2b = M4b = -\frac{2EI\theta}{L} = -1666.7 \text{ in-lb}$$

$$M3a = M3b = 0.0$$

where: M is the applied load, E is the Young's modulus, I the area moment of inertia and L is the total length of each of the elements.

### Calculated Solution:

The *GENESIS* solution is:

$$\theta = 0.1043 \text{ Rad}$$

$$M1a = M2a = M4a = -3333.3 \text{ in-lb}$$

$$M1b = M2b = M4b = -1666.7 \text{ in-lb}$$

$$M3a = M3b = 0.0$$

### Comparison Between Calculated Solutions and Reference Solution:

There are no differences between the analytical solution and the *GENESIS* solution.

---

## 2.14.1 Input Data

```
ID A014
SOL COMPAT1
CEND
TITLE = PLANAR GRILLAGE WITH INTERNAL PIN
ECHO=NONE
SET 5=5
LINE=64,80
LOADCASE 1
    LOAD = 100
    DISP=5
    FORCE = ALL
BEGIN BULK
$
$   GRID POINTS
$
GRID   1           -72.0   0.     0.0           123456
GRID   2             0.0  -72.    0.0           123456
GRID   3            72.0   0.     0.0           123456
GRID   4             0.    72.    0.0           123456
GRID   5             0.     0.     0.0           12345
$
$   ELEMENT DEFINITIONS
$
CBAR   1           1       5       1       0.0     0.0     -1.0
CBAR   2           1       5       2       0.0     0.0     -1.0
CBAR   3           1       5       3       0.0     0.0     -1.0
+      5
CBAR   4           1       5       4       0.0     0.0     -1.0
$
$   PROPERTY DEFINITIONS
$
PBAR   1           51       1.0     .08333  0.08333
$
$   MATERIAL DEFINITIONS
$
MAT1   51           6.902+6       0.33    1.6636-4
$
$   LOAD DEFINITIONS
$
MOMENT 100         5           10000.00.0     0.0     1.0
$
ENDDATA
```



---

## 2.14.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A014  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
PLANAR GRILLAGE WITH INTERNAL PIN

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 5
NUMBER OF CBAR ELEMENTS: 4
TOTAL NUMBER OF NON RIGID ELEMENTS: 4
```

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 1

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 PLANAR GRILLAGE WITH INTERNAL PIN

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 PLANAR GRILLAGE WITH INTERNAL PIN

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 4.791168E-02  
 SYSTEM VOLUME 2.880000E+02  
 SYSTEM MASS/VOLUME 1.663600E-04

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 PLANAR GRILLAGE WITH INTERNAL PIN

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 0.000000E+00 ; STRAIN ENERGY : 5.216088E+02  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 PLANAR GRILLAGE WITH INTERNAL PIN

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 1.0000E+04 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 PLANAR GRILLAGE WITH INTERNAL PIN

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3          |
|----------|------------|------------|------------|------------|------------|-------------|
| 1        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -1.0000E+04 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 PLANAR GRILLAGE WITH INTERNAL PIN

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE                          | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|-----------------------------------|-------------|------------|-----------------|------------|------------|---|
| 1                                 |             | 0.0000E+00 | 0.0000E+00      | 0.0000E+00 | 0.0000E+00 |   |
| 1GENESIS                          | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 8 |
| PLANAR GRILLAGE WITH INTERNAL PIN |             |            |                 |            |            |   |

# M A X I M U M   S P C   F O R C E

| LOADCASE                          | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|-----------------------------------|-------------|------------|-----------------|------------|------------|---|
| 1                                 | 1           | 6.9444E+01 | 0.0000E+00      | 6.9444E+01 | 0.0000E+00 |   |
| 1GENESIS                          | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 9 |
| PLANAR GRILLAGE WITH INTERNAL PIN |             |            |                 |            |            |   |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE                          | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |    |
|-----------------------------------|-------------|------------|-----------------|------------|------------|----|
| 1                                 |             | 0.0000E+00 | 0.0000E+00      | 0.0000E+00 | 0.0000E+00 |    |
| 1GENESIS                          | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 10 |
| PLANAR GRILLAGE WITH INTERNAL PIN |             |            |                 |            |            |    |

LOADCASE 1

# G R I D   D I S P L A C E M E N T S

| GRID ID                           | T1           | T2              | T3           |         |
|-----------------------------------|--------------|-----------------|--------------|---------|
|                                   | R1           | R2              | R3           |         |
| 5                                 | 0.000000E+00 | 0.000000E+00    | 0.000000E+00 |         |
|                                   | 0.000000E+00 | 0.000000E+00    | 1.043218E-01 |         |
| 1GENESIS                          | VERSION 7.2  | DATE 12-13-2002 | TIME 12:49   | PAGE 11 |
| PLANAR GRILLAGE WITH INTERNAL PIN |              |                 |              |         |

LOADCASE 1

# F O R C E S   I N   B A R   E L E M E N T S

| BAR ID | END | BENDING MOMENT 1 | BENDING MOMENT 2 | AXIAL FORCE  |
|--------|-----|------------------|------------------|--------------|
|        |     | SHEAR FORCE 1    | SHEAR FORCE 2    | TORQUE       |
| 1      | A   | 0.000000E+00     | -3.333333E+03    | 0.000000E+00 |
|        |     | 0.000000E+00     | -6.944444E+01    | 0.000000E+00 |
|        | B   | 0.000000E+00     | 1.666667E+03     | 0.000000E+00 |
|        |     | 0.000000E+00     | -6.944444E+01    | 0.000000E+00 |
| 2      | A   | 0.000000E+00     | -3.333333E+03    | 0.000000E+00 |

|   |   |              |               |              |
|---|---|--------------|---------------|--------------|
|   |   | 0.000000E+00 | -6.944444E+01 | 0.000000E+00 |
|   | B | 0.000000E+00 | 1.666667E+03  | 0.000000E+00 |
|   |   | 0.000000E+00 | -6.944444E+01 | 0.000000E+00 |
| 3 | A | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
|   |   | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
|   | B | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
|   |   | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
| 4 | A | 0.000000E+00 | -3.333333E+03 | 0.000000E+00 |
|   |   | 0.000000E+00 | -6.944444E+01 | 0.000000E+00 |
|   | B | 0.000000E+00 | 1.666667E+03  | 0.000000E+00 |
|   |   | 0.000000E+00 | -6.944444E+01 | 0.000000E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.15 Rod Modeled with Tetra Elements

### Example ID:

A015

### Analysis Data Used:

GRID, CTETRA, PSOLID, MAT1, LOADCASE, LOAD, FORCE, SPC, SPC1, STRESS, DISP

### Special Features Used:

NONE. This example shows how to assemble a brick using CTETRA elements

### Problem Statement:

Find the tip displacements and the stresses at the root of a cantilever rod loaded in its free end with a tension load of 4800 lb.

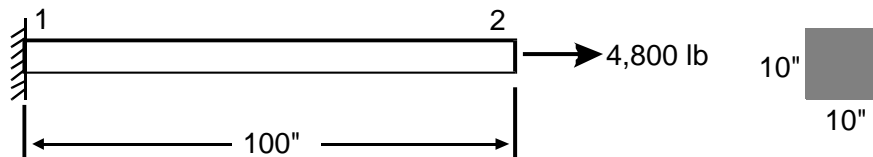


Figure 2-16

### Structural Analysis Model:

1. Tridimensional model of the rod assembled with 30 CTETRA elements.
2. Material:  $E=1.0E+7$ psi.
3. One Load cases:  $P=4800$ lb.

The stresses for solid elements are printed out in the material coordinate system. In this case the material coordinate system is the basic coordinate system (the default). Other coordinate system can be used as the material coordinate system. See the description of the PSOLID element in Volume II for more details.

### Analysis Data Relationships:

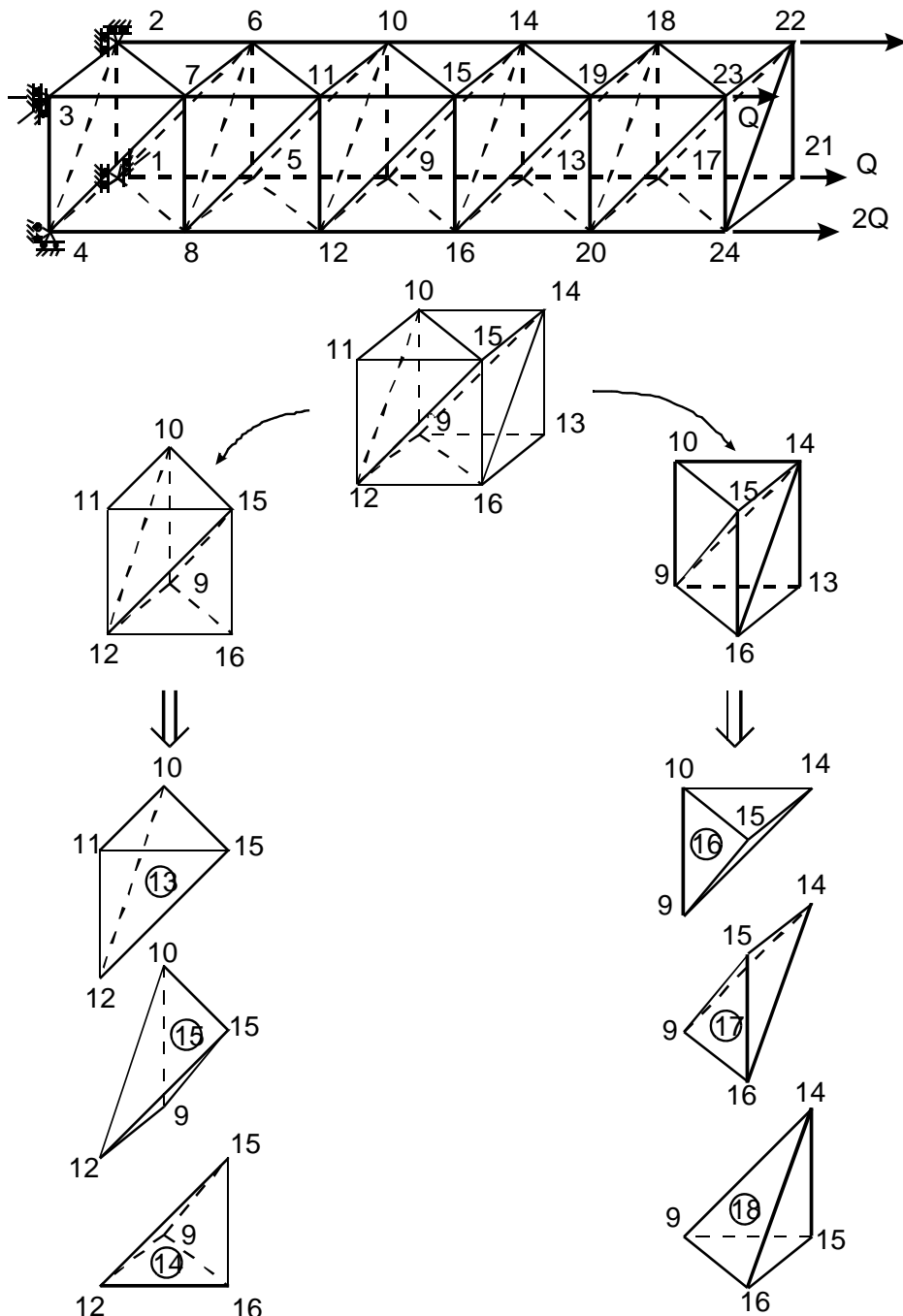


In this example the solution control commands SET 5=21,22,23,24 and SET 6=1,2,3,4,5,6 were used to limit the output.

## Special Modeling Techniques:

It is important to correctly model the boundary conditions. In this problem they have to globally prevent the structure from moving in the six degrees of freedoms of the fixed boundary condition (see figure below). Please note that if all 4 nodes 1,2,3 and 4 were constrained against all movement, slightly different answers will be obtained. Also it is important to load the structure with the correct consistent load vector. Please also note, that if the structure were loaded with 4 loads of 1200 lb in each of the free end corners, incorrect answers would be obtained.

The procedure to assemble a brick with 6 CTETRA elements are shown in **Figure 2-17**. There are alternative ways to do this; another way is to divide the brick in 5 CTETRA elements. See the Design Capabilities discussion (Ch. 4) in Volume I.



**Figure 2-17**

## Reference Solutions:

The analytical solutions for this problem are:

$$U_x = \frac{PL}{EA} = 0.00048 \text{ in}$$

$$\sigma = \frac{P}{A} = 48 \text{ psi}$$

where: P is the applied load, E is the Young's modulus, A the area and L is the total length of the beam.

## Calculated Solution:

The *GENESIS* solution is:

$$U_x = 0.00048 \text{ in}$$

$$\sigma = \frac{P}{A} = 48 \text{ psi}$$

## Comparison Between Calculated Solution and Reference Solution:

There are no differences between the analytical solution and the *GENESIS* solution.



---

## 2.15.1 Input Data

```
ID A015
SOL COMPAT1
CEND
TITLE = TENSION ROD ASSEMBLED WITH TETRA ELEMENTS
LINE=64,80
SET 5=21,22,23,24
SET 6=1,2,3,4,5,6
LOADCASE 1
    LOAD = 42
    FORCE = ALL
    SPC=77
    DISPLACEMENT=5
    STRESS=6

$
BEGIN BULK
$
$    BOUNDARY CONDITIONS
$
SPC1    77        12        1
SPC1    77        13        2
SPC1    77        12        3
SPC1    77        13        4
$
$    GRID POINTS
$
GRID    1          0.0      10.      0.0
GRID    2          0.0      10.      10.0
GRID    3          0.0      0.0      10.0
GRID    4          0.0      0.0      0.0
$
GRID    5          20.0     10.      0.0
GRID    6          20.0     10.      10.0
GRID    7          20.0     0.0      10.0
GRID    8          20.0     0.0      0.0
$
GRID    9          40.0     10.      0.0
GRID   10          40.0     10.      10.0
GRID   11          40.0     0.0      10.0
GRID   12          40.0     0.0      0.0
$
GRID   13          60.0     10.      0.0
GRID   14          60.0     10.      10.0
GRID   15          60.0     0.0      10.0
GRID   16          60.0     0.0      0.0
$
GRID   17          80.0     10.      0.0
GRID   18          80.0     10.      10.0
GRID   19          80.0     0.0      10.0
GRID   20          80.0     0.0      0.0
$
GRID   21         100.0     10.      0.0
GRID   22         100.0     10.      10.0
```

|                         |    |        |        |     |      |     |
|-------------------------|----|--------|--------|-----|------|-----|
| GRID                    | 23 |        | 100.0  | 0.0 | 10.0 |     |
| GRID                    | 24 |        | 100.0  | 0.0 | 0.0  |     |
| \$                      |    |        |        |     |      |     |
| \$ ELEMENT DEFINITIONS  |    |        |        |     |      |     |
| \$                      |    |        |        |     |      |     |
| CTETRA                  | 1  | 1      | 3      | 7   | 2    | 4   |
| CTETRA                  | 2  | 1      | 4      | 8   | 1    | 7   |
| CTETRA                  | 3  | 1      | 2      | 7   | 1    | 4   |
| CTETRA                  | 4  | 1      | 2      | 6   | 7    | 1   |
| CTETRA                  | 5  | 1      | 7      | 6   | 8    | 1   |
| CTETRA                  | 6  | 1      | 8      | 5   | 1    | 6   |
| \$                      |    |        |        |     |      |     |
| CTETRA                  | 7  | 1      | 7      | 11  | 6    | 8   |
| CTETRA                  | 8  | 1      | 8      | 12  | 5    | 11  |
| CTETRA                  | 9  | 1      | 6      | 11  | 5    | 8   |
| CTETRA                  | 10 | 1      | 6      | 10  | 11   | 5   |
| CTETRA                  | 11 | 1      | 11     | 10  | 12   | 5   |
| CTETRA                  | 12 | 1      | 12     | 9   | 5    | 10  |
| \$                      |    |        |        |     |      |     |
| CTETRA                  | 13 | 1      | 11     | 15  | 10   | 12  |
| CTETRA                  | 14 | 1      | 12     | 16  | 9    | 15  |
| CTETRA                  | 15 | 1      | 10     | 15  | 9    | 12  |
| CTETRA                  | 16 | 1      | 10     | 14  | 15   | 9   |
| CTETRA                  | 17 | 1      | 15     | 14  | 16   | 9   |
| CTETRA                  | 18 | 1      | 16     | 13  | 9    | 14  |
| \$                      |    |        |        |     |      |     |
| CTETRA                  | 19 | 1      | 15     | 19  | 14   | 16  |
| CTETRA                  | 20 | 1      | 16     | 20  | 13   | 19  |
| CTETRA                  | 21 | 1      | 14     | 19  | 13   | 16  |
| CTETRA                  | 22 | 1      | 14     | 18  | 19   | 13  |
| CTETRA                  | 23 | 1      | 19     | 18  | 20   | 13  |
| CTETRA                  | 24 | 1      | 20     | 17  | 13   | 18  |
| \$                      |    |        |        |     |      |     |
| CTETRA                  | 25 | 1      | 19     | 23  | 18   | 20  |
| CTETRA                  | 26 | 1      | 20     | 24  | 17   | 23  |
| CTETRA                  | 27 | 1      | 18     | 23  | 17   | 20  |
| CTETRA                  | 28 | 1      | 18     | 22  | 23   | 17  |
| CTETRA                  | 29 | 1      | 23     | 22  | 24   | 17  |
| CTETRA                  | 30 | 1      | 24     | 21  | 17   | 22  |
| \$                      |    |        |        |     |      |     |
| \$ PROPERTY DEFINITIONS |    |        |        |     |      |     |
| \$                      |    |        |        |     |      |     |
| PSOLID                  | 1  | 51     |        |     |      |     |
| \$                      |    |        |        |     |      |     |
| \$ MATERIAL DEFINITIONS |    |        |        |     |      |     |
| \$                      |    |        |        |     |      |     |
| MAT1                    | 51 | 10.0E6 | 0.3    |     |      |     |
| \$                      |    |        |        |     |      |     |
| \$ LOAD DEFINITIONS     |    |        |        |     |      |     |
| \$                      |    |        |        |     |      |     |
| FORCE                   | 42 | 21     | 800.0  | 1.0 | 0.0  | 0.0 |
| FORCE                   | 42 | 22     | 1600.0 | 1.0 | 0.0  | 0.0 |
| FORCE                   | 42 | 23     | 800.0  | 1.0 | 0.0  | 0.0 |
| FORCE                   | 42 | 24     | 1600.0 | 1.0 | 0.0  | 0.0 |
| \$                      |    |        |        |     |      |     |

**ENDDATA**

---

## 2.15.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A015

RUN STARTED: Dec 13, 2002 12:49

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
TENSION ROD ASSEMBLED WITH TETRA ELEMENTS

### ANALYSIS PROBLEM SUMMARY

|                                     |    |
|-------------------------------------|----|
| NUMBER OF GRID POINTS:              | 24 |
| NUMBER OF CTETRA ELEMENTS:          | 30 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 30 |

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 64

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 TENSION ROD ASSEMBLED WITH TETRA ELEMENTS

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 TENSION ROD ASSEMBLED WITH TETRA ELEMENTS

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 1.000000E+04  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 TENSION ROD ASSEMBLED WITH TETRA ELEMENTS

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 1.590384E-15 ; STRAIN ENERGY : 1.152000E+00  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 TENSION ROD ASSEMBLED WITH TETRA ELEMENTS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3          |
|----------|------------|------------|------------|------------|------------|-------------|
| 1        | 4.8000E+03 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 2.4000E+04 | -2.4000E+04 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 TENSION ROD ASSEMBLED WITH TETRA ELEMENTS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1          | T2          | T3         | R1         | R2          | R3         |
|----------|-------------|-------------|------------|------------|-------------|------------|
| 1        | -4.8000E+03 | -1.2830E-12 | 3.0340E-12 | 4.4646E-11 | -2.4000E+04 | 2.4000E+04 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 TENSION ROD ASSEMBLED WITH TETRA ELEMENTS

| LOADCASE                                  | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |  |
|---|-------------|------------|-----------------|------------|------------|---|--|
| 1   | 22          | 1.6000E+03 | 1.6000E+03      | 0.0000E+00 | 0.0000E+00 |   |  |
| 1GENESIS                                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 8 |  |
| TENSION ROD ASSEMBLED WITH TETRA ELEMENTS |             |            |                 |            |            |   |  |

| LOADCASE                                  | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |  |
|---|-------------|------------|-----------------|------------|------------|---|--|
| 1   | 2           | 1.6000E+03 | -1.6000E+03     | 1.2790E-13 | 1.4381E-11 |   |  |
| 1GENESIS                                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 9 |  |
| TENSION ROD ASSEMBLED WITH TETRA ELEMENTS |             |            |                 |            |            |   |  |

| LOADCASE                                  | GRID ID     | MAGNITUDE  | T1              | T2          | T3          |    |  |
|---|-------------|------------|-----------------|-------------|-------------|----|--|
| 1   | 22          | 4.8022E-04 | 4.8000E-04      | -1.4400E-05 | -1.2455E-17 |    |  |
| 1GENESIS                                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49  | PAGE        | 10 |  |
| TENSION ROD ASSEMBLED WITH TETRA ELEMENTS |             |            |                 |             |             |    |  |

| TETRA ID | NORMAL    | SHEAR      | PRINCIPAL |          |
|----------|-----------|------------|-----------|----------|
|          | VON MISES | OCTAHEDRAL | MAX SHEAR | PRESSURE |

|   |   |               |    |               |   |               |               |
|---|---|---------------|----|---------------|---|---------------|---------------|
| 1 | X | 4.800000E+01  | XY | -8.401915E-13 | 1 | 4.800000E+01  |               |
|   | Y | -7.283063E-14 | YZ | 1.262405E-13  | 2 | 6.750156E-14  |               |
|   | Z | 6.750156E-14  | ZX | -1.258333E-13 | 3 | -7.283063E-14 |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 2 | X | 4.800000E+01  | XY | -8.091608E-13 | 1 | 4.800000E+01  |               |
|   | Y | 2.451372E-13  | YZ | 2.083375E-13  | 2 | 2.451372E-13  |               |
|   | Z | -9.592327E-14 | ZX | 2.932037E-13  | 3 | -9.592327E-14 |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 3 | X | 4.800000E+01  | XY | -7.423755E-13 | 1 | 4.800000E+01  |               |
|   | Y | 3.961276E-13  | YZ | 7.492984E-14  | 2 | 3.961276E-13  |               |
|   | Z | -1.421085E-13 | ZX | -5.375401E-14 | 3 | -1.421085E-13 |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 4 | X | 4.800000E+01  | XY | 4.235165E-13  | 1 | 4.800000E+01  |               |
|   | Y | 1.829648E-13  | YZ | -1.588187E-13 | 2 | 3.375078E-13  |               |
|   | Z | 3.375078E-13  | ZX | -1.708319E-13 | 3 | 1.829648E-13  |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 5 | X | 4.800000E+01  | XY | 3.534734E-13  | 1 | 4.800000E+01  |               |
|   | Y | 3.197442E-14  | YZ | -2.606255E-14 | 2 | 3.836931E-13  |               |
|   | Z | 3.836931E-13  | ZX | 1.787728E-13  | 3 | 3.197442E-14  |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 6 | X | 4.800000E+01  | XY | 7.506015E-13  | 1 | 4.800000E+01  |               |
|   | Y | 4.956036E-13  | YZ | 7.330093E-15  | 2 | 4.956036E-13  |               |
|   | Z | 2.557954E-13  | ZX | -7.818766E-14 | 3 | 2.557954E-13  |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*





---

## 2.16 Rod Modeled with Penta Elements

### Example ID:

A016

### Analysis Data:

GRID, CPENTA, PSOLID, MAT1, LOADCASE, LOAD, FORCE, SPC, SPC1, STRESS, DIS-  
PLACEMENT

### Special Features Used:

NONE. This example shows how to assemble a brick using CPENTA elements.

### Problem Statement:

Find the tip displacements and the stresses at the root of a cantilever rod loaded in its free end with a tension load of 4800 lb.

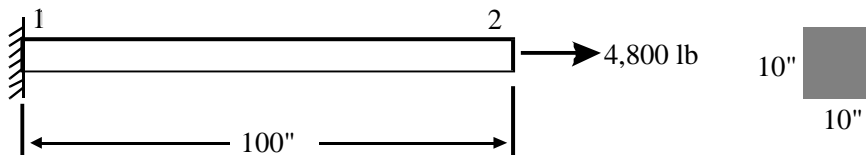


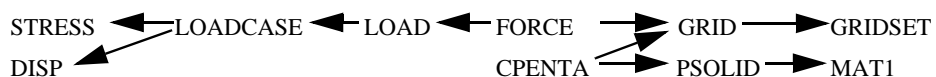
Figure 2-18

### Structural Data Relationships:

1. Tridimensional model of the rod assembled with 10 CPENTA elements.
2. Material:  $E=1.0E+7$ psi.
3. One Load cases:  $P=4800$ lb.

The stresses for solid elements are printed out in the material coordinate system. In this case the material coordinate system is the basic coordinate system (the default). Other coordinate systems can be used as the material coordinate system. See the description of the PSOLID element in Volume II for more details.

### Analysis Data Relationships:



In this example the solution control commands SET 5=21,22,23,24 and SET 6=1,2,3,4,5,6 were used to limit the output.

## Special Modeling Techniques:

It is important to correctly model the boundary conditions. In this problem the structure is globally prevented from moving in the six rigid body degrees of freedom at the fixed boundary. Please note that if all four boundary nodes (1, 2, 3 and 4) are fully constrained, slightly different answers will be obtained. Also it is important to load the structure with the correct consistent load vector (i.e., if the structure is loaded with 4 loads of 1200 lb in each of the free end corners, incorrect answers will be obtained).

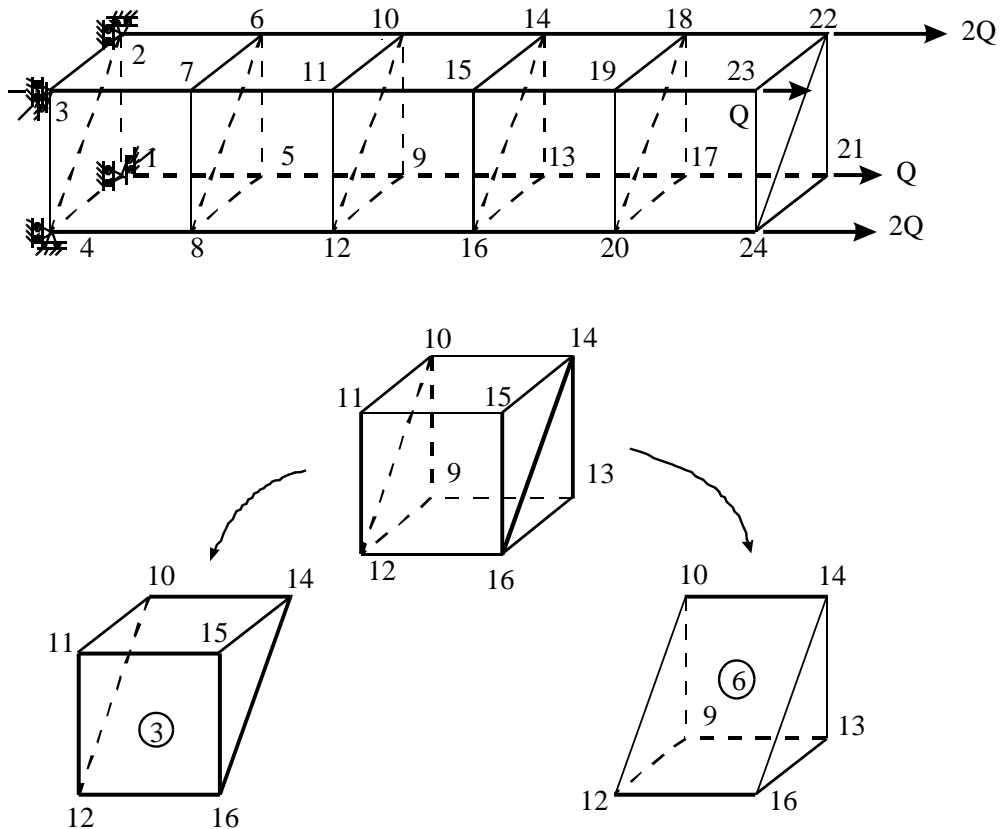


Figure 2-19

## Reference Solution:

The analytical solution for this problem is:

$$U_x = \frac{PL}{EA} = 0.00048 \text{ IN}$$

$$\sigma = \frac{P}{A} = 48 \text{ psi}$$

where: P is the applied load, E is the Young's modulus, A the area and L is the total length of the beam.

**Calculated Solution:**

The *GENESIS* solution is:

$$U_x = 0.00048 \text{ IN}$$

$$\sigma = \frac{P}{A} = 48 \text{ psi}$$

**Comparison Between Calculated Solution and Reference:**

There are no differences between the analytical solution and the *GENESIS* solution.

---

## 2.16.1 Input Data

```
ID A016
SOL COMPAT1
CEND
TITLE = TENSION ROD ASSEMBLED WITH PENTA ELEMENTS
LINE=64,80
SET 5=21,22,23,24
SET 6=1,2,3,4,5,6
LOADCASE 1
    LOAD = 42
    FORCE = ALL
    SPC=77
    DISPLACEMENT=5
    STRESS=6

$
BEGIN BULK
$
$    BOUNDARY CONDITIONS
$
SPC1    77        12        1
SPC1    77        13        2
SPC1    77        12        3
SPC1    77        13        4
$
$    GRID POINTS
$
GRID    1          0.0      10.      0.0
GRID    2          0.0      10.      10.0
GRID    3          0.0      0.0      10.0
GRID    4          0.0      0.0      0.0
$
GRID    5          20.0     10.      0.0
GRID    6          20.0     10.      10.0
GRID    7          20.0     0.0      10.0
GRID    8          20.0     0.0      0.0
$
GRID    9          40.0     10.      0.0
GRID   10          40.0     10.      10.0
GRID   11          40.0     0.0      10.0
GRID   12          40.0     0.0      0.0
$
GRID   13          60.0     10.      0.0
GRID   14          60.0     10.      10.0
GRID   15          60.0     0.0      10.0
GRID   16          60.0     0.0      0.0
$
GRID   17          80.0     10.      0.0
GRID   18          80.0     10.      10.0
GRID   19          80.0     0.0      10.0
GRID   20          80.0     0.0      0.0
$
GRID   21          100.0    10.      0.0
GRID   22          100.0    10.      10.0
```

```

GRID    23                100.0  0.0   10.0
GRID    24                100.0  0.0   0.0
$
$      ELEMENT DEFINITIONS
$
CPENTA  1      1      2      3      4      6      7      8
CPENTA  3      1      6      7      8     10     11     12
CPENTA  5      1     10     11     12     14     15     16
CPENTA  7      1     14     15     16     18     19     20
CPENTA  9      1     18     19     20     22     23     24
$
CPENTA  2      1      4      1      2      8      5      6
CPENTA  4      1      8      5      6     12      9     10
CPENTA  6      1     12      9     10     16     13     14
CPENTA  8      1     16     13     14     20     17     18
CPENTA 10      1     20     17     18     24     21     22
$
$      PROPERTY DEFINITIONS
$
PSOLID  1      51
$
$      MATERIAL DEFINITIONS
$
MAT1    51      10.0E6      0.3
$
$      LOAD DEFINITIONS
$
FORCE   42      21                800.0  1.0   0.0   0.0
FORCE   42      22                1600.0 1.0   0.0   0.0
FORCE   42      23                800.0  1.0   0.0   0.0
FORCE   42      24                1600.0 1.0   0.0   0.0
$
ENDDATA

```

---

## 2.16.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A016  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
TENSION ROD ASSEMBLED WITH PENTA ELEMENTS

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS:                24
NUMBER OF CPENTA ELEMENTS:            10
TOTAL NUMBER OF NON RIGID ELEMENTS:    10
```

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 64

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 TENSION ROD ASSEMBLED WITH PENTA ELEMENTS

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 TENSION ROD ASSEMBLED WITH PENTA ELEMENTS

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 1.000000E+04  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 TENSION ROD ASSEMBLED WITH PENTA ELEMENTS

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 1.162389E-14 ; STRAIN ENERGY : 1.152000E+00  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 TENSION ROD ASSEMBLED WITH PENTA ELEMENTS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3          |
|----------|------------|------------|------------|------------|------------|-------------|
| 1        | 4.8000E+03 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 2.4000E+04 | -2.4000E+04 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 TENSION ROD ASSEMBLED WITH PENTA ELEMENTS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1          | T2         | T3         | R1         | R2          | R3         |
|----------|-------------|------------|------------|------------|-------------|------------|
| 1        | -4.8000E+03 | 4.8885E-12 | 6.3345E-12 | 2.6326E-11 | -2.4000E+04 | 2.4000E+04 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 TENSION ROD ASSEMBLED WITH PENTA ELEMENTS

| LOADCASE                                  | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |  |
|---|-------------|------------|-----------------|------------|------------|---|--|
| 1   | 22          | 1.6000E+03 | 1.6000E+03      | 0.0000E+00 | 0.0000E+00 |   |  |
| 1GENESIS                                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE       | 8 |  |
| TENSION ROD ASSEMBLED WITH PENTA ELEMENTS |             |            |                 |            |            |   |  |

| LOADCASE                                  | GRID ID     | MAGNITUDE  | T1              | T2         | T3          |   |  |
|---|-------------|------------|-----------------|------------|-------------|---|--|
| 1   | 4           | 1.6000E+03 | -1.6000E+03     | 5.9716E-13 | -1.1340E-07 |   |  |
| 1GENESIS                                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE        | 9 |  |
| TENSION ROD ASSEMBLED WITH PENTA ELEMENTS |             |            |                 |            |             |   |  |

| LOADCASE                                  | GRID ID     | MAGNITUDE  | T1              | T2         | T3          |
|---|-------------|------------|-----------------|------------|-------------|
| 1   | 23          | 4.8022E-04 | 4.8000E-04      | 2.4490E-14 | -1.4400E-05 |
| 1GENESIS                                  | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:49 | PAGE 10     |
| TENSION ROD ASSEMBLED WITH PENTA ELEMENTS |             |            |                 |            |             |

|    |              |              |               |
|----|--------------|--------------|---------------|
| 24 | 4.800000E-04 | 1.440000E-05 | -4.959690E-15 |
|    | 0.000000E+00 | 0.000000E+00 | 0.000000E+00  |

| PENTA ID | NORMAL<br>VON MISES | SHEAR<br>OCTAHEDRAL | PRINCIPAL<br>MAX SHEAR | PRESSURE |
|----------|---------------------|---------------------|------------------------|----------|
|----------|---------------------|---------------------|------------------------|----------|



|   |   |               |    |               |   |               |               |
|---|---|---------------|----|---------------|---|---------------|---------------|
| 1 | X | 4.800000E+01  | XY | -6.895663E-09 | 1 | 4.800000E+01  |               |
|   | Y | -1.695472E-09 | YZ | 1.190896E-09  | 2 | 1.395897E-10  |               |
|   | Z | 1.395897E-10  | ZX | 6.570976E-09  | 3 | -1.695472E-09 |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 2 | X | 4.800000E+01  | XY | 6.895747E-09  | 1 | 4.800000E+01  |               |
|   | Y | -1.696007E-09 | YZ | 1.190805E-09  | 2 | 1.396039E-10  |               |
|   | Z | 1.396039E-10  | ZX | -6.570875E-09 | 3 | -1.696007E-09 |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 3 | X | 4.800000E+01  | XY | 9.713318E-09  | 1 | 4.800000E+01  |               |
|   | Y | 3.494018E-09  | YZ | -1.768138E-09 | 2 | 3.494018E-09  |               |
|   | Z | 1.725656E-09  | ZX | -9.432481E-09 | 3 | 1.725656E-09  |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 4 | X | 4.800000E+01  | XY | -9.713227E-09 | 1 | 4.800000E+01  |               |
|   | Y | 3.493644E-09  | YZ | -1.768237E-09 | 2 | 3.493644E-09  |               |
|   | Z | 1.725653E-09  | ZX | 9.432581E-09  | 3 | 1.725653E-09  |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 5 | X | 4.800000E+01  | XY | -1.660786E-08 | 1 | 4.800000E+01  |               |
|   | Y | -6.780555E-09 | YZ | 3.142942E-09  | 2 | -3.710976E-09 |               |
|   | Z | -3.710976E-09 | ZX | 1.620281E-08  | 3 | -6.780555E-09 |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |
| 6 | X | 4.800000E+01  | XY | 1.660801E-08  | 1 | 4.800000E+01  |               |
|   | Y | -6.780787E-09 | YZ | 3.142863E-09  | 2 | -3.711008E-09 |               |
|   | Z | -3.711008E-09 | ZX | -1.620275E-08 | 3 | -6.780787E-09 |               |
|   |   | 4.800000E+01  |    | 2.262742E+01  |   | 2.400000E+01  | -1.600000E+01 |

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.17 Anisotropic Plate with Thermal Load - A

### Example ID:

A017

### Analysis Data Used:

GRID, CQUAD4, PSHELL, MAT8, LOADCASE, TEMP, TEMPD, SPC, SPC, SPC1, FORCE, STRESS, STRAIN, DISP

### Special Features Used:

Utilization of orthotropic material MAT8

### Problem Statement:

Find the elements forces, stresses and strains of the anisotropic plate shown in **Figure 2-20**. The plate has its four sides clamped and the material is orthotropic with its principal axis rotated in 21.7 degrees from the element coordinate system.

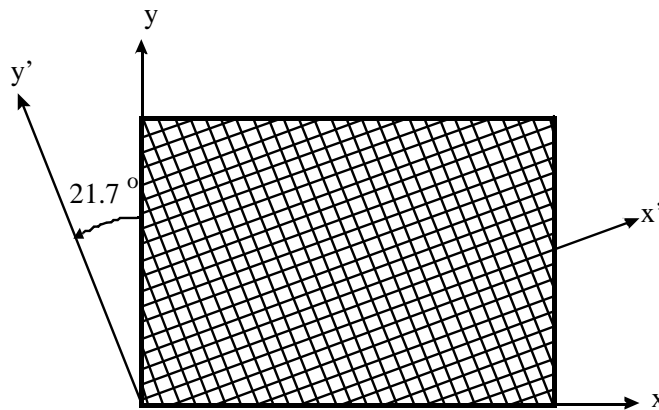
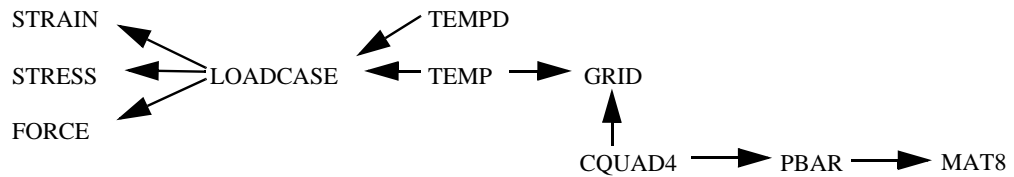


Figure 2-20

### Structural Analysis Model:

1. Plate assembled with 4 CQUAD4 elements.
2. Section properties of the plate: thickness=2.0.
3. Material:  $E1=7.8E6$ ,  $E2=2.6E6$ ,  $\nu_{12}=0.25$ ,  $G=1.3E6$ ,  $A1=3.5E-6$ ,  $A2=11.4E-6$ ,  $A3=0.0$  (Data is in the material coordinate system).
4. One temperature Load case:  $T-T0=20.0$  degrees.

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE.

## Reference Solutions:

The analytical solution for this problem can be obtained by using the stress-strain relationship in the element coordinate system.

The strains are zero because the plate is constrained to move.

Stresses:

$$\sigma_x = -701.2$$

$$\sigma_y = -659.7$$

$$\tau_{xy} = -19.6$$

Element forces:

$$N_x = 2.0\sigma_x = -1402.3$$

$$N_y = 2.0\sigma_y = -1319.4$$

$$N_{xy} = 2.0\tau_{xy} = -39.2$$

### Calculated Solutions:

$$\varepsilon_{xx} = \varepsilon_{yy} = \gamma_{xy} = 0.0$$

$$\sigma_x = -701.2$$

$$\sigma_y = -659.7$$

$$\tau_{xy} = -19.6$$

$$N_x = -1402.3$$

$$N_y = -1319.4$$

$$N_{xy} = -39.2$$

### Comparison Between Calculated Solutions and Reference Solutions:

There are no differences between the analytical solutions and the *GENESIS* solutions.

---

## 2.17.1 Input Data

```
ID A017
SOL COMPAT1
CEND
TITLE=ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I
SUBTITLE=USE OF MAT8
ECHO=NONE
LINE=64,80
LOADCASE 1
    TEMP = 1
    STRESS=ALL
    STRAIN=ALL
    FORCE=ALL

$
BEGIN BULK
$
$    GRID POINTS
$
GRID    1          0.0    0.0    0.0          123456
GRID    2          10.0    0.0    0.0          123456
GRID    3          20.0    0.0    0.0          123456
$
GRID    12         0.0    0.0    5.0          123456
GRID    13         10.0    0.0    5.0          123456
GRID    14         20.0    0.0    5.0          123456
$
GRID    23         0.0    0.0   10.0          123456
GRID    24         10.0    0.0   10.0          123456
GRID    25         20.0    0.0   10.0          123456
$
$    ELEMENT DEFINITIONS
$
CQUAD4  1          1          1          2          13          12          21.7
CQUAD4  2          1          2          3          14          13          21.7
CQUAD4  3          1          12         13         24          23          21.7
CQUAD4  4          1          13         14         25          24          21.7
$
$    PROPERTY DEFINITIONS
$
PSHELL  1          1          2.0    1          0.0
$
$    MATERIAL DEFINITIONS
$
MAT8    1          7.8+6    2.6+6    0.25    1.3+6
+        3.50-6    11.40-6    80.0
$
$    TEMPERATURE DEFINITION
$
TEMPD   1          100.0
$
ENDDATA
```



---

## 2.17.2 Output

1

```
  GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N  E      S      I      S
G  GG  EEEE   N N N  EEEE   SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A017  
RUN STARTED: Dec 13, 2002 12:49  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 1  
ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  
USE OF MAT8

### ANALYSIS PROBLEM SUMMARY

```
NUMBER OF GRID POINTS: 9
NUMBER OF CQUAD4->PSHELL ELEMENTS: 4
TOTAL NUMBER OF NON RIGID ELEMENTS: 4
```

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 6

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 2  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  
 USE OF MAT8

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 3  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  
 USE OF MAT8

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 4.000000E+02  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 4  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  
 USE OF MAT8

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -1.999995E+00 ; STRAIN ENERGY : 2.512098E-32  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 5  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  
 USE OF MAT8

#### O L O A D R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

|   |            |            |            |            |             |            |
|---|------------|------------|------------|------------|-------------|------------|
| 1 | 0.0000E+00 | 0.0000E+00 | 1.8190E-12 | 0.0000E+00 | -4.3656E-11 | 0.0000E+00 |
|---|------------|------------|------------|------------|-------------|------------|

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 6  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  
 USE OF MAT8

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1 | T2 | T3 | R1 | R2 | R3 |
|----------|----|----|----|----|----|----|
|----------|----|----|----|----|----|----|

|   |            |            |             |            |            |            |
|---|------------|------------|-------------|------------|------------|------------|
| 1 | 0.0000E+00 | 0.0000E+00 | -4.5475E-12 | 0.0000E+00 | 5.8208E-11 | 0.0000E+00 |
|---|------------|------------|-------------|------------|------------|------------|

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 7  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  
 USE OF MAT8



# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE   | GRID ID | MAGNITUDE  | T1         | T2         | T3         |  |
|--|---------|------------|------------|------------|------------|--|
| 1  | 13      | 9.0949E-13 | 0.0000E+00 | 0.0000E+00 | 9.0949E-13 |  |
| 1GENESIS    VERSION    7.2                      DATE 12-13-2002    TIME 12:49    PAGE    8 |         |            |            |            |            |  |
| ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I   |         |            |            |            |            |  |
| USE OF MAT8  |         |            |            |            |            |  |

# M A X I M U M   S P C   F O R C E

| LOADCASE   | GRID ID | MAGNITUDE  | T1          | T2         | T3          |  |
|--|---------|------------|-------------|------------|-------------|--|
| 1  | 24      | 1.3200E+04 | -3.9225E+02 | 0.0000E+00 | -1.3194E+04 |  |
| 1GENESIS    VERSION    7.2                      DATE 12-13-2002    TIME 12:49    PAGE    9 |         |            |             |            |             |  |
| ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I   |         |            |             |            |             |  |
| USE OF MAT8  |         |            |             |            |             |  |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE  | GRID ID | MAGNITUDE  | T1          | T2         | T3         |  |
|---|---------|------------|-------------|------------|------------|--|
| 1   | 13      | 5.6171E-20 | -1.0177E-20 | 0.0000E+00 | 5.5241E-20 |  |
| 1GENESIS    VERSION    7.2                      DATE 12-13-2002    TIME 12:49    PAGE    10 |         |            |             |            |            |  |
| ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  |         |            |             |            |            |  |
| USE OF MAT8   |         |            |             |            |            |  |

LOADCASE                      1

# F O R C E S   I N   Q U A D 4   E L E M E N T S

| QUAD4 ID  | - NX -                        | - NY -                        | - NXY -                       |
|---|-------------------------------|-------------------------------|-------------------------------|
|   | - MX -                        | - MY -                        | - MXY -                       |
| 1   | -1.402331E+03<br>0.000000E+00 | -1.319371E+03<br>0.000000E+00 | -3.922539E+01<br>0.000000E+00 |
| 2   | -1.402331E+03<br>0.000000E+00 | -1.319371E+03<br>0.000000E+00 | -3.922539E+01<br>0.000000E+00 |
| 3   | -1.402331E+03<br>0.000000E+00 | -1.319371E+03<br>0.000000E+00 | -3.922539E+01<br>0.000000E+00 |
| 4   | -1.402331E+03<br>0.000000E+00 | -1.319371E+03<br>0.000000E+00 | -3.922539E+01<br>0.000000E+00 |
| 1GENESIS    VERSION    7.2                      DATE 12-13-2002    TIME 12:49    PAGE    11 |                               |                               |                               |
| ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  |                               |                               |                               |
| USE OF MAT8   |                               |                               |                               |

LOADCASE                      1

# S T R E S S E S   I N   Q U A D 4   E L E M E N T S

| QUAD4 ID | SURFACE | MAX SHEAR | VON MISES | MAJOR  | MINOR |
|----------|---------|-----------|-----------|--------|-------|
|          |         | SIGMA X   | SIGMA Y   | TAU XY |       |

|   |   |               |               |               |               |
|---|---|---------------|---------------|---------------|---------------|
| 1 | 1 | 2.854468E+01  | 6.822194E+02  | -6.518809E+02 | -7.089702E+02 |
|   |   | -7.011654E+02 | -6.596857E+02 | -1.961269E+01 |               |
|   | 2 | 2.854468E+01  | 6.822194E+02  | -6.518809E+02 | -7.089702E+02 |
|   |   | -7.011654E+02 | -6.596857E+02 | -1.961269E+01 |               |
| 2 | 1 | 2.854468E+01  | 6.822194E+02  | -6.518809E+02 | -7.089702E+02 |
|   |   | -7.011654E+02 | -6.596857E+02 | -1.961269E+01 |               |
|   | 2 | 2.854468E+01  | 6.822194E+02  | -6.518809E+02 | -7.089702E+02 |
|   |   | -7.011654E+02 | -6.596857E+02 | -1.961269E+01 |               |
| 3 | 1 | 2.854468E+01  | 6.822194E+02  | -6.518809E+02 | -7.089702E+02 |
|   |   | -7.011654E+02 | -6.596857E+02 | -1.961269E+01 |               |
|   | 2 | 2.854468E+01  | 6.822194E+02  | -6.518809E+02 | -7.089702E+02 |
|   |   | -7.011654E+02 | -6.596857E+02 | -1.961269E+01 |               |
| 4 | 1 | 2.854468E+01  | 6.822194E+02  | -6.518809E+02 | -7.089702E+02 |
|   |   | -7.011654E+02 | -6.596857E+02 | -1.961269E+01 |               |
|   | 2 | 2.854468E+01  | 6.822194E+02  | -6.518809E+02 | -7.089702E+02 |
|   |   | -7.011654E+02 | -6.596857E+02 | -1.961269E+01 |               |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:49 PAGE 12  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-I  
 USE OF MAT8

LOADCASE 1

# S T R A I N S I N Q U A D 4 E L E M E N T S

| QUAD4 ID | SURFACE | MAX SHEAR<br>EPSILON X<br>(EPSILON-X)<br>(KAPPA-Y) | VON MISES<br>EPSILON Y<br>(EPSILON-Y)<br>(KAPPA-XY) | MAJOR<br>GAMMA XY<br>(GAMMA-XY) | MINOR<br>(KAPPA-X) |
|----------|---------|--|---|---------------------------------|--------------------|
| 1        | 1       | 6.27271E-21  | 3.98814E-21   | 5.64168E-21                     | -6.31026E-22       |
|          |         | -5.07315E-22                                       | 5.51797E-21   | 1.74435E-21                     |                    |
|          | 2       | 6.27271E-21  | 3.98814E-21   | 5.64168E-21                     | -6.31026E-22       |
|          |         | -5.07315E-22                                       | 5.51797E-21   | 1.74435E-21                     |                    |
|          | 0       | (-5.07315E-22)(                                    | 5.51797E-21)(                                       | 1.74435E-21)(                   | 0.00000E+00)       |
|          |         | (0.00000E+00)(                                     | 0.00000E+00)  |                                 |                    |
| 2        | 1       | 6.28629E-21  | 4.15004E-21   | 6.16196E-21                     | -1.24326E-22       |
|          |         | 5.07315E-22  | 5.53032E-21   | -3.77979E-21                    |                    |
|          | 2       | 6.28629E-21  | 4.15004E-21   | 6.16196E-21                     | -1.24326E-22       |
|          |         | 5.07315E-22  | 5.53032E-21   | -3.77979E-21                    |                    |
|          | 0       | (5.07315E-22)(                                     | 5.53032E-21)(                                       | -3.77979E-21)(                  | 0.00000E+00)       |
|          |         | (0.00000E+00)(                                     | 0.00000E+00)  |                                 |                    |
| 3        | 1       | 6.27396E-21  | 4.14232E-21   | 1.22792E-22                     | -6.15117E-21       |
|          |         | -5.10403E-22                                       | -5.51797E-21  | 3.77979E-21                     |                    |
|          | 2       | 6.27396E-21  | 4.14232E-21   | 1.22792E-22                     | -6.15117E-21       |
|          |         | -5.10403E-22                                       | -5.51797E-21  | 3.77979E-21                     |                    |
|          | 0       | (-5.10403E-22)(                                    | -5.51797E-21)(                                      | 3.77979E-21)(                   | 0.00000E+00)       |
|          |         | (0.00000E+00)(                                     | 0.00000E+00)  |                                 |                    |
| 4        | 1       | 6.28754E-21  | 3.99721E-21   | 6.33810E-22                     | -5.65373E-21       |

```

      5.10403E-22  -5.53032E-21  -1.74435E-21
2      6.28754E-21   3.99721E-21   6.33810E-22  -5.65373E-21
      5.10403E-22  -5.53032E-21  -1.74435E-21
0      ( 5.10403E-22)(-5.53032E-21)(-1.74435E-21)( 0.00000E+00)
      ( 0.00000E+00)( 0.00000E+00)

```

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.18 Anisotropic Plate with Thermal Load - B

### Example ID:

A018

### Analysis Data Used:

GRID, CQUAD4, PSHELL, MAT2, LOADCASE, TEMP, TEMPD, SPC, SPC, SPC1, FORCE, STRESS, STRAIN, DISP

### Special Features Used:

Utilization of anisotropic material MAT2.

### Problem Statement:

Find the elements forces, stresses and strains of the anisotropic plate shown in **Figure 2-21**. The plate has its four sides clamped and the material is orthotropic with principal axis rotated in 21.7 degrees from the element coordinate system. The mechanical properties are given below and are in the element coordinate system.

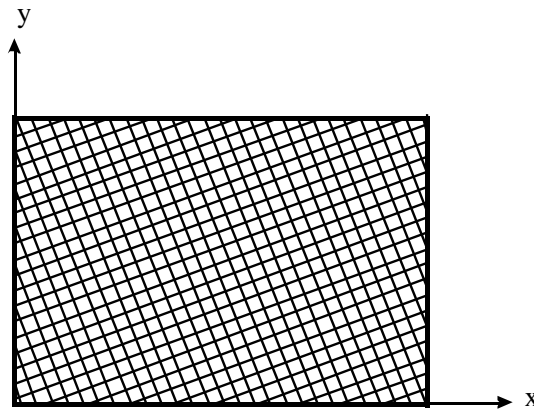
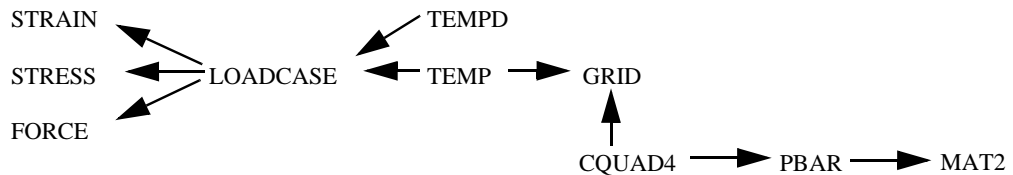


Figure 2-21

## Structural Analysis Model:

1. Plate assembled with 4 CQUAD4 elements.
2. Section properties of plate: thickness=2.0.
3. Material:  
G11=6.756E6, G12=1.147E6, G13=1.423E6  
G22=2.898E6, G23=4.013E5, G33=1.783E6  
A1=4.58E-6, A2=10.32E-6, A12=-5.428E-6  
(Data is in the element coordinate system and corresponds to the orthotropic material used in A017.)
4. One temperature Load case: T-T0=20.0 degrees.

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE. Since the mechanical properties of the plate are given in the element coordinate system, the angle 21.7 should not be included in the CQUAD4 data.

## Reference Solutions:

Same as problem [A017](#).

**Calculated Solutions:**

$$\varepsilon_{xx} = \varepsilon_{yy} = \gamma_{xy} = 0.0$$

$$\sigma_x = -701.2$$

$$\sigma_y = -659.7$$

$$\tau_{xy} = -19.6$$

$$N_x = -1402.3$$

$$N_y = -1319.4$$

$$N_{xy} = -39.2$$

**Comparison Between Calculated Solutions and Reference Solutions:**

There is no difference between the analytical solutions and the *GENESIS* solutions.

---

## 2.18.1 Input Data

```
ID A018
SOL COMPAT1
CEND
TITLE=ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II
SUBTITLE=USE OF MAT2
ECHO=NONE
LINE=64,80
LOADCASE 1
    TEMP = 1
    STRESS=ALL
    STRAIN=ALL
    FORCE=ALL

$
BEGIN BULK
$
$    GRID POINTS
$
GRID    1          0.0    0.0    0.0          123456
GRID    2          10.0    0.0    0.0          123456
GRID    3          20.0    0.0    0.0          123456
$
GRID    12         0.0    0.0    5.0          123456
GRID    13         10.0    0.0    5.0          123456
GRID    14         20.0    0.0    5.0          123456
$
GRID    23         0.0    0.0   10.0          123456
GRID    24         10.0    0.0   10.0          123456
GRID    25         20.0    0.0   10.0          123456
$
$    ELEMENT DEFINITIONS
$
CQUAD4  1          1          1          2          13          12
CQUAD4  2          1          2          3          14          13
CQUAD4  3          1          12         13         24          23
CQUAD4  4          1          13         14         25          24
$
$    PROPERTY DEFINITIONS
$
PSHELL  1          1          2.0    1          0.0
$
$    MATERIAL DEFINITIONS
$
$1-----2-----3-----4-----5-----6-----7-----8-----9
MAT2    1          6756788.1146968.1423123.2898213.401314. 1783138.
+        4.58-6   10.32-6  -5.428-6  80.0.0
$
$    TEMPERATURE DEFINITION
$
TEMPD   1          100.0
$
ENDDATA
```

---

## 2.18.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A018  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II  
USE OF MAT2

### ANALYSIS PROBLEM SUMMARY

|                                     |   |
|-------------------------------------|---|
| NUMBER OF GRID POINTS:              | 9 |
| NUMBER OF CQUAD4->PSHELL ELEMENTS:  | 4 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 4 |



NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 6

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II  
 USE OF MAT2

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II  
 USE OF MAT2

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 4.000000E+02  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II  
 USE OF MAT2

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -1.999665E+00 ; STRAIN ENERGY : 2.512512E-32  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II  
 USE OF MAT2

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3          | R1         | R2          | R3         |
|----------|------------|------------|-------------|------------|-------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | -9.0949E-13 | 0.0000E+00 | -4.3656E-11 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 6  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II  
 USE OF MAT2

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1         | T2         | T3          | R1         | R2         | R3         |
|----------|------------|------------|-------------|------------|------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | -1.8190E-12 | 0.0000E+00 | 1.4552E-11 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 7  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II  
 USE OF MAT2

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE                                    | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|---|-------------|------------|-----------------|------------|------------|---|
| 1   | 13          | 9.0949E-13 | 0.0000E+00      | 0.0000E+00 | 9.0949E-13 |   |
| 1GENESIS                                    | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:50 | PAGE       | 8 |
| ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II |             |            |                 |            |            |   |
| USE OF MAT2                                 |             |            |                 |            |            |   |

# M A X I M U M   S P C   F O R C E

| LOADCASE                                    | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|---|-------------|------------|-----------------|------------|------------|---|
| 1   | 2           | 1.3200E+04 | 3.9224E+02      | 0.0000E+00 | 1.3194E+04 |   |
| 1GENESIS                                    | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:50 | PAGE       | 9 |
| ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II |             |            |                 |            |            |   |
| USE OF MAT2                                 |             |            |                 |            |            |   |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE                                    | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |    |
|---|-------------|------------|-----------------|------------|------------|----|
| 1   | 13          | 5.6171E-20 | -1.0177E-20     | 0.0000E+00 | 5.5241E-20 |    |
| 1GENESIS                                    | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:50 | PAGE       | 10 |
| ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II |             |            |                 |            |            |    |
| USE OF MAT2                                 |             |            |                 |            |            |    |

LOADCASE 1

# F O R C E S   I N   Q U A D 4   E L E M E N T S

| QUAD4 ID                                    | - NX -                        | - NY -                        | - NXY -                       |
|---|-------------------------------|-------------------------------|-------------------------------|
|   | - MX -                        | - MY -                        | - MXY -                       |
| 1   | -1.402323E+03<br>0.000000E+00 | -1.319374E+03<br>0.000000E+00 | -3.922363E+01<br>0.000000E+00 |
| 2   | -1.402323E+03<br>0.000000E+00 | -1.319374E+03<br>0.000000E+00 | -3.922363E+01<br>0.000000E+00 |
| 3   | -1.402323E+03<br>0.000000E+00 | -1.319374E+03<br>0.000000E+00 | -3.922363E+01<br>0.000000E+00 |
| 4   | -1.402323E+03<br>0.000000E+00 | -1.319374E+03<br>0.000000E+00 | -3.922363E+01<br>0.000000E+00 |
| 1GENESIS                                    | VERSION 7.2                   | DATE 12-13-2002               | TIME 12:50 PAGE 11            |
| ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II |                               |                               |                               |
| USE OF MAT2                                 |                               |                               |                               |

LOADCASE 1

# S T R E S S E S   I N   Q U A D 4   E L E M E N T S

| QUAD4 ID | SURFACE | MAX SHEAR<br>SIGMA X | VON MISES<br>SIGMA Y | MAJOR<br>TAU XY | MINOR |
|----------|---------|----------------------|----------------------|-----------------|-------|
|----------|---------|----------------------|----------------------|-----------------|-------|

|   |   |               |               |               |               |
|---|---|---------------|---------------|---------------|---------------|
| 1 | 1 | 2.854236E+01  | 6.822178E+02  | -6.518819E+02 | -7.089666E+02 |
|   |   | -7.011617E+02 | -6.596868E+02 | -1.961182E+01 |               |
|   | 2 | 2.854236E+01  | 6.822178E+02  | -6.518819E+02 | -7.089666E+02 |
|   |   | -7.011617E+02 | -6.596868E+02 | -1.961182E+01 |               |
| 2 | 1 | 2.854236E+01  | 6.822178E+02  | -6.518819E+02 | -7.089666E+02 |
|   |   | -7.011617E+02 | -6.596868E+02 | -1.961182E+01 |               |
|   | 2 | 2.854236E+01  | 6.822178E+02  | -6.518819E+02 | -7.089666E+02 |
|   |   | -7.011617E+02 | -6.596868E+02 | -1.961182E+01 |               |
| 3 | 1 | 2.854236E+01  | 6.822178E+02  | -6.518819E+02 | -7.089666E+02 |
|   |   | -7.011617E+02 | -6.596868E+02 | -1.961182E+01 |               |
|   | 2 | 2.854236E+01  | 6.822178E+02  | -6.518819E+02 | -7.089666E+02 |
|   |   | -7.011617E+02 | -6.596868E+02 | -1.961182E+01 |               |
| 4 | 1 | 2.854236E+01  | 6.822178E+02  | -6.518819E+02 | -7.089666E+02 |
|   |   | -7.011617E+02 | -6.596868E+02 | -1.961182E+01 |               |
|   | 2 | 2.854236E+01  | 6.822178E+02  | -6.518819E+02 | -7.089666E+02 |
|   |   | -7.011617E+02 | -6.596868E+02 | -1.961182E+01 |               |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 12  
 ANISOTROPIC PLANTE WITH TEMPERATURE LOAD-II  
 USE OF MAT2

LOADCASE 1

# S T R A I N S I N Q U A D 4 E L E M E N T S

| QUAD4 ID | SURFACE | MAX SHEAR<br>EPSILON X<br>(EPSILON-X)<br>(KAPPA-Y) | VON MISES<br>EPSILON Y<br>(EPSILON-Y)<br>(KAPPA-XY) | MAJOR<br>GAMMA XY<br>(GAMMA-XY) | MINOR<br>(KAPPA-X) |
|----------|---------|--|---|---------------------------------|--------------------|
| 1        | 1       | 6.22081E-21  | 3.95641E-21   | 5.59952E-21                     | -6.21292E-22       |
|          |         | -4.96507E-22                                       | 5.47474E-21   | 1.74435E-21                     |                    |
|          | 2       | 6.22081E-21  | 3.95641E-21   | 5.59952E-21                     | -6.21292E-22       |
|          |         | -4.96507E-22                                       | 5.47474E-21   | 1.74435E-21                     |                    |
|          | 0       | (-4.96507E-22)(                                    | 5.47474E-21)(                                       | 1.74435E-21)(                   | 0.00000E+00)       |
|          |         | (0.00000E+00)(                                     | 0.00000E+00)  |                                 |                    |
| 2        | 1       | 6.32955E-21  | 4.17713E-21   | 6.19981E-21                     | -1.29745E-22       |
|          |         | 4.96507E-22  | 5.57355E-21   | -3.77979E-21                    |                    |
|          | 2       | 6.32955E-21  | 4.17713E-21   | 6.19981E-21                     | -1.29745E-22       |
|          |         | 4.96507E-22  | 5.57355E-21   | -3.77979E-21                    |                    |
|          | 0       | (4.96507E-22)(                                     | 5.57355E-21)(                                       | -3.77979E-21)(                  | 0.00000E+00)       |
|          |         | (0.00000E+00)(                                     | 0.00000E+00)  |                                 |                    |
| 3        | 1       | 6.23091E-21  | 4.11534E-21   | 1.17481E-22                     | -6.11343E-21       |
|          |         | -5.21212E-22                                       | -5.47474E-21  | 3.77979E-21                     |                    |
|          | 2       | 6.23091E-21  | 4.11534E-21   | 1.17481E-22                     | -6.11343E-21       |
|          |         | -5.21212E-22                                       | -5.47474E-21  | 3.77979E-21                     |                    |
|          | 0       | (-5.21212E-22)(                                    | -5.47474E-21)(                                      | 3.77979E-21)(                   | 0.00000E+00)       |
|          |         | (0.00000E+00)(                                     | 0.00000E+00)  |                                 |                    |
| 4        | 1       | 6.33948E-21  | 4.02896E-21   | 6.43566E-22                     | -5.69591E-21       |

```

      5.21212E-22  -5.57355E-21  -1.74435E-21
2      6.33948E-21   4.02896E-21   6.43566E-22  -5.69591E-21
      5.21212E-22  -5.57355E-21  -1.74435E-21
0      ( 5.21212E-22)(-5.57355E-21)(-1.74435E-21)( 0.00000E+00)
      ( 0.00000E+00)( 0.00000E+00)

```

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.19 Straight Uniform Fin Analysis

### Example ID:

A019

### Analysis Data Used:

GRID, CROD, CHBDY (LINE AND POINT), PROD, PHBDY, MAT4, MAT1(DUMMY), LOADCASE, HEAT, SPCD, THERMAL, SPC and SPC1.

### Special Features Used:

Convection using boundary elements LINE and POINT.

### Problem Statement:

Find the temperature distribution in the fin shown in the figure below. Consider convection through the perimeter and through the tip.

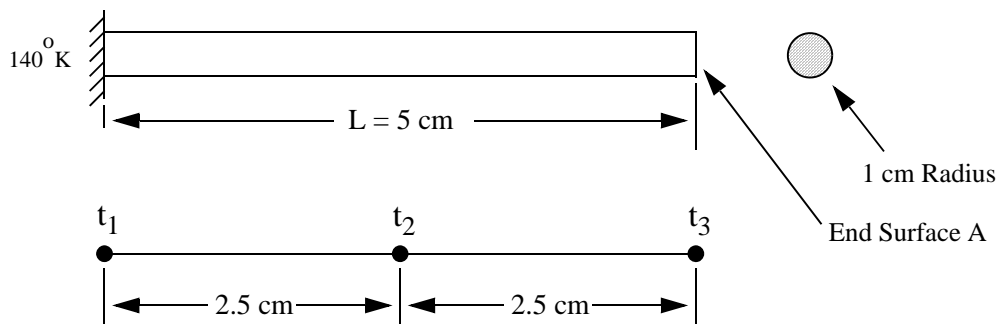
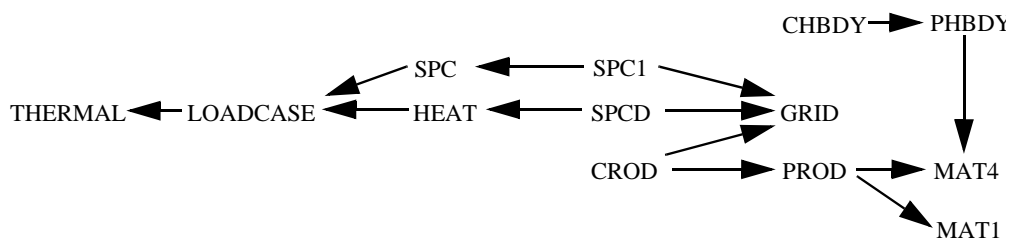


Figure 2-22

### Heat Transfer Analysis Model:

1. Fin structure assembled with 2 conductive rod elements and 3 CHBDY boundary elements (2 LINE and 1 POINT).
2. Section properties:  $\text{Area} = 3.141592\text{ cm}^2$ .
3. Materials:
  - $k = 70\text{ Watts/cm-}^\circ\text{K}$  (conduction)
  - $h = 5\text{ Watts/cm}^2\text{-}^\circ\text{K}$  (convection)In addition, dummy structural material MAT1 is included in the input data. Structural material data is necessary to run *GENESIS*.
4. One load case: Convection due to an ambient temperature of  $40^\circ\text{K}$ , and an enforced temperature of  $140.0^\circ\text{K}$  at the root.

## Analysis Data Relationships:



## Special Modeling Techniques:

The convection through the perimeter is modeled using two CHBDY (LINE) boundary elements. In this case the AREA FACTOR parameter corresponds to the circular perimeter. The convection through the tip is modeled with one POINT boundary element.

## Reference Solutions:

Rao,S.S, “The Finite Element Method In Engineering.” Pergamon Press, second edition, page 452. 1989.

| Grid | Temperature |
|------|-------------|
| 1    | 140.00      |
| 2    | 80.44       |
| 3    | 63.36       |

## Calculated Solutions:.

| Grid | Temperature |
|------|-------------|
| 1    | 140.00      |
| 2    | 80.45       |
| 3    | 63.32       |

## Comparison Between Calculated Solutions and Reference Solutions:

The calculated solution and the reference solution are very similar. The reason for the small difference is that the number of digits used in both solutions are different.

---

## 2.19.1 Input Data

```
ID A019
SOL COMPAT1
CEND
summary=no
TITLE = STRAIGHT UNIFORM FIN ANALYSIS.
LINE=64,80
ECHO=NONE
LOADCASE 1
LABEL= CONVECTION FIN ANALYSIS USING 1 POINT-CHBDY & 2 LINE-CHBDYS.
SPC=101
HEAT=101
THERMAL=ALL
$
BEGIN BULK
$
$   HEAT TRANSFER BOUNDARY CONDITIONS
$
SPC1    101          1
SPCD    101      1          140.0
$
$   AMBIENT TEMPERATURE
$
SPC1    101          999
SPCD    101      999          40.0
GRID    999      0      0.0      0.0      3.0
$
$   GRID DATA
$
GRID    1      0      0.0      0.0      0.0      0      123456
GRID    2      0      2.5      0.0      0.0      0      0
GRID    3      0      5.0      0.0      0.0      0      0
$
$   ELEMENT DEFINITIONS
$
CROD    1      22      1      2
CROD    2      22      2      3
CHBDY   11      109     LINE   1      2
+       999      999          1.0      0.0      0.0
CHBDY   12      109     LINE   2      3
+       999      999          1.0      0.0      0.0
CHBDY   13      309     POINT   3
+       999          1.0      0.0      0.0
$
$   PROPERTY DEFINITIONS
$
PROD    22      70      3.141592
PHBDY   109      5      6.283185      1.0
PHBDY   309      5      3.141592      1.0
$
$
$   MATERIAL DATA
$
```

|         |    |       |      |
|---------|----|-------|------|
| MAT1    | 70 | 1.0+7 | 0.30 |
| MAT4    | 70 | 70.0  |      |
| \$      |    |       |      |
| MAT1    | 5  | 1.0+7 | 0.30 |
| MAT4    | 5  | 5.0   |      |
| \$      |    |       |      |
| ENDDATA |    |       |      |



---

## 2.19.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A019  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
STRAIGHT UNIFORM FIN ANALYSIS.

```

*****
*   D E S I G N   C Y C L E           0 (ANALYSIS)*
*****
```

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
STRAIGHT UNIFORM FIN ANALYSIS.

# M A S S / V O L U M E   S U M M A R Y

SYSTEM MASS                    0.000000E+00  
 SYSTEM VOLUME                1.570796E+01  
 SYSTEM MASS/VOLUME        0.000000E+00  
 1GENESIS    VERSION 7.2                    DATE 12-13-2002   TIME 12:50   PAGE    3  
 STRAIGHT UNIFORM FIN ANALYSIS.

# H E A T   S O L U T I O N   R E S I D U A L S

LOADCASE            1 ; RESIDUAL : 0.000000E+00 ;    HEAT ENERGY 8.929663E+05  
 1GENESIS    VERSION 7.2                    DATE 12-13-2002   TIME 12:50   PAGE    4  
 STRAIGHT UNIFORM FIN ANALYSIS.

CONVECTION FIN ANALYSIS USING 1 POINT-CHBDY   & 2 LINE-CHBDYS LOADCASE                    1

# G R I D   T E M P E R A T U R E S

| GRID ID | TEMP         |
|---------|--------------|
| 1       | 1.400000E+02 |
| 2       | 8.044754E+01 |
| 3       | 6.332257E+01 |
| 999     | 4.000000E+01 |

1                                    \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.20 Square Plate With Volume Heat Generation and Enforced Temperature

### Example ID:

A020

### Analysis Data Used:

GRID, CTRIA3, PSHELL, MAT4, MAT1(DUMMY), LOADCASE, HEAT, QVOL, SPCD, THERMAL, SPC and SPC1.

### Special Features Used:

Enforced temperature.

### Problem Statement:

Find the temperature distribution in the square plate with uniform energy generation as shown in the figure bellow. Assume there is no variation of the temperature in the z direction. Consider  $k = 30 \text{ watts/cm-}^{\circ}\text{K}$ ,  $L = 10.0 \text{ cm.}$ ,  $T = 50^{\circ}\text{K}$  at boundaries defined by  $(L,y)$  and  $(x,L)$ . Consider  $Q_v = 100 \text{ Watt/cm}^3$ .

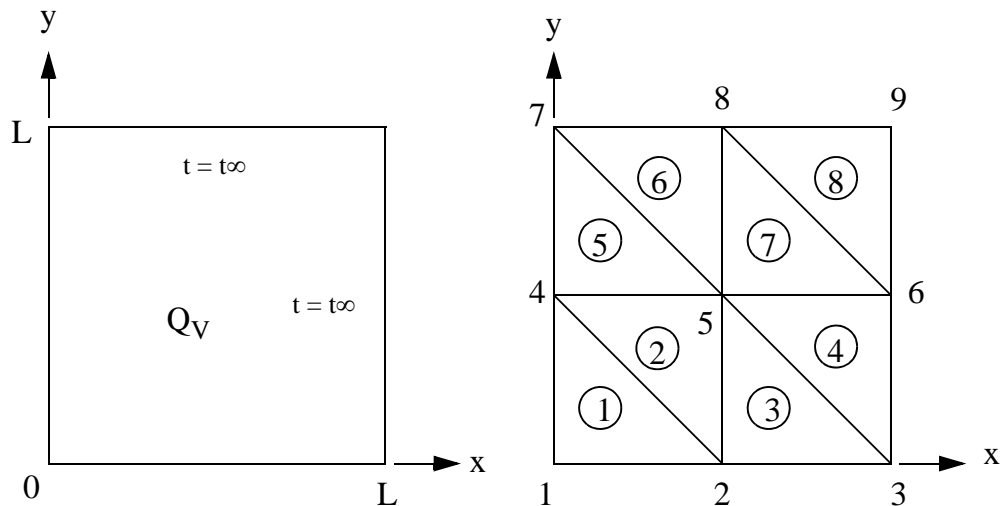


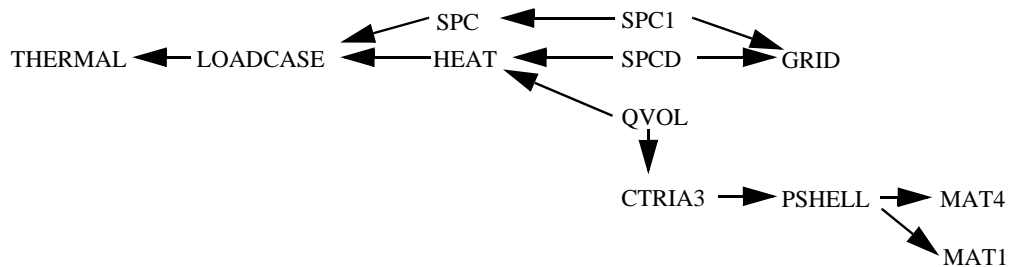
Figure 2-23

## Heat Transfer Analysis Model:

1. Plate structure assembled with 8 conductive CTRIA3 elements.
2. Section properties: PSHELL: thickness=1.0 cm.
3. Materials:
  - $k=30$  Watts/cm- $^{\circ}$ K (conduction).In addition, dummy structural material MAT1 is included in the input data.  
Structural material data is necessary to run *GENESIS*.
4. One load case: Conduction due to volume energy generation  
 $Q_v=100.0$  Watt/cm $^3$ .
  - boundary conditions: enforced temperature  $T = 50$   $^{\circ}$ K at (L,y) and (x,L).

The enforced temperature are specified with SPCD data statements. In addition, it is necessary to constrain the corresponding grids. The grids are constrained with SPC1 data statements.

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE

## Reference Solutions:

Rao,S.S, “The Finite Element Method In Engineering Pergamon Press, second edition, page 477. 1989.

| Grid ID | Temperature |
|---------|-------------|
| 1       | 133.3       |
| 2       | 119.4       |
| 3       | 50.0        |
| 4       | 119.4       |
| 5       | 105.6       |
| 6       | 50.0        |
| 7       | 50.0        |
| 8       | 50.0        |
| 9       | 50.0        |

## Calculated Solutions:.

| Grid ID | Temperature |
|---------|-------------|
| 1       | 133.3       |
| 2       | 119.4       |
| 3       | 50.0        |
| 4       | 119.4       |
| 5       | 105.6       |
| 6       | 50.0        |
| 7       | 50.0        |
| 8       | 50.0        |
| 9       | 50.0        |

## Comparison Between Calculated Solution And Reference Solution:

There is no difference between the calculated solution and the reference solution.

---

## 2.20.1 Input Data

```
ID A020
SOL COMPAT1
CEND
TITLE= SQUARE PLATE WITH VOLUME HEAT GENERATION AND ENFORCED TEMPERATURE.
LINE=64,80
ECHO=NONE
LOADCASE 1
LABEL= REF. T=(133.3,119.4,50,119.4,105.6,50.0,50.0,50,50.0 CELSIUS)
$   REFERENCE SOLUTION: Rao,S.S, "The Finite Element Method In Eng."
$                           Pergamon Press,second edition, pag.477. 1989.
    HEAT=200
    SPC=22
    THERMAL=ALL
BEGIN BULK
$
$   BOUNDARY CONDITIONS
$   ENFORCED TEMPERATURE DATA
$
SPCD    200    3                50.0
SPCD    200    6                50.0
SPCD    200    7                50.0
SPCD    200    8                50.0
SPCD    200    9                50.0
$
SPC1    22                7      8      9
SPC1    22                3      6
$
$   GRID DATA
$
GRID    1      0      0.      0.      0.0      0
GRID    2      0      5.0     0.      0.0      0
GRID    3      0      10.     0.      0.0      0
GRID    4      0      0.      5.0     0.0      0
GRID    5      0      5.0     5.0     0.0      0
GRID    6      0      10.     5.0     0.0      0
GRID    7      0      0.      10.     0.0      0
GRID    8      0      5.0     10.     0.0      0
GRID    9      0      10.     10.     0.0      0
$
$   ELEMENT DEFINITIONS
$
CTRIA3  1      22      1      2      4
CTRIA3  2      22      4      2      5
CTRIA3  3      22      2      3      5
CTRIA3  4      22      5      3      6
CTRIA3  5      22      4      5      7
CTRIA3  6      22      7      5      8
CTRIA3  7      22      5      6      8
CTRIA3  8      22      8      6      9
$
$   PROPERTY DEFINITIONS
$
```

```

PSHELL  22      33      1.
$
$  MATERIAL DATA
$
MAT1     33      1.0+6      0.25
MAT4     33      30.0
$
$  HEAT TRANSFER LOADS
$
QVOL     200      100.0  1      2      3      4
QVOL     200      100.0  5      6      7      8
$
ENDDATA

```

---

## 2.20.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G   GG   EEEE  N N N   EEEE  SSSS   I      SSSS
G   G   E      N  NN  E      S      I      S
GGGG   EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A020  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
SQUARE PLATE WITH VOLUME HEAT GENERATION AND ENFORCED TEMPERATURE.

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 9
NUMBER OF CTRIA3->PSHELL ELEMENTS: 8
TOTAL NUMBER OF NON RIGID ELEMENTS: 8
```



NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF THERMAL DEGREES OF FREEDOM: 9

#### LOAD CASES SUMMARY

NUMBER OF HEAT TRANSFER LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 SQUARE PLATE WITH VOLUME HEAT GENERATION AND ENFORCED TEMPERATURE.

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 SQUARE PLATE WITH VOLUME HEAT GENERATION AND ENFORCED TEMPERATURE.

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 1.000000E+02  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 SQUARE PLATE WITH VOLUME HEAT GENERATION AND ENFORCED TEMPERATURE.

#### H E A T S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 2.263114E-15 ; HEAT ENERGY 1.736111E+05  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 SQUARE PLATE WITH VOLUME HEAT GENERATION AND ENFORCED TEMPERATURE.

REF. T=(133.3,119.4,50,119.4,105.6,50.0,50.0,50,50.0 CELSIUS LOADCASE 1

#### G R I D T E M P E R A T U R E S

| GRID ID | TEMP         |
|---------|--------------|
| 1       | 1.333333E+02 |
| 2       | 1.194444E+02 |
| 3       | 5.000000E+01 |
| 4       | 1.194444E+02 |
| 5       | 1.055556E+02 |
| 6       | 5.000000E+01 |
| 7       | 5.000000E+01 |
| 8       | 5.000000E+01 |
| 9       | 5.000000E+01 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.21 Conduction on an Elliptical Bar Heated by a Distant Source

### Example ID:

A021

### Analysis Data Used:

GRID, CBAR, CHBDY (ELCYL), PBAR, PHBDY, MAT4, MAT1(DUMMY), LOADCASE, HEAT, QVECT, THERMAL, SPC and SPC1.

### Special Features Used:

Utilization of boundary element ELCYL and vector flux loading.

### Problem Statement:

Find the temperature distribution in the bar shown in the figure below. Consider only conduction through the bar, neglecting convection.

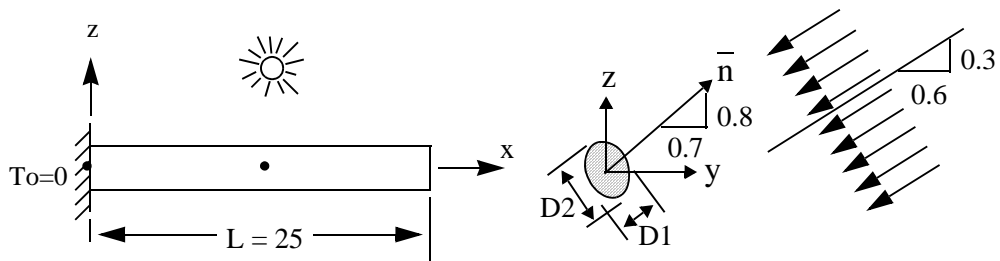
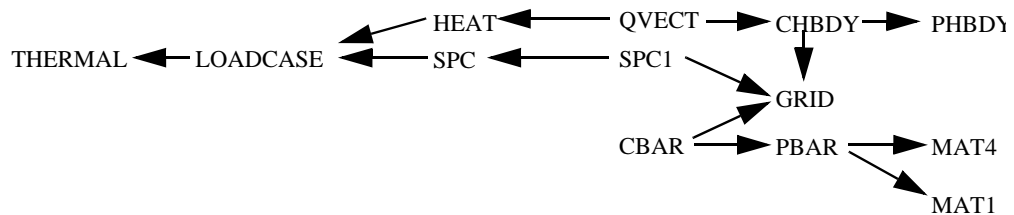


Figure 2-24

### Heat Transfer Analysis Model:

1. Bar structure assembled with 2 conductive CBAR elements and 2 CHBDY (ELCYL) boundary elements.
2. Section properties:  
PBAR: AREA=2.945243,  
PHBDY: D1=1.5, D2=2.5, ALPHA = 1.0
3. Materials:  
- $k=70$  Watts/cm- $^{\circ}$ K (conduction).  
In addition, structural material MAT1 is included in the input data.  
Structural material data is necessary to run *GENESIS*.
4. One load case: Conduction due to vector flux  $Q=100 \cdot (0.0, 0.6, 0.3)$ . Boundary condition is zero temperature at root.

## Analysis Data Relationships:



## Special Modeling Techniques:

The CHBDY (ELCYL) elements are used to compute the heat loading only. Their properties do no reference any material, in this way convection is not considered in this problem.

## Reference Solutions:

$$T(X) = \frac{qv \cdot X \left( L - \frac{X}{2} \right)}{k}$$

where;

qv = Power/volume

Power = 2611.0 (calculated using analytical integration)

volume = 2.9452\*25.0:

| X    | Temperature |
|------|-------------|
| 0.0  | 0.0         |
| 12.5 | 118.7       |
| 25.0 | 158.3       |

## Calculated Solutions:

| X    | Temperature |
|------|-------------|
| 0.0  | 0.0         |
| 12.5 | 118.7       |
| 25.0 | 158.3       |

## Comparison Between Calculated Solution And Reference Solution:

There is no difference between the calculated solution and the analytical solution.

---

## 2.21.1 Input Data

```
ID A021
SOL COMPAT1
CEND
TITLE= CONDUCTION ON AN ELLIPTICAL BAR HEATED BY A DISTANT SOURCE.
LINE=64,80
ECHO=NONE
LOADCASE 1
    LABEL= CONDUCTION USING 2 BAR. LOADING USING 2 CHBDY (ELCYL) and 2 QVECT.
    SPC=101
    HEAT=102
    THERMAL=ALL
BEGIN BULK
$
$    HEAT TRANSFER BOUNDARY CONDITIONS
$
SPC1    101            1
$
$    GRID DATA
$
GRID    1            0            0.0        0.0        0.0        0
GRID    2            0            12.5        0.0        0.0        0
GRID    3            0            25.0        0.0        0.0        0
$
$    ELEMENT DEFINITIONS
$
CBAR    1            22            1            2            0.0        1.0        0.0
CBAR    2            22            2            3            0.0        1.0        0.0
CHBDY   11            109        ELCYL    1            2            0.0        0.7        0.8
+
CHBDY   12            109        ELCYL    2            3            0.0        0.7        0.8
+
$
$    PROPERTY DEFINITIONS: DUMMY INERTIAS.
$
PBAR    22            70            2.9452431.667E-56.667E-5
PHBDY   109            1.0        1.5        2.5
$
$    MATERIAL DATA
$
MAT1    70            1.0+7            0.30
MAT4    70            70.0
$
$    HEAT LOADING: HEAT FLUX VECTOR
$
QVECT   102            100.0        .0        0.6        0.3        11
QVECT   102            100.0        .0        0.6        0.3        12
$
ENDDATA
```

---

## 2.21.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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PROJECT NAME: A021

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

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1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
CONDUCTION ON AN ELLIPTICAL BAR HEATED BY A DISTANT SOURCE.

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 3
NUMBER OF CBAR ELEMENTS: 2
NUMBER OF CHBDY ELEMENTS: 2
```

|                                       |   |
|---------------------------------------|---|
| TOTAL NUMBER OF NON RIGID ELEMENTS:   | 4 |
| NUMBER OF ELEMENT PROPERTIES:         | 2 |
| NUMBER OF MATERIALS:                  | 1 |
| NUMBER OF THERMAL DEGREES OF FREEDOM: | 2 |

#### LOAD CASES SUMMARY

|                                     |   |
|-------------------------------------|---|
| NUMBER OF HEAT TRANSFER LOAD CASES: | 1 |
| TOTAL NUMBER OF LOAD CASES:         | 1 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
CONDUCTION ON AN ELLIPTICAL BAR HEATED BY A DISTANT SOURCE.

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
CONDUCTION ON AN ELLIPTICAL BAR HEATED BY A DISTANT SOURCE.

#### M A S S / V O L U M E S U M M A R Y

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 0.000000E+00 |
| SYSTEM VOLUME      | 7.363107E+01 |
| SYSTEM MASS/VOLUME | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
CONDUCTION ON AN ELLIPTICAL BAR HEATED BY A DISTANT SOURCE.

#### H E A T S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -3.483986E-16 ; HEAT ENERGY 1.291185E+05  
1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
CONDUCTION ON AN ELLIPTICAL BAR HEATED BY A DISTANT SOURCE.

CONDUCTION USING 2 BAR. LOADING USING 2 CHBDY (ELCYL) AND 2 LOADCASE 1

#### G R I D T E M P E R A T U R E S

| GRID ID | TEMP         |
|---------|--------------|
| 1       | 0.000000E+00 |
| 2       | 1.187069E+02 |
| 3       | 1.582758E+02 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.22 Heat Conduction in an Anisotropic Plate

### Example ID:

A022

### Analysis Data Used:

GRID, CQUAD4, PSHELL, MAT5, MAT8(DUMMY), LOADCASE, SPC, SPC, SPC1, HEAT, QHBDY, THERMAL.

### Special Features Used:

Utilization of anisotropic material MAT5 with PSHELL.

### Problem Statement:

Find the temperature distribution in the anisotropic plate shown in the figure below. The plate is loaded in one edge with a constant flux load of 100.0 units. The boundary at the opposite edge of the loading has a constant temperature of zero degrees.

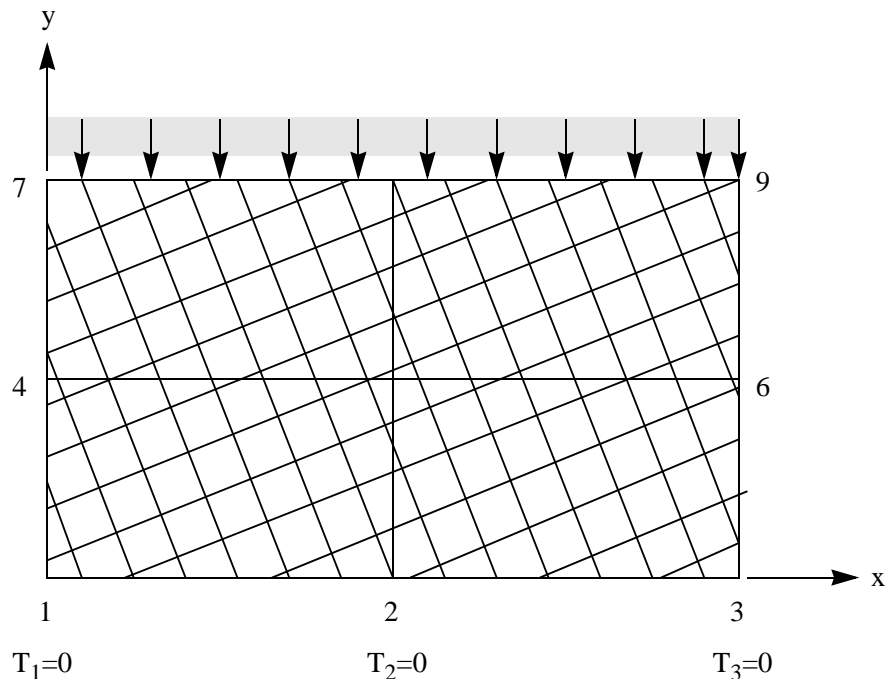


Figure 2-25

### Heat Transfer Analysis Model:

1. Plate assembled with 4 QUAD4 elements.

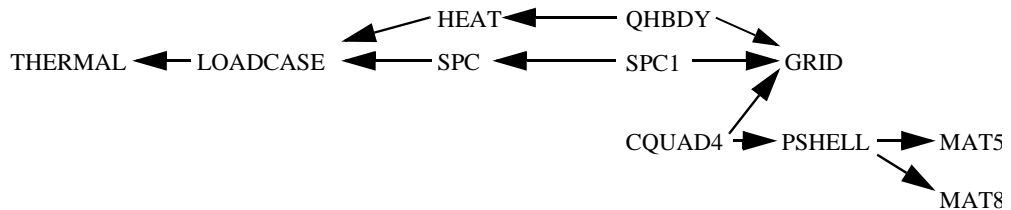
2. Section properties of plate: thickness=2.0
3. Material:
  - KXX=65.215
  - KXY=12.0241
  - KYY=39.78501

This material correspond to an orthotropic material of  $k_{xx}=70$  and  $k_{yy}=35$  rotated 21.7 degrees.

In addition, dummy structural material MAT8 is included in the input data.

Structural material data is necessary to run *GENESIS*.
4. One HEAT FLUX Load case: a)  $Q_0 = 100.0$  units.

### Analysis Data Relationships:



### Special Modeling Techniques:

NONE.

### Reference Solutions:

See problem A023



### Calculated Solutions:

| Grid ID | Temperature |
|---------|-------------|
| 1       | 0.000       |
| 2       | 0.000       |
| 3       | 0.000       |
| 4       | 16.171      |
| 5       | 12.636      |
| 6       | 9.791       |
| 7       | 29.939      |
| 8       | 25.839      |
| 9       | 22.046      |

### Comparison Between Calculated Solution And Reference Solution:

There are no differences between the reference solution and the *GENESIS* solution.

---

## 2.22.1 Input Data

```
ID A022
SOL COMPAT1
CEND
TITLE=HEAT CONDUCTION IN AN ANISOTROPIC PLATE.
SUBTITLE=USE OF 4 CONDUCTIVE QUAD4 AND 2 QHBDY(LINE). HEAT FLUX LOADING.
ECHO=NONE
LINE=64,80
LOADCASE 10
    HEAT = 1
    THERMAL=ALL
    SPC=1
$
BEGIN BULK
$
$    HEAT TRANSFER BOUNDARY CONDITIONS
$
SPC1      1          1          2          3
$    GRID POINTS
$
GRID      1          0.0        0.0        0.0
GRID      2          10.0        0.0        0.0
GRID      3          20.0        0.0        0.0
$
GRID      4          0.0         5.0        0.0
GRID      5          10.0        5.0        0.0
GRID      6          20.0        5.0        0.0
$
GRID      7          0.0         10.0       0.0
GRID      8          10.0        10.0       0.0
GRID      9          20.0        10.0       0.0
$
$    ELEMENT DEFINITIONS
$
CQUAD4    1          1          1          2          5          4
CQUAD4    2          1          2          3          6          5
CQUAD4    3          1          4          5          8          7
CQUAD4    4          1          5          6          9          8
$
$    PROPERTY DEFINITIONS
$
PSHELL    1          1          2.0
$
$    MATERIAL DEFINITIONS
$
MAT8      1          7.8+6      2.6+6      0.25      1.3+6
+          3.50-6      11.40-6  80.0
MAT5      1          65.215     12.0241  0.0        39.78501
$
$    HEAT FLUX LOADING DEFINITION
$
QHBDY     1          LINE      100.0      2.0        7          8
QHBDY     1          LINE      100.0      2.0        8          9
```

\$  
ENDDATA

---

## 2.22.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A022  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
HEAT CONDUCTION IN AN ANISOTROPIC PLATE.  
USE OF 4 CONDUCTIVE QUAD4 AND 2 QHBDY(LINE). HEAT FLUX LOADING.

### ANALYSIS PROBLEM SUMMARY

|                                     |   |
|-------------------------------------|---|
| NUMBER OF GRID POINTS:              | 9 |
| NUMBER OF CQUAD4->PSHELL ELEMENTS:  | 4 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 4 |

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF THERMAL DEGREES OF FREEDOM: 6

#### LOAD CASES SUMMARY

NUMBER OF HEAT TRANSFER LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 HEAT CONDUCTION IN AN ANISOTROPIC PLATE.  
 USE OF 4 CONDUCTIVE QUAD4 AND 2 QHBDY(LINE). HEAT FLUX LOADING.

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 HEAT CONDUCTION IN AN ANISOTROPIC PLATE.  
 USE OF 4 CONDUCTIVE QUAD4 AND 2 QHBDY(LINE). HEAT FLUX LOADING.

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 4.000000E+02  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 HEAT CONDUCTION IN AN ANISOTROPIC PLATE.  
 USE OF 4 CONDUCTIVE QUAD4 AND 2 QHBDY(LINE). HEAT FLUX LOADING.

#### H E A T S O L U T I O N R E S I D U A L S

LOADCASE 10 ; RESIDUAL : -5.362542E-16 ; HEAT ENERGY 5.183068E+04  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 HEAT CONDUCTION IN AN ANISOTROPIC PLATE.  
 USE OF 4 CONDUCTIVE QUAD4 AND 2 QHBDY(LINE). HEAT FLUX LOADING.  
 LOADCASE 10

#### G R I D T E M P E R A T U R E S

| GRID ID | TEMP         |
|---------|--------------|
| 1       | 0.000000E+00 |
| 2       | 0.000000E+00 |
| 3       | 0.000000E+00 |
| 4       | 1.617075E+01 |
| 5       | 1.263648E+01 |
| 6       | 9.790604E+00 |
| 7       | 2.993856E+01 |
| 8       | 2.583861E+01 |
| 9       | 2.204558E+01 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.23 Heat Conduction in an Anisotropic Solid

### Example ID:

A023

### Analysis Data Used:

GRID, CHEXA, PSOLID, MAT5, MAT1(DUMMY), LOADCASE, HEAT, QHBDY, SPC, SPC1, THERMAL.

### Special Features Used:

Utilization of anisotropic material MAT5 with PSOLID.

### Problem Statement:

Find the temperature distribution in the anisotropic solid shown in the figure below. The solid is loaded in one face with a constant flux load of 100.0 units. The boundary at the face opposite of the loading has a constant temperature of zero degrees.

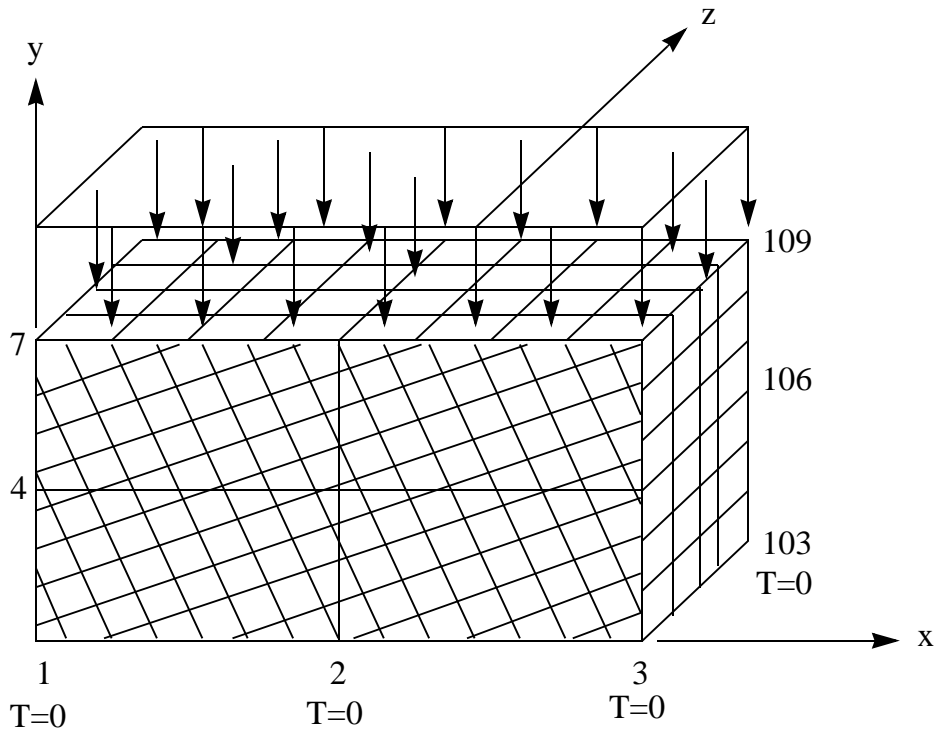


Figure 2-26

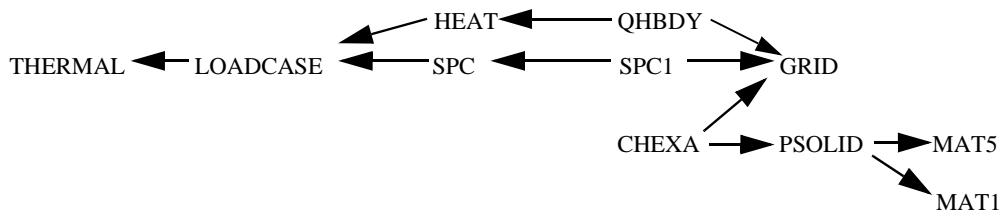
## Heat Transfer Analysis Model:

1. Solid assembled with 4 conductive CHEXA elements.
2. Material:
  - KXX=65.21500
  - KXY=12.02410
  - KYY=39.78501
  - KZZ=50.00000

This material corresponds to an orthotropic material of  $k_{xx}=70$ ,  $k_{yy}=35$  and  $k_{zz}=50$  rotated 21.7 degrees in the z-axis.

In addition, dummy structural material MAT1 is included in the input data. Structural material data is necessary to run *GENESIS*.
3. One HEAT FLUX Load case:  $Q_0 = 100.0$  units.

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE.

## Reference Solutions:

See problem A022, in that problem the solid is modeled as a plate. Since the heat flux gradient is zero in the z direction the plate solution can capture the 3-D temperature field. Calculated Solutions:

| Grid ID | Temperature |
|---------|-------------|
| 1       | 0.000       |
| 2       | 0.000       |
| 3       | 0.000       |
| 4       | 16.171      |
| 5       | 12.636      |
| 6       | 9.791       |
| 7       | 29.939      |
| 8       | 25.839      |
| 9       | 22.046      |
| 101     | 0.000       |
| 102     | 0.000       |
| 103     | 0.000       |
| 104     | 16.171      |
| 105     | 12.636      |
| 106     | 9.791       |
| 107     | 29.939      |
| 108     | 25.839      |
| 109     | 22.046      |

## Comparison Between Calculated Solution And Reference Solution:

There are no differences between the reference solutions and the calculated solution.



---

## 2.23.1 Input Data

```
ID A023
SOL COMPAT1
CEND
TITLE    =HEAT CONDUCTION IN AN ANISOTROPIC SOLID.
ECHO=NONE
LINE=64,80
LOADCASE 10
    HEAT = 1
    SPC=1
    THERMAL=ALL
BEGIN BULK
$
$    HEAT TRANSFER BOUNDARY CONDITIONS
$
SPC1      1          1          2          3
SPC1      1          101        102        103
$
$    GRID DATA
$
GRID      1          0.0        0.00        0.0
GRID      2          10.0        0.00        0.0
GRID      3          20.0        0.00        0.0
$
GRID      4          0.0        5.00        0.0
GRID      5          10.0        5.00        0.0
GRID      6          20.0        5.00        0.0
$
GRID      7          0.0        10.00       0.0
GRID      8          10.0        10.00       0.0
GRID      9          20.0        10.00       0.0
$
GRID      101        0.0        0.00        2.0
GRID      102        10.0        0.00        2.0
GRID      103        20.0        0.00        2.0
$
GRID      104        0.0        5.00        2.0
GRID      105        10.0        5.00        2.0
GRID      106        20.0        5.00        2.0
$
GRID      107        0.0        10.00       2.0
GRID      108        10.0        10.00       2.0
GRID      109        20.0        10.00       2.0
$
$    ELEMENT DEFINITIONS
$
CHEXA     1          1          1          2          5          4          101        102
+         105        104
CHEXA     2          1          2          3          6          5          102        103
+         106        105
CHEXA     3          1          4          5          8          7          104        105
+         108        107
CHEXA     4          1          5          6          9          8          105        106
```

```

+      109      108
$
$      PROPERTY DEFINITIONS
$
PSOLID  1      1
$
$      MATERIAL DEFINITIONS
$
MAT1    1      7.777      0.3
MAT5    1      65.215  12.0241  0.0      39.78501  0.0      50.0
$
$
$      HEAT FLUX DEFINITION
$
QHBDY   1      AREA4    100.0      7      8      108      107
QHBDY   1      AREA4    100.0      8      9      109      108
$
ENDDATA

```

---

## 2.23.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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PROJECT NAME: A023

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

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1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
HEAT CONDUCTION IN AN ANISOTROPIC SOLID.

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 18
NUMBER OF CHEXA ELEMENTS: 4
TOTAL NUMBER OF NON RIGID ELEMENTS: 4
```

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF THERMAL DEGREES OF FREEDOM: 12

#### LOAD CASES SUMMARY

NUMBER OF HEAT TRANSFER LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 HEAT CONDUCTION IN AN ANISOTROPIC SOLID.

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 HEAT CONDUCTION IN AN ANISOTROPIC SOLID.

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 4.000000E+02  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 HEAT CONDUCTION IN AN ANISOTROPIC SOLID.

#### H E A T S O L U T I O N R E S I D U A L S

LOADCASE 10 ; RESIDUAL : -1.637177E-15 ; HEAT ENERGY 5.183068E+04  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 HEAT CONDUCTION IN AN ANISOTROPIC SOLID.

LOADCASE 10

#### G R I D T E M P E R A T U R E S

| GRID ID | TEMP         |
|---------|--------------|
| 1       | 0.000000E+00 |
| 2       | 0.000000E+00 |
| 3       | 0.000000E+00 |
| 4       | 1.617075E+01 |
| 5       | 1.263648E+01 |
| 6       | 9.790604E+00 |
| 7       | 2.993856E+01 |
| 8       | 2.583861E+01 |
| 9       | 2.204558E+01 |
| 101     | 0.000000E+00 |
| 102     | 0.000000E+00 |
| 103     | 0.000000E+00 |

```
104      1.617075E+01
105      1.263648E+01
106      9.790604E+00
107      2.993856E+01
108      2.583861E+01
109      2.204558E+01
1          ***** END OF OUTPUT *****
```

---

## 2.24 Convection-Conduction Heat Transfer in the Straight Cantilever Problem

### Example ID:

A024

### Analysis Data Used:

GRID, CQUAD4, CHBDY (AREA4), PSHELL, PHBDY, MAT4, MAT1(DUMMY), LOAD-CASE, HEAT, SPCD, THERMAL, SPC and SPC1.

### Special Features Used:

Utilization of ambient temperature loading and use of boundary element AREA4.

### Problem Statement:

Find the temperature distribution in the straight cantilever shown in the figure bellow. The beam is subjected to an ambient temperature of 100 degrees. Consider  $k = 90$  watts/cm-°K for conduction and  $h = 10$  watts/cm<sup>2</sup>-°K for convection. The length of the beam is 6.00 cm and its cross section 0.2x0.1 cm<sup>2</sup>. The temperature at the root is zero. Consider convection through one of the 6\*0.2 faces.

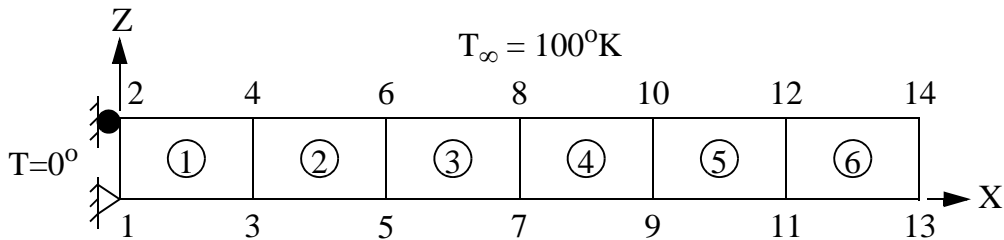


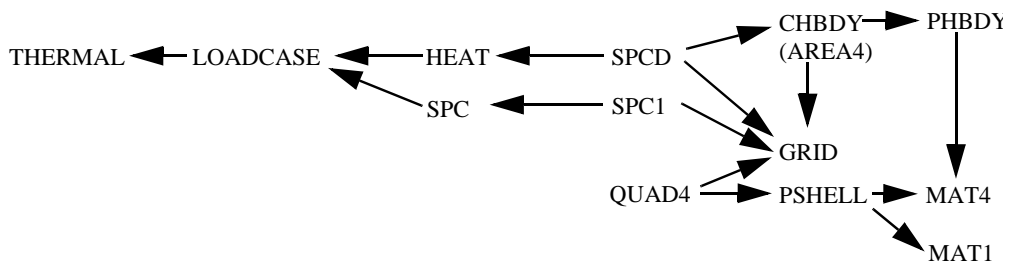
Figure 2-27

## Heat Transfer Analysis Model:

1. Beam structure assembled with 6 conductive QUAD4 elements and 6 convective CHBDY (AREA4) boundary elements.
2. Section properties:
  - thickness=0.1 cm.
3. Materials:
  - $k=90 \text{ Watts/cm-}^{\circ}\text{K}$  (conduction).
  - $h=10.0 \text{ Watts/cm}^2\text{-}^{\circ}\text{K}$  (convection).In addition, dummy structural material MAT1 is included in the input data.  
Structural material data is necessary to run *GENESIS*.
4. One load case: Conduction and convection due to ambient temperature of 100.0 degrees.

The ambient temperature is specified with a SPCD data statement. In addition, it is necessary to constrain the corresponding grid (999). The grid is constrained with a SPC1 data statement.

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE

## Reference Solutions:

NONE.

### Calculated Solutions:

| Location | Temperature |
|----------|-------------|
| x=0.0    | 0.000       |
| x=1.0    | 67.04       |
| x=2.0    | 89.14       |
| x=3.0    | 96.42       |
| x=4.0    | 98.81       |
| x=5.0    | 99.57       |
| x=6.0    | 99.74       |



---

## 2.24.1 Input Data

```
ID A024
SOL COMPAT1
CEND
TITLE= CONDUCTION-CONVECTION ANALYSIS OF THE STRAIGHT CANTILEVER BEAM
SUBTITLE= 1 HEAT TRANSFER LOAD CONDITIONS, 6 QUAD4 & 6 CHBDY (AREA4).
LINE=64,80
ECHO=NONE
$
LOADCASE 1
    HEAT=101
    SPC=101
    THERMAL=ALL
$
BEGIN BULK
$
$    HEAT TRANSFER BOUNDARY CONDITIONS
$
SPC1    101            1
SPC1    101            2
$
$    AMBIENT TEMPERATURE
$
SPC1    101            999
SPCD    101    999    100.0
GRID    999    0    0.0    0.0    3.0    0    0
$
$    GRID DATA
$
GRID    1    0    0.0    0.0    0.0    0    0
GRID    2    0    0.0    0.0    0.2    0    0
GRID    3    0    1.0    0.0    0.0    0    0
GRID    4    0    1.0    0.0    0.2    0    0
GRID    5    0    2.0    0.0    0.0    0    0
GRID    6    0    2.0    0.0    0.2    0    0
GRID    7    0    3.0    0.0    0.0    0    0
GRID    8    0    3.0    0.    0.2    0    0
GRID    9    0    4.0    0.    0.0    0    0
GRID    10    0    4.0    0.0    0.2    0    0
GRID    11    0    5.0    0.0    0.0    0    0
GRID    12    0    5.0    0.0    0.2    0    0
GRID    13    0    6.0    0.0    0.0    0    0
GRID    14    0    6.0    0.0    0.2    0    0
$
$    ELEMENT DEFINITIONS
$
CQUAD4  1    22    1    3    4    2
CQUAD4  2    22    3    5    6    4
CQUAD4  3    22    5    7    8    6
CQUAD4  4    22    7    9    10    8
CQUAD4  5    22    9    11    12    10
CQUAD4  6    22    11    13    14    12
$
```

|                         |     |       |       |     |     |     |     |
|-------------------------|-----|-------|-------|-----|-----|-----|-----|
| CHBDY                   | 101 | 109   | AREA4 | 1   | 3   | 4   | 2   |
| +                       | 999 | 999   | 999   | 999 | 1.0 | 0.0 | 0.0 |
| CHBDY                   | 102 | 109   | AREA4 | 3   | 5   | 6   | 4   |
| +                       | 999 | 999   | 999   | 999 | 1.0 | 0.0 | 0.0 |
| CHBDY                   | 103 | 109   | AREA4 | 5   | 7   | 8   | 6   |
| +                       | 999 | 999   | 999   | 999 | 1.0 | 0.0 | 0.0 |
| CHBDY                   | 104 | 109   | AREA4 | 7   | 9   | 10  | 8   |
| +                       | 999 | 999   | 999   | 999 | 1.0 | 0.0 | 0.0 |
| CHBDY                   | 105 | 109   | AREA4 | 9   | 11  | 12  | 10  |
| +                       | 999 | 999   | 999   | 999 | 1.0 | 0.0 | 0.0 |
| CHBDY                   | 106 | 109   | AREA4 | 11  | 13  | 14  | 12  |
| +                       | 999 | 999   | 999   | 999 | 1.0 | 0.0 | 0.0 |
| \$                      |     |       |       |     |     |     |     |
| \$ PROPERTY DEFINITIONS |     |       |       |     |     |     |     |
| \$                      |     |       |       |     |     |     |     |
| PSHELL                  | 22  | 33    | 0.1   | 33  |     |     |     |
| PHBDY                   | 109 | 109   |       |     |     |     |     |
| \$                      |     |       |       |     |     |     |     |
| \$ MATERIAL DATA        |     |       |       |     |     |     |     |
| \$                      |     |       |       |     |     |     |     |
| MAT1                    | 33  | 1.0+7 | 0.30  |     |     |     |     |
| MAT4                    | 33  | 90.0  |       |     |     |     |     |
| MAT4                    | 109 | 10.0  |       |     |     |     |     |
| \$                      |     |       |       |     |     |     |     |
| ENDDATA                 |     |       |       |     |     |     |     |

---

## 2.24.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A024  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
CONDUCTION-CONVECTION ANALYSIS OF THE STRAIGHT CANTILEVER BEAM  
1 HEAT TRANSFER LOAD CONDITIONS, 6 QUAD4 & 6 CHBDY (AREA4).

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 15
NUMBER OF CQUAD4->PSHELL ELEMENTS: 6
NUMBER OF CHBDY ELEMENTS: 6
```

TOTAL NUMBER OF NON RIGID ELEMENTS: 12  
 NUMBER OF ELEMENT PROPERTIES: 2  
 NUMBER OF MATERIALS: 1  
 NUMBER OF THERMAL DEGREES OF FREEDOM: 13

#### LOAD CASES SUMMARY

NUMBER OF HEAT TRANSFER LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 CONDUCTION-CONVECTION ANALYSIS OF THE STRAIGHT CANTILEVER BEAM  
 1 HEAT TRANSFER LOAD CONDITIONS, 6 QUAD4 & 6 CHBDY (AREA4).

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 CONDUCTION-CONVECTION ANALYSIS OF THE STRAIGHT CANTILEVER BEAM  
 1 HEAT TRANSFER LOAD CONDITIONS, 6 QUAD4 & 6 CHBDY (AREA4).

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 0.000000E+00  
 SYSTEM VOLUME 1.200000E-01  
 SYSTEM MASS/VOLUME 0.000000E+00

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 CONDUCTION-CONVECTION ANALYSIS OF THE STRAIGHT CANTILEVER BEAM  
 1 HEAT TRANSFER LOAD CONDITIONS, 6 QUAD4 & 6 CHBDY (AREA4).

#### H E A T S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 6.636613E-15 ; HEAT ENERGY 5.008372E+04  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 CONDUCTION-CONVECTION ANALYSIS OF THE STRAIGHT CANTILEVER BEAM  
 1 HEAT TRANSFER LOAD CONDITIONS, 6 QUAD4 & 6 CHBDY (AREA4).  
 LOADCASE 1

#### G R I D T E M P E R A T U R E S

| GRID ID | TEMP         |
|---------|--------------|
| 1       | 0.000000E+00 |
| 2       | 0.000000E+00 |
| 3       | 6.704024E+01 |
| 4       | 6.704024E+01 |
| 5       | 8.913534E+01 |
| 6       | 8.913534E+01 |
| 7       | 9.641499E+01 |
| 8       | 9.641499E+01 |
| 9       | 9.880599E+01 |
| 10      | 9.880599E+01 |
| 11      | 9.956880E+01 |

```
12      9.956880E+01
13      9.974361E+01
14      9.974361E+01
999      1.000000E+02
1      ***** END OF OUTPUT *****
```

---

## 2.25 Dynamic Response of the Cantilevered Beam

### Example ID:

A025

### Analysis Data Used:

GRID, CBAR, PBAR, MAT1, LOADCASE, METHOD, DLOAD, FREQ, MODES, DISP, EIGR, RLOAD1, DAREA, DELAY, TABLED1

### Special Feature Used:

Frequency response analysis using direct dynamic analysis and modal dynamic analysis.

### Problem Statement:

Find the nodal displacements of the cantilever beam shown in the figure below. The beam is loaded with a tip point load of 100.0 N and it is applied at three different frequencies. The first frequency is close to zero to simulate a static loading ( $f=1.0\text{e-}8$  hz), the second frequency is  $f=8.8$  hz and the third is  $f=57.1$  hz. The last two frequencies are very close to the first and second natural modes to simulate loadings close to resonance. Use direct dynamic analysis and modal analysis. For modal analysis consider the first three modes. Do not consider damping.

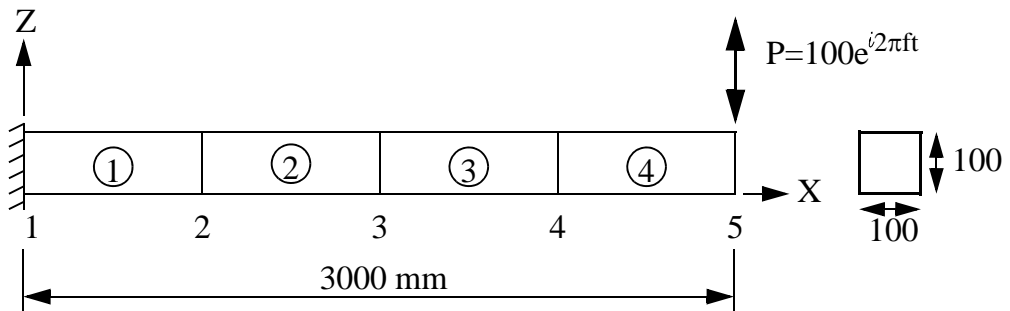
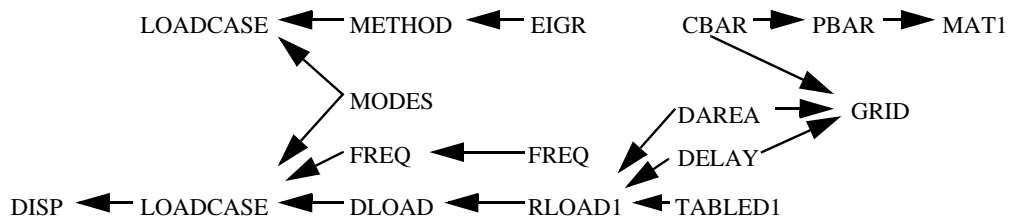


Figure 2-28

## Structural Analysis Model:

1. Four BAR elements.
2. The section properties are: area=10000 mm<sup>2</sup>. I=8.3+6 mm<sup>4</sup>
3. Material: E=2.07E+5 MPa. density=8.0-9 N-sec<sup>2</sup>/mm<sup>4</sup>
4. One frequency load case, necessary for modal dynamic analysis.
5. One direct dynamic response load case. P=100 N
6. One modal dynamic response load case. P=100 N
7. Excitation frequencies for the problem are: f1=1.0e-8 hz, f2=8.8 hz and f3=57.1 hz.

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE.

## Reference Solutions:

None for the displacements. Their values can be verified by comparing load cases two and three which are obtained using different methodologies.

Note that the theoretical solutions for the natural frequencies and the static solution at the tip are:

$$f1 = 0.5596 \sqrt{EI/\rho AL^4} = 9.13 \text{ Hz}$$

$$f2 = 3.5069 \sqrt{EI/\rho AL^4} = 57.21 \text{ Hz}$$

$$Y = PL^3/3EI = 0.52176 \text{ mm}$$

where E is the young's modulus, A the cross section area, I the inertia,  $\rho$  the mass density and L the length.

## Calculated Solutions:

| Grid | Direct Dynamic Response (mm) | Modal Dynamic Response (mm) |
|------|------------------------------|-----------------------------|
|      | f = 1.0e-8 hz                |                             |
| 1    | 0.0000                       | 0.0000                      |
| 2    | 0.0448                       | 0.0448                      |
| 3    | 0.1631                       | 0.1631                      |
| 4    | 0.3302                       | 0.3302                      |
| 5    | 0.5218                       | 0.5218                      |
|      | f = 8.80 hz                  |                             |
| 1    | 0.0000                       | 0.0000                      |
| 2    | 0.6930                       | 0.6930                      |
| 3    | 2.4254                       | 2.4254                      |
| 4    | 4.7134                       | 4.7134                      |
| 5    | 7.1863                       | 7.1863                      |
|      | f = 57.1 hz                  |                             |
| 1    | 0.0000                       | 0.0000                      |
| 2    | -1.5875                      | -1.5875                     |
| 3    | -2.7210                      | -2.7210                     |
| 4    | -0.5268                      | -0.5268                     |
| 5    | 3.7934                       | 3.7934                      |

The solutions for the natural frequencies are:

f1 = 9.13 hz

f2 = 57.20 hz



## **Comparison Between Calculated Solutions And Reference Solutions:**

The two approaches, direct and modal analysis, give the same results for the two load cases.

The differences between analytical solutions and calculated solutions for natural frequencies and static tip displacement are:

f1: 0%

f2: 0.05%

y : 0%

---

## 2.25.1 Input Data

```
ID A025
SOL COMPAT1
CEND
TITLE=DYNAMIC RESPONSE OF THE CANTILEVER BEAM
LINE=64,80
ECHO=NONE
$
DYNOUTPUT = REAL
$
LOADCASE 1
LABEL = NATURAL FREQUENCIES (TO BE USED IN LOADCASE 3)
METHOD=4
$
LOADCASE 2
LABEL = SOLUTION USING:  DIRECT DYNAMIC RESPONSE
FREQ = 32
DLOAD = 23
DISP=ALL
$
LOADCASE 3
LABEL = SOLUTION USING:  MODAL DYNAMIC RESPONSE
FREQ = 32
DLOAD = 23
MODES=1
DISP=ALL
$
BEGIN BULK
$
$   FREQUENCIES (FIRST FREQ. IS CLOSE STATIC SOLUTION, NEXT 2 ARE
$   CLOSE TO THE FIRST 2 NATURAL FREQUENCIES I.E. CLOSE TO RESONANCE)
$
FREQ      32      1.0E-8  8.8      57.1
$
$   GRID DATA (ONLY DISPLACEMENTS IN Z AND ROTATIONS IN Y ARE REQUESTED)
$
GRID      1          0.00  0.      0.0      123456
GRID      2          750.0  0.      0.0      1246
GRID      3          1500.0  0.      0.0      1246
GRID      4          2250.0  0.      0.0      1246
GRID      5          3000.0  0.      0.0      1246
$
$   ELEMENT DEFINITIONS
$
CBAR      1          1          1          2          0.      1.0      0.
CBAR      2          1          2          3          0.      1.0      0.
CBAR      3          1          3          4          0.      1.0      0.
CBAR      4          1          4          5          0.      1.0      0.
$
$   PROPERTY DEFINITIONS
$
PBAR      1          1          1.0+4  8.333+6  8.333+6  0.0
$
```

```

$ MATERIAL DATA
$
MAT1      1      2.07+5      0.3      8.0-9
$
$ EIGENVALUES DATA
$
EIGR      4      SUB      3      MAX
$
$ LOAD DEFINITION
$
RLOAD1    23      1      2      4
$
DAREA     1      5      3      100.0
DELAY     2      5      3      60.0
$
$ C(W) = 1.0 = CONSTANT (SAME LOAD FOR EACH EXITING FREQUENCY)
$
TABLED1   4
+         1.0      1.0      2.0      1.0
$
ENDDATA

```

---

## 2.25.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A025

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 5
NUMBER OF CBAR ELEMENTS: 4
TOTAL NUMBER OF NON RIGID ELEMENTS: 4
```

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 8

#### LOAD CASES SUMMARY

NUMBER OF USER FREQUENCY LOAD CASES: 1  
 NUMBER OF DIRECT DYNAMIC LOAD CASES: 1  
 NUMBER OF MODAL DYNAMIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 3

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 2.400000E-01  
 SYSTEM VOLUME 3.000000E+07  
 SYSTEM MASS/VOLUME 8.000000E-09

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM

NATURAL FREQUENCIES (TO BE USED IN LOADCASE 3) LOADCASE 1

#### E I G E N V A L U E S

| MODE | CYCLES       | EIGENVALUE   | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|--------------|--------------|---------------------|--------------------------|
| 1    | 9.128302E+00 | 3.289575E+03 | 5.735481E+01 | 6.001558E-02        | 1.974257E+02             |
| 2    | 5.719718E+01 | 1.291543E+05 | 3.593805E+02 | 5.994695E-02        | 7.742408E+03             |
| 3    | 1.608664E+02 | 1.021622E+06 | 1.010753E+03 | 5.897694E-02        | 6.025216E+04             |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM

SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

G R I D D Y N A M I C D I S P L A C E M E N T S (FREQUENCY = 1.0000E-08)

| GRID ID | T1 | T2 | T3 |
|---------|----|----|----|
| R1      |    | R2 | R3 |

(REAL/IMAGINARY)

|   |      |              |              |              |
|---|------|--------------|--------------|--------------|
| 1 | REAL | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|---|------|--------------|--------------|--------------|

|   |      |              |               |              |
|---|------|--------------|---------------|--------------|
|   | REAL | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
| 2 | REAL | 0.000000E+00 | 0.000000E+00  | 4.483875E-02 |
|   | REAL | 0.000000E+00 | -1.141350E-04 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 1.690381E-07 |
|   | IMAG | 0.000000E+00 | -4.302788E-10 | 0.000000E+00 |
| 3 | REAL | 0.000000E+00 | 0.000000E+00  | 1.630500E-01 |
|   | REAL | 0.000000E+00 | -1.956600E-04 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 6.146840E-07 |
|   | IMAG | 0.000000E+00 | -7.376208E-10 | 0.000000E+00 |
| 4 | REAL | 0.000000E+00 | 0.000000E+00  | 3.301763E-01 |
|   | REAL | 0.000000E+00 | -2.445750E-04 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 1.244735E-06 |
|   | IMAG | 0.000000E+00 | -9.220260E-10 | 0.000000E+00 |
| 5 | REAL | 0.000000E+00 | 0.000000E+00  | 5.217600E-01 |
|   | REAL | 0.000000E+00 | -2.608800E-04 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 1.966989E-06 |
|   | IMAG | 0.000000E+00 | -9.834944E-10 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 6  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM

SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 8.8000E+00)

| GRID ID |      | T1<br>R1         | T2<br>R2      | T3<br>R3     |
|---------|------|------------------|---------------|--------------|
|         |      | (REAL/IMAGINARY) |               |              |
| 1       | REAL | 0.000000E+00     | 0.000000E+00  | 0.000000E+00 |
|         | REAL | 0.000000E+00     | 0.000000E+00  | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 0.000000E+00 |
| 2       | REAL | 0.000000E+00     | 0.000000E+00  | 6.929855E-01 |
|         | REAL | 0.000000E+00     | -1.731171E-03 | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 3.043009E-13 |
|         | IMAG | 0.000000E+00     | -7.601845E-16 | 0.000000E+00 |
| 3       | REAL | 0.000000E+00     | 0.000000E+00  | 2.425352E+00 |
|         | REAL | 0.000000E+00     | -2.779265E-03 | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 1.065010E-12 |
|         | IMAG | 0.000000E+00     | -1.220419E-15 | 0.000000E+00 |
| 4       | REAL | 0.000000E+00     | 0.000000E+00  | 4.713387E+00 |
|         | REAL | 0.000000E+00     | -3.237488E-03 | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 2.069722E-12 |
|         | IMAG | 0.000000E+00     | -1.421632E-15 | 0.000000E+00 |

|   |      |              |               |              |
|---|------|--------------|---------------|--------------|
| 5 | REAL | 0.000000E+00 | 0.000000E+00  | 7.186289E+00 |
|   | REAL | 0.000000E+00 | -3.319345E-03 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 3.155613E-12 |
|   | IMAG | 0.000000E+00 | -1.457577E-15 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 7  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM

SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 5.7100E+01)

| GRID ID          |      | T1<br>R1     | T2<br>R2      | T3<br>R3      |
|------------------|------|--------------|---------------|---------------|
| (REAL/IMAGINARY) |      |              |               |               |
| 1                | REAL | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
|                  | REAL | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
|                  | IMAG | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
|                  | IMAG | 0.000000E+00 | 0.000000E+00  | 0.000000E+00  |
| 2                | REAL | 0.000000E+00 | 0.000000E+00  | -1.587532E+00 |
|                  | REAL | 0.000000E+00 | 2.902462E-03  | 0.000000E+00  |
|                  | IMAG | 0.000000E+00 | 0.000000E+00  | -4.050359E-13 |
|                  | IMAG | 0.000000E+00 | 7.405213E-16  | 0.000000E+00  |
| 3                | REAL | 0.000000E+00 | 0.000000E+00  | -2.720996E+00 |
|                  | REAL | 0.000000E+00 | -5.631984E-04 | 0.000000E+00  |
|                  | IMAG | 0.000000E+00 | 0.000000E+00  | -6.942228E-13 |
|                  | IMAG | 0.000000E+00 | -1.436919E-16 | 0.000000E+00  |
| 4                | REAL | 0.000000E+00 | 0.000000E+00  | -5.267900E-01 |
|                  | REAL | 0.000000E+00 | -4.930351E-03 | 0.000000E+00  |
|                  | IMAG | 0.000000E+00 | 0.000000E+00  | -1.344028E-13 |
|                  | IMAG | 0.000000E+00 | -1.257908E-15 | 0.000000E+00  |
| 5                | REAL | 0.000000E+00 | 0.000000E+00  | 3.793373E+00  |
|                  | REAL | 0.000000E+00 | -6.071034E-03 | 0.000000E+00  |
|                  | IMAG | 0.000000E+00 | 0.000000E+00  | 9.678243E-13  |
|                  | IMAG | 0.000000E+00 | -1.548937E-15 | 0.000000E+00  |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 8  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM

SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 1.0000E-08)

| GRID ID          |  | T1<br>R1 | T2<br>R2 | T3<br>R3 |
|------------------|--|----------|----------|----------|
| (REAL/IMAGINARY) |  |          |          |          |

|   |      |              |               |              |
|---|------|--------------|---------------|--------------|
| 1 | REAL | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
|   | REAL | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 0.000000E+00 |
| 2 | REAL | 0.000000E+00 | 0.000000E+00  | 4.483875E-02 |
|   | REAL | 0.000000E+00 | -1.141350E-04 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 1.690381E-07 |
|   | IMAG | 0.000000E+00 | -4.302788E-10 | 0.000000E+00 |
| 3 | REAL | 0.000000E+00 | 0.000000E+00  | 1.630500E-01 |
|   | REAL | 0.000000E+00 | -1.956600E-04 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 6.146840E-07 |
|   | IMAG | 0.000000E+00 | -7.376208E-10 | 0.000000E+00 |
| 4 | REAL | 0.000000E+00 | 0.000000E+00  | 3.301763E-01 |
|   | REAL | 0.000000E+00 | -2.445750E-04 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 1.244735E-06 |
|   | IMAG | 0.000000E+00 | -9.220260E-10 | 0.000000E+00 |
| 5 | REAL | 0.000000E+00 | 0.000000E+00  | 5.217600E-01 |
|   | REAL | 0.000000E+00 | -2.608800E-04 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 1.966989E-06 |
|   | IMAG | 0.000000E+00 | -9.834944E-10 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 9  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM

SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 8.8000E+00)

| GRID ID |      | T1               | T2            | T3           |
|---------|------|------------------|---------------|--------------|
|         |      | R1               | R2            | R3           |
|         |      | (REAL/IMAGINARY) |               |              |
| 1       | REAL | 0.000000E+00     | 0.000000E+00  | 0.000000E+00 |
|         | REAL | 0.000000E+00     | 0.000000E+00  | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 0.000000E+00 |
| 2       | REAL | 0.000000E+00     | 0.000000E+00  | 6.929856E-01 |
|         | REAL | 0.000000E+00     | -1.731171E-03 | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 3.043009E-13 |
|         | IMAG | 0.000000E+00     | -7.601844E-16 | 0.000000E+00 |
| 3       | REAL | 0.000000E+00     | 0.000000E+00  | 2.425352E+00 |
|         | REAL | 0.000000E+00     | -2.779265E-03 | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 1.065010E-12 |
|         | IMAG | 0.000000E+00     | -1.220419E-15 | 0.000000E+00 |
| 4       | REAL | 0.000000E+00     | 0.000000E+00  | 4.713387E+00 |
|         | REAL | 0.000000E+00     | -3.237488E-03 | 0.000000E+00 |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 2.069722E-12 |



|   |      |              |               |              |
|---|------|--------------|---------------|--------------|
|   | IMAG | 0.000000E+00 | -1.421632E-15 | 0.000000E+00 |
| 5 | REAL | 0.000000E+00 | 0.000000E+00  | 7.186289E+00 |
|   | REAL | 0.000000E+00 | -3.319345E-03 | 0.000000E+00 |
|   | IMAG | 0.000000E+00 | 0.000000E+00  | 3.155613E-12 |
|   | IMAG | 0.000000E+00 | -1.457577E-15 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 10  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM

SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 5.7100E+01)

| GRID ID |      | T1<br>R1         | T2<br>R2      | T3<br>R3      |
|---------|------|------------------|---------------|---------------|
|         |      | (REAL/IMAGINARY) |               |               |
| 1       | REAL | 0.000000E+00     | 0.000000E+00  | 0.000000E+00  |
|         | REAL | 0.000000E+00     | 0.000000E+00  | 0.000000E+00  |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 0.000000E+00  |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 0.000000E+00  |
| 2       | REAL | 0.000000E+00     | 0.000000E+00  | -1.587528E+00 |
|         | REAL | 0.000000E+00     | 2.902470E-03  | 0.000000E+00  |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | -4.050349E-13 |
|         | IMAG | 0.000000E+00     | 7.405232E-16  | 0.000000E+00  |
| 3       | REAL | 0.000000E+00     | 0.000000E+00  | -2.721000E+00 |
|         | REAL | 0.000000E+00     | -5.632021E-04 | 0.000000E+00  |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | -6.942238E-13 |
|         | IMAG | 0.000000E+00     | -1.436929E-16 | 0.000000E+00  |
| 4       | REAL | 0.000000E+00     | 0.000000E+00  | -5.267867E-01 |
|         | REAL | 0.000000E+00     | -4.930345E-03 | 0.000000E+00  |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | -1.344020E-13 |
|         | IMAG | 0.000000E+00     | -1.257906E-15 | 0.000000E+00  |
| 5       | REAL | 0.000000E+00     | 0.000000E+00  | 3.793373E+00  |
|         | REAL | 0.000000E+00     | -6.071053E-03 | 0.000000E+00  |
|         | IMAG | 0.000000E+00     | 0.000000E+00  | 9.678243E-13  |
|         | IMAG | 0.000000E+00     | -1.548941E-15 | 0.000000E+00  |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.26 Dynamic Response of the Cantilevered Beam With Element Structural Damping

### Example ID:

A026

### Analysis Data Used:

GRID, CBAR, PBAR, MAT1, LOADCASE, METHOD, DLOAD, FREQ, MODES, DISP, EIGR, RLOAD1, DAREA, DELAY, TABLED1, FREQ

### Special Feature Used:

Element structural damping (ge values in MAT1 data).

### Problem Statement:

Find the magnitude of the tip displacement of the cantilever beam shown in the figure below. The beam is loaded with a tip point load of 100.0 N which is applied at 6 different frequencies. The frequencies cover a range that includes the first natural frequency of the beam. Use both direct dynamic analysis and modal analysis. For the modal analysis consider the first three modes. Consider also, that each beam element has a structural damping (ge) equal to 5%.

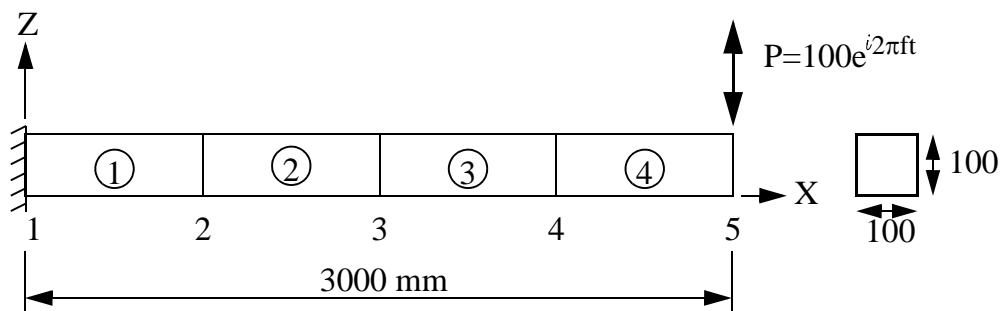


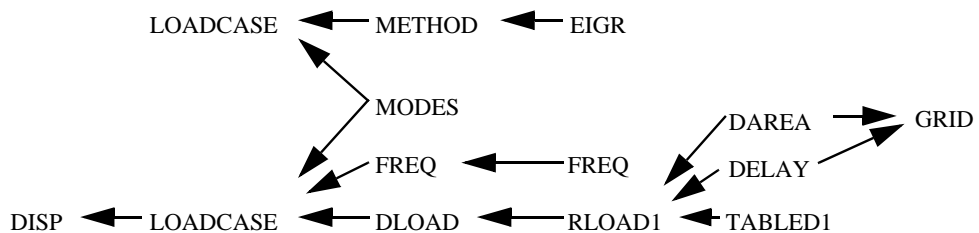
Figure 2-29

## Structural Analysis Model:

1. Four CBAR elements.
2. The section properties are: Area=10000 mm<sup>2</sup>. I=8.3+6 mm<sup>4</sup>
3. Material: E=2.07E+5 MPa. density=8.0-9 N-sec<sup>2</sup>/mm<sup>4</sup>, ge = 0.05
4. One frequency load case, necessary for modal dynamic analysis.
5. One direct dynamic response load case. P=100 N
6. One modal dynamic response load case. P=100 N
7. Excitation frequencies for the problem are: f1=8.6 hz, f2=8.8 hz, f3=9.0 hz, f4=9.1283 hz, f5=9.2 hz, f6=9.4 hz.
8. The magnitude of the displacements are selected by using DYNOUTPUT=POLAR in the solution control.

NOTE: Because the damping is uniform, the same results can be obtained by using the command: PARAM,G,0.05. In this case the three coefficient in MAT1 must be blank.

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE.

## Reference Solutions:

None for the displacements. Their values can be verified comparing load cases two and three which are obtained using the two different methodologies.

### Calculated Solutions:

| <b>f<br/>(Hz)</b> | <b>Direct Dynamic<br/>Response (mm)</b> | <b>Modal Dynamic<br/>Response (mm)</b> |
|-------------------|---|--|
| 8.6               | 4.1319                                  | 4.1319                                 |
| 8.8               | 5.8660                                  | 5.8660                                 |
| 9.0               | 8.8536                                  | 8.8536                                 |
| 9.1283            | 10.1312                                 | 10.1312*                               |
| 9.2               | 9.6573                                  | 9.6573                                 |
| 9.4               | 6.4475                                  | 6.4475                                 |

\* close to the natural frequency.

### Comparison Between Calculated Solutions and Reference Solutions:

Both methodologies give same results.

---

## 2.26.1 Input Data

```
ID A026
SOL COMPAT1
CEND
TITLE=DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING
SUBTITLE=FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY
LINE=64,80
ECHO=NONE
$
DYNOUTPUT = POLAR
SET 5 = 5
$
LOADCASE 1
LABEL = NATURAL FREQUENCIES (TO BE USED IN LOADCASE 3)
METHOD=4
$
LOADCASE 2
LABEL = SOLUTION USING:  DIRECT DYNAMIC RESPONSE
DLOAD = 23
FREQ = 32
DISP=5
$
LOADCASE 3
LABEL = SOLUTION USING:  MODAL DYNAMIC RESPONSE
DLOAD = 23
FREQ = 32
MODES=1
DISP=5
$
BEGIN BULK
$
$   FREQUENCIES (FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY)
$
FREQ      32      8.6      8.8      9.0      9.1283  9.2      9.4
$
$   GRID DATA (ONLY DISPLACEMENTS IN Z AND ROTATIONS IN Y ARE REQUESTED)
$
GRID      1      0.00      0.      0.0      123456
GRID      2      750.0      0.      0.0      1246
GRID      3      1500.0      0.      0.0      1246
GRID      4      2250.0      0.      0.0      1246
GRID      5      3000.0      0.      0.0      1246
$
$   ELEMENT DEFINITIONS
$
CBAR      1      1      1      2      0.      1.0      0.
CBAR      2      1      2      3      0.      1.0      0.
CBAR      3      1      3      4      0.      1.0      0.
CBAR      4      1      4      5      0.      1.0      0.
$
$   PROPERTY DEFINITIONS
$
PBAR      1      1      1.0+4      8.333+6 8.333+6 0.0
```

```

$
$ MATERIAL DATA
$
MAT1      1      2.07+5      0.3      8.0-9      0.05
$
$ EIGENVALUES DATA
$
EIGR      4      SUB      3      MAX
$
$ LOAD DEFINITION
$
RLOAD1    23      1      2      4
$
DAREA     1      5      3      100.0
DELAY     2      5      3      60.0
$
$ C(W) = 1.0 = CONSTANT (SAME LOAD FOR EACH EXITING FREQUENCY)
$
TABLED1   4
+         1.0      1.0      2.0      1.0
$
$ ENDDATA

```

---

## 2.26.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A026  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 5
NUMBER OF CBAR ELEMENTS: 4
TOTAL NUMBER OF NON RIGID ELEMENTS: 4
```

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 8

#### LOAD CASES SUMMARY

NUMBER OF USER FREQUENCY LOAD CASES: 1  
 NUMBER OF DIRECT DYNAMIC LOAD CASES: 1  
 NUMBER OF MODAL DYNAMIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 3

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 2.400000E-01  
 SYSTEM VOLUME 3.000000E+07  
 SYSTEM MASS/VOLUME 8.000000E-09

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 NATURAL FREQUENCIES (TO BE USED IN LOADCASE 3) LOADCASE 1

#### E I G E N V A L U E S

| MODE | CYCLES       | EIGENVALUE   | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|--------------|--------------|---------------------|--------------------------|
| 1    | 9.128302E+00 | 3.289575E+03 | 5.735481E+01 | 6.001558E-02        | 1.974257E+02             |
| 2    | 5.719718E+01 | 1.291543E+05 | 3.593805E+02 | 5.994695E-02        | 7.742408E+03             |
| 3    | 1.608664E+02 | 1.021622E+06 | 1.010753E+03 | 5.897694E-02        | 6.025216E+04             |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

#### G R I D D Y N A M I C D I S P L A C E M E N T S (FREQUENCY = 8.6000E+00)

| GRID ID | T1 | T2 | T3 |
|---------|----|----|----|
| R1      |    | R2 | R3 |

(MAGNITUDE/PHASE)

|   |      |              |              |              |
|---|------|--------------|--------------|--------------|
| 5 | MAGN | 0.000000E+00 | 0.000000E+00 | 4.131863E+00 |
|---|------|--------------|--------------|--------------|



|       |              |              |              |
|-------|--------------|--------------|--------------|
| MAGN  | 0.000000E+00 | 1.916311E-03 | 0.000000E+00 |
| PHASE | 0.000000E+00 | 0.000000E+00 | 3.360961E+02 |
| PHASE | 0.000000E+00 | 1.563295E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 6  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 8.8000E+00)

| GRID ID | T1 | T2 | T3 |
|---------|----|----|----|
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 5.865974E+00 |
|   | MAGN  | 0.000000E+00 | 2.710126E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.247887E+02 |
|   | PHASE | 0.000000E+00 | 1.450351E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 7  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0000E+00)

| GRID ID | T1 | T2 | T3 |
|---------|----|----|----|
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 8.853634E+00 |
|   | MAGN  | 0.000000E+00 | 4.074254E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.992584E+02 |
|   | PHASE | 0.000000E+00 | 1.195181E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 8  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1283E+00)

| GRID ID | T1 | T2 | T3 |
|---------|----|----|----|
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.013120E+01 |
|   | MAGN  | 0.000000E+00 | 4.650014E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.613684E+02 |
|   | PHASE | 0.000000E+00 | 3.416371E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 9  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.657305E+00 |
|                   | MAGN 0.000000E+00  | 4.425950E-03 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 2.525835E+02 |
|                   | PHASE 0.000000E+00 | 7.285720E+01 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 10  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: DIRECT DYNAMIC RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.4000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 6.447469E+00 |
|                   | MAGN 0.000000E+00  | 2.942451E-03 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 2.197053E+02 |
|                   | PHASE 0.000000E+00 | 3.999351E+01 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 11  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 8.6000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 4.131863E+00 |
|                   | MAGN 0.000000E+00  | 1.916311E-03 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.360961E+02 |
|                   | PHASE 0.000000E+00 | 1.563295E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 12  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 8.8000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 5.865974E+00 |
|   | MAGN  | 0.000000E+00 | 2.710126E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.247887E+02 |
|   | PHASE | 0.000000E+00 | 1.450351E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 13  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 8.853634E+00 |
|   | MAGN  | 0.000000E+00 | 4.074255E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.992584E+02 |
|   | PHASE | 0.000000E+00 | 1.195182E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 14  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1283E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.013120E+01 |
|   | MAGN  | 0.000000E+00 | 4.650014E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.613684E+02 |
|   | PHASE | 0.000000E+00 | 3.416371E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 15  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
 SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|---------|----|----|----|

|   |       | R1                | R2           | R3           |
|---|-------|-------------------|--------------|--------------|
|   |       | (MAGNITUDE/PHASE) |              |              |
| 5 | MAGN  | 0.000000E+00      | 0.000000E+00 | 9.657305E+00 |
|   | MAGN  | 0.000000E+00      | 4.425950E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00      | 0.000000E+00 | 2.525835E+02 |
|   | PHASE | 0.000000E+00      | 7.285721E+01 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 16  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH ELEMENT STRUCTURAL DAMPING  
FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY  
SOLUTION USING: MODAL DYNAMIC RESPONSE LOADCASE 3

G R I D D Y N A M I C D I S P L A C E M E N T S (FREQUENCY = 9.4000E+00)

| GRID ID | T1                | T2           | T3           |              |
|---------|-------------------|--------------|--------------|--------------|
|         | R1                | R2           | R3           |              |
|         | (MAGNITUDE/PHASE) |              |              |              |
| 5       | MAGN              | 0.000000E+00 | 0.000000E+00 | 6.447469E+00 |
|         | MAGN              | 0.000000E+00 | 2.942451E-03 | 0.000000E+00 |
|         | PHASE             | 0.000000E+00 | 0.000000E+00 | 2.197053E+02 |
|         | PHASE             | 0.000000E+00 | 3.999352E+01 | 0.000000E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.27 Dynamic Response of the Cantilevered Beam With and Without Modal Damping

### Example ID:

A027

### Analysis data used:

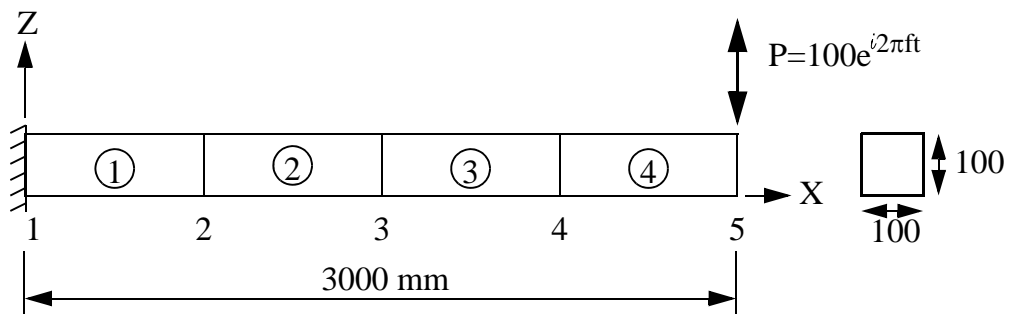
GRID, CBAR, PBAR, MAT1, LOADCASE, METHOD, DLOAD, FREQ, MODES, SDAMP-  
ING, DISP, EIGR, RLOAD1, DAREA, DELAY, TABLED1, TABDMP1

### Special feature used:

Modal structural damping.

### Problem statement:

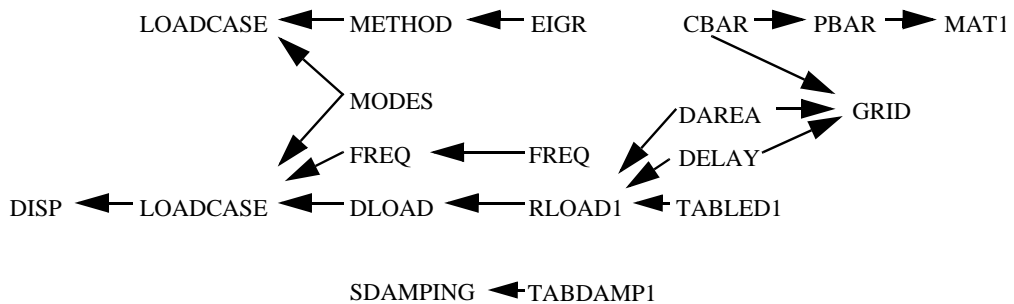
Find the magnitude of the tip displacement of the cantilever beam shown in the figure below. The beam is loaded with a tip point load of 100.0 N which is applied at 6 different frequencies. The frequencies cover a range that includes the first natural frequency of the beam. Use modal analysis with modal damping and use modal analysis without any kind of damping. Consider the first three modes in the two modal analyses.



## Structural Analysis Model:

1. Four CBAR elements.
2. The section properties are: Area=10000 mm<sup>2</sup>. I=8.3+6 mm<sup>4</sup>
3. Material: E=2.07E+5 MPa. density=8.0-9 N-sec<sup>2</sup>/mm<sup>4</sup>.
4. One frequency load case, necessary for modal dynamic analysis.
5. One modal dynamic response load case. P=100 N with modal damping.
6. One modal dynamic response load case. P=100 N
7. Excitation frequencies for the problem are: f1=8.6 hz, f2=8.8 hz, f3=9.0 hz, f4=9.1283 hz, f5=9.2 hz, f6=9.4 hz.
8. The magnitude of the displacements are selected by using DYNOUTPUT=POLAR in the solution control. Modal structural damping is selected with parameter KDAMP = -1.
9. Damping is  $ge(f) = \text{constant} = 0.05$

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE.

## Reference Solutions:

None. However, the damped solution can be compared with the modal damped solution of example A026. Because the damping is uniform in both examples the solution should be very close. The undamped solution can be verified at the frequency  $f=9.1283$ . The displacements there should be extremely large because that frequency is very close to the first natural frequency (9.128302).

## Calculated Solutions:

| Magnitude of Tip Displacement Using Modal Analysis |                    |                 |
|--|--------------------|-----------------|
| Hz   | Modal Damping (mm) | Undamped (mm)   |
| 8.6  | 4.1319             | 4.5219          |
| 8.8  | 5.8660             | 7.1863          |
| 9.0  | 8.8536             | 18.1618         |
| 9.1283   | 10.1312            | 1,080,290.0000* |
| 9.2  | 9.6573             | 32.1024         |
| 9.4  | 6.4474             | 8.3685          |

\* Resonance

## Comparison Between Calculated Solutions and Reference Solutions:

The modal damped solution is very close to the damped solution in example A026 (differences are less than 0.01%). The undamped solution captures the resonance condition.

---

## 2.27.1 Input Data

```
ID A027
SOL COMPAT1
CEND
TITLE=DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING
SUBTITLE=FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY.
LINE=64,80
ECHO=NONE
$
DYNOUTPUT = POLAR
SET 5 = 5
$
LOADCASE 1
LABEL = NATURAL FREQUENCIES (TO BE USED IN LOADCASES 2 & 3)
METHOD=4
$
LOADCASE 2
LABEL = LOADCASE USING: MODAL DYNAMIC RESPONSE WITH MODAL DAMPING
DLOAD = 23
FREQ = 32
MODES=1
SDAMPING=1
DISP=5
$
LOADCASE 3
LABEL = LOADCASE USING: MODAL DYNAMIC RESPONSE WITH OUT DAMPING
DLOAD = 23
FREQ = 32
MODES=1
DISP=5
$
BEGIN BULK
$
PARAM KDAMP -1
$
$ MODAL DAMPING
$
$ G(W) = 0.05 = CONSTANT (SAME DAMPING FOR EACH EXISTING FREQUENCY)
$
TABDMP1 1 G
+ 1.0 0.05 2.0 0.05
$
$ FREQUENCIES (FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIES
$ I.E. CLOSE TO FIRST RESONANCE MODE.)
$
FREQ 32 8.6 8.8 9.0 9.1283 9.2 9.4
$
$ GRID DATA (ONLY DISPLACEMENT IN Z AND ROTATIONS IN Y ARE REQUESTED)
$
GRID 1 0.00 0. 0.0 123456
GRID 2 750.0 0. 0.0 1246
GRID 3 1500.0 0. 0.0 1246
GRID 4 2250.0 0. 0.0 1246
```



```

GRID      5              3000.0  0.      0.0              1246
$
$  ELEMENT DEFINITIONS
$
CBAR      1      1      1      2      0.      1.0      0.
CBAR      2      1      2      3      0.      1.0      0.
CBAR      3      1      3      4      0.      1.0      0.
CBAR      4      1      4      5      0.      1.0      0.
$
$  PROPERTY DEFINITIONS
$
PBAR      1      1      1.0+4  8.333+6 8.333+6 0.0
$
$  MATERIAL DATA
$
MAT1      1      2.07+5      0.3      8.0-9
$
$  EIGENVALUES DATA
$
EIGR      4      SUB              3      MAX
$
$  LOAD DEFINITION
$
RLOAD1    23      1      2              4
$
DAREA      1      5      3      100.0
DELAY      2      5      3      60.0
$
$  C(W) = 1.0 = CONSTANT (SAME VALUE FOR EACH EXITING FREQUENCY)
$
TABLED1    4
+      1.0      1.0      2.0      1.0
$
ENDDATA

```

---

## 2.27.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A027

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.

### ANALYSIS PROBLEM SUMMARY

|                                     |   |
|-------------------------------------|---|
| NUMBER OF GRID POINTS:              | 5 |
| NUMBER OF CBAR ELEMENTS:            | 4 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 4 |

NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 8

#### LOAD CASES SUMMARY

NUMBER OF USER FREQUENCY LOAD CASES: 1  
 NUMBER OF MODAL DYNAMIC LOAD CASES: 2  
 TOTAL NUMBER OF LOAD CASES: 3

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 2.400000E-01  
 SYSTEM VOLUME 3.000000E+07  
 SYSTEM MASS/VOLUME 8.000000E-09

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.  
 NATURAL FREQUENCIES (TO BE USED IN LOADCASES 2 & 3) LOADCASE 1

#### E I G E N V A L U E S

| MODE | CYCLES       | EIGENVALUE   | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|--------------|--------------|---------------------|--------------------------|
| 1    | 9.128302E+00 | 3.289575E+03 | 5.735481E+01 | 6.001558E-02        | 1.974257E+02             |
| 2    | 5.719718E+01 | 1.291543E+05 | 3.593805E+02 | 5.994695E-02        | 7.742408E+03             |
| 3    | 1.608664E+02 | 1.021622E+06 | 1.010753E+03 | 5.897694E-02        | 6.025216E+04             |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.  
 LOADCASE USING: MODAL DYNAMIC RESPONSE WITH MODAL DAMPING LOADCASE 2

G R I D D Y N A M I C D I S P L A C E M E N T S (FREQUENCY = 8.6000E+00)

| GRID ID | T1 | T2 | T3 |
|---------|----|----|----|
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |      |              |              |              |
|---|------|--------------|--------------|--------------|
| 5 | MAGN | 0.000000E+00 | 0.000000E+00 | 4.131851E+00 |
|   | MAGN | 0.000000E+00 | 1.916249E-03 | 0.000000E+00 |

|       |              |              |              |
|-------|--------------|--------------|--------------|
| PHASE | 0.000000E+00 | 0.000000E+00 | 3.360965E+02 |
| PHASE | 0.000000E+00 | 1.563344E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 6  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY.  
 LOADCASE USING: MODAL DYNAMIC RESPONSE WITH MODAL DAMPING LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 8.8000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 5.865956E+00 |
|   | MAGN  | 0.000000E+00 | 2.710034E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.247890E+02 |
|   | PHASE | 0.000000E+00 | 1.450382E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 7  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY.  
 LOADCASE USING: MODAL DYNAMIC RESPONSE WITH MODAL DAMPING LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 8.853606E+00 |
|   | MAGN  | 0.000000E+00 | 4.074108E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.992585E+02 |
|   | PHASE | 0.000000E+00 | 1.195195E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 8  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCY.  
 LOADCASE USING: MODAL DYNAMIC RESPONSE WITH MODAL DAMPING LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1283E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.013117E+01 |
|   | MAGN  | 0.000000E+00 | 4.649840E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.613684E+02 |
|   | PHASE | 0.000000E+00 | 3.416372E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 9

DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.

LOADCASE USING: MODAL DYNAMIC RESPONSE WITH MODAL DAMPING LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 9.657273E+00 |
|   | MAGN  | 0.000000E+00 | 4.425781E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.525834E+02 |
|   | PHASE | 0.000000E+00 | 7.285665E+01 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 10

DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.

LOADCASE USING: MODAL DYNAMIC RESPONSE WITH MODAL DAMPING LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.4000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 6.447446E+00 |
|   | MAGN  | 0.000000E+00 | 2.942332E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.197051E+02 |
|   | PHASE | 0.000000E+00 | 3.999103E+01 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 11

DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.

LOADCASE USING: MODAL DYNAMIC RESPONSE WITH OUT DAMPING LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 8.6000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 4.521913E+00 |
|   | MAGN  | 0.000000E+00 | 2.096748E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.600000E+02 |
|   | PHASE | 0.000000E+00 | 1.800000E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 12

DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.

LOADCASE USING: MODAL DYNAMIC RESPONSE WITH OUT DAMPING LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 8.8000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 7.186289E+00 |
|   | MAGN  | 0.000000E+00 | 3.319345E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.515948E-11 |
|   | PHASE | 0.000000E+00 | 1.800000E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 13  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.  
 LOADCASE USING: MODAL DYNAMIC RESPONSE WITH OUT DAMPING LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.816175E+01 |
|   | MAGN  | 0.000000E+00 | 8.355583E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.600000E+02 |
|   | PHASE | 0.000000E+00 | 1.800000E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 14  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.  
 LOADCASE USING: MODAL DYNAMIC RESPONSE WITH OUT DAMPING LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1283E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.080290E+06 |
|   | MAGN  | 0.000000E+00 | 4.957038E+02 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.512800E+02 |
|   | PHASE | 0.000000E+00 | 7.128000E+01 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 15  
 DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
 FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.  
 LOADCASE USING: MODAL DYNAMIC RESPONSE WITH OUT DAMPING LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 3.210236E+01 |
|   | MAGN  | 0.000000E+00 | 1.470868E-02 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.800000E+02 |
|   | PHASE | 0.000000E+00 | 3.600000E+02 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 16  
DYNAMIC RESPONSE OF THE CANTILEVER BEAM WITH AND WITHOUT MODAL DAMPING  
FREQUENCIES ARE CLOSE TO THE FIRST NATURAL FREQUENCIE.  
LOADCASE USING: MODAL DYNAMIC RESPONSE WITH OUT DAMPING LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.4000E+00)

| GRID ID | T1 | T2 | T3 |
|---------|----|----|----|
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 8.368456E+00 |
|   | MAGN  | 0.000000E+00 | 3.818078E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.800000E+02 |
|   | PHASE | 0.000000E+00 | 6.741409E-12 | 0.000000E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

## 2.28 Dynamic Response of a CELAS1/CDAMP1/CMASS1 Structure

### Example ID:

A028

### Analysis Data Used:

SPOINT, CELAS1, CDAMP1, CMASS1, PELAS, PDAMP, PMASS, LOADCASE, FREQ, SPC, DLOAD, DISP, VELO, ACCE, FORCE, STRESS, FREQ, RLOAD3, SPC1

### Special Features Used:

Utilization of scalar point (SPOINT) and the scalar elements CDAMP1 and CMASS1.

### Problem Statement:

Find the displacement, velocity and acceleration at the free degree of freedom of the scalar point 2 shown in the figure below. Obtain the elastic stress and force of the elastic element and the force acting in the damper. Verify that the forces of the structure are in dynamic equilibrium.

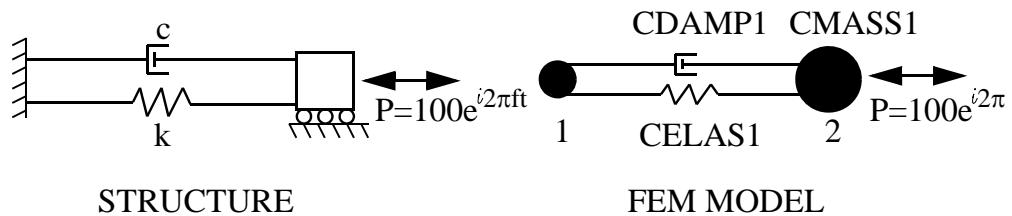


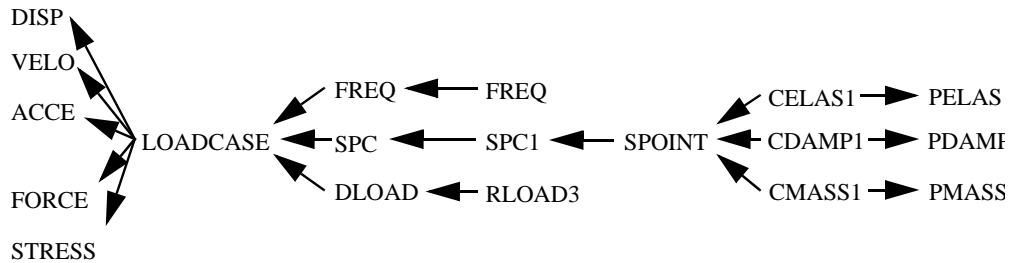
Figure 2-30

### Structural Analysis Model:

1. One degree of freedom structure assembled with one CELAS1 element one CDAMP1 element and one CMASS1 element. Elements are connected using two scalar points (SPOINT).
2. Properties of CELAS1:  $k=10$ ,  $S=0.1$
3. Properties of CDAMP1:  $c=2.5$
4. Properties of CMASS1:  $m=2.5$
5. One Load case:  $P=100.0$ , applied at a frequency  $f=0.159155$  hz.



## Analysis Data Relationships:



## Special Modeling Techniques:

NONE

## Reference Solutions:

The analytical solution for this problem is:

$$u2 = \frac{P}{k - ((2\pi f)^2 m + (2\pi f)ci)} = 12 - 4i$$

$$v2 = (2\pi f)u1 = 4 + 12i$$

$$a2 = (2\pi f)v1 = -12 + 4i$$

$$\text{Force elas1} = k(u1 - u2) = -120 + 40i$$

$$\text{Stress elas1} = S \times \text{Force elas1} = -12 + 4i$$

$$\text{Force damp1} = c(v1 - v2) = -10.0 - 30i$$

$$\text{Inertial Force} = m(a1 - a2) = 30 - 10i$$

where: P is the applied load, k is the elas1 stiffness, m is the mass, c the viscous coefficient, i is the imaginary number (square root of -1) and pi is 3.141592.

## Calculated solution:

$$u2 = 12 - 4i$$

$$v2 = 4 + 12i$$

$$a2 = -12 + 4i$$

$$\text{Force elas1} = -120 + 40i$$

$$\text{Stress elas1} = -12 + 4i$$

$$\text{Force damp1} = -10.0 - 30i$$

$$\text{Inertial Force} = m(a1 - a2) = 30 - 10i (*)$$

Note: Inertial forces are not calculated in the program. a1=0.

## Comparison Between Calculated Solution And Reference Solution

There are no differences between the analytical solution and calculated solution.

---

## 2.28.1 Input Data

```
ID A028
SOL COMPAT1
CEND
$
TITLE=DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE
LINE=64,80
ECHO=NONE
$
LOADCASE 1
LABEL = DIRECT DYNAMIC RESPONSE.
FREQ = 100
SPC=1
DLOAD = 200
DISP=ALL
VELO=ALL
ACCE=ALL
FORCE=ALL
STRESS=ALL
$
BEGIN BULK
$
$   FREQUENCIES
$
FREQ    100      0.159155
$
$   BOUNDARY CONDITION
$
SPC1    1          1
$
$   SPOINT DATA
$
SPOINT  1          2
$
$   ELEMENT DEFINITIONS
$
CELAS1  1          1          1          2
CDAMP1  2          2          1          2
CMASS1  3          3          2
$
$
$   PROPERTY DEFINITIONS
$
PELAS   1          10.0          0.1
PDAMP   2          2.5
PMASS   3          2.5
$
$   LOAD DEFINITION
$
RLOAD3  200        2          100.0
$
ENDDATA
```

---

## 2.28.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A028

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF SPOINTS: 2
NUMBER OF CELAS1/2 ELEMENTS: 1
NUMBER OF CDAMP1/2 ELEMENTS: 1
```

|                                     |   |
|-------------------------------------|---|
| NUMBER OF CMASS1/2 ELEMENTS:        | 1 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 3 |
| NUMBER OF ELEMENT PROPERTIES:       | 3 |
| NUMBER OF DEGREES OF FREEDOM:       | 1 |

#### LOAD CASES SUMMARY

|                                      |   |
|--------------------------------------|---|
| NUMBER OF DIRECT DYNAMIC LOAD CASES: | 1 |
| TOTAL NUMBER OF LOAD CASES:          | 1 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

#### M A S S / V O L U M E S U M M A R Y

|               |              |
|---------------|--------------|
| SYSTEM MASS   | 2.500000E+00 |
| SYSTEM VOLUME | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

DIRECT DYNAMIC RESPONSE. LOADCASE 1

S P O I N T D Y N A M I C D I S P L A C E M E N T S (FREQUENCY = 1.5915E-01)

| SPOINT ID | REAL         | IMAGINARY     |
|-----------|--------------|---------------|
| 1         | 0.000000E+00 | 0.000000E+00  |
| 2         | 1.200000E+01 | -4.000003E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

DIRECT DYNAMIC RESPONSE. LOADCASE 1

S P O I N T D Y N A M I C V E L O C I T I E S (FREQUENCY = 1.5915E-01)

| SPOINT ID | REAL         | IMAGINARY    |
|-----------|--------------|--------------|
| 1         | 0.000000E+00 | 0.000000E+00 |
| 2         | 4.000004E+00 | 1.200001E+01 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 6  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

DIRECT DYNAMIC RESPONSE. LOADCASE 1

S P O I N T D Y N A M I C A C C E L E R A T I O N S (FREQUENCY = 1.5915E-01)

| SPOINT ID | REAL          | IMAGINARY    |
|-----------|---------------|--------------|
| 1         | 0.000000E+00  | 0.000000E+00 |
| 2         | -1.200001E+01 | 4.000006E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 7  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

DIRECT DYNAMIC RESPONSE. LOADCASE 1

F O R C E S I N E L A S E L E M E N T S (FREQUENCY = 1.5915E-01)

| ELAS1 ID | REAL FORCE    | IMAG FORCE   |
|----------|---------------|--------------|
| 1        | -1.200000E+02 | 4.000003E+01 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 8  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

DIRECT DYNAMIC RESPONSE. LOADCASE 1

F O R C E S I N D A M P E L E M E N T S (FREQUENCY = 1.5915E-01)

| DAMP1 ID | REAL FORCE    | IMAG FORCE    |
|----------|---------------|---------------|
| 2        | -1.000001E+01 | -3.000001E+01 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 9  
DYNAMIC RESPONSE OF THE ELAS1/DAMP1/MASS1 STRUCTURE

DIRECT DYNAMIC RESPONSE. LOADCASE 1

S T R E S S E S I N E L A S E L E M E N T S (FREQUENCY = 1.5915E-01)

| ELAS1 ID | STRESS             | STRESS            |
|----------|--------------------|-------------------|
| 1        | REAL -1.200000E+01 | IMAG 4.000003E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.29 Three Rod Truss - Analysis Using Multidisciplinary Load Cases

### Example ID:

A029

### Analysis data used:

GRID, GRDSET, CROD, PROD, MAT1, MAT4, LOADCASE, HEAT, TEMP, LOAD, METHOD, FREQ, DLOAD, MODES, SDAMPING, SPC, QHBDY, SPCD, SPC1, FORCE, EIGR, FREQ, RLOAD1, TABLED1, TABDMP1, THERMAL, DISP, STRESS

### Special Feature Used:

Multiple load case types

### Problem Statement:

The three rod truss of [Figure 2-31](#) is subjected to five different loadcases. The first load case corresponds to a heat transfer analysis. The second to a static analysis where the results of the heat transfer analysis are used to create thermal loads. The third load case corresponds to a frequency calculation analysis. This load case is used to calculate the first two natural frequencies and also is used for the fifth load case. The forth load case corresponds to a direct dynamic response analysis in which a point load is applied at a frequency of 20 hz. The last loadcase is a modal dynamic response analysis. In this load case another point load is applied also at a frequency of 20 hz.

Considering the geometry described for example A001 and modal damping of 5% for the fifth load case find:

1. For the heat transfer loadcase, the temperature at grid 4.
2. For the static load case, the displacements u4 and v4 at grid 4 and the stresses at the three rods.
3. For the normal mode analysis, the first natural frequency. Do not consider an extra mass element as it is in the A009 example.
4. For the direct frequency response analysis, find the magnitudes of the dynamic displacements u4 and v4 of grid 4.

- For the modal frequency response analysis find the magnitudes of the dynamic displacements  $u_4$  and  $v_4$  of grid 4.

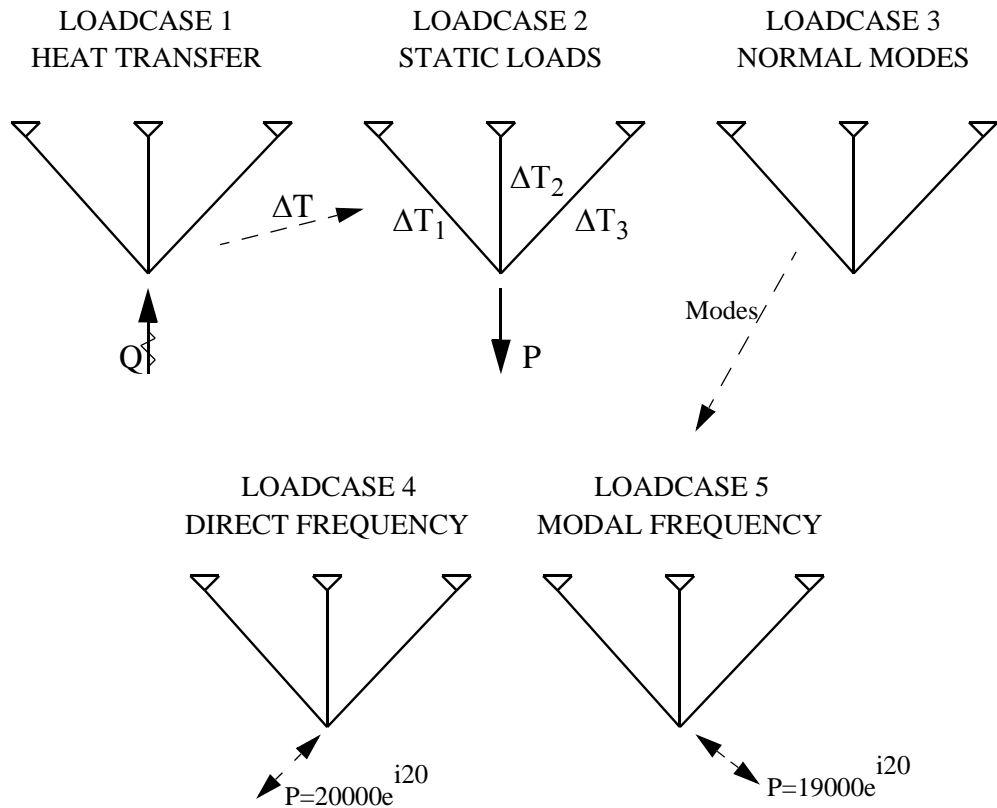


Figure 2-31

### Structural Analysis Model:

- Three CROD elements.
- Section properties:  $A_1=A_3=1.0$ ,  $A_2=2.0$
- Material:  $E=1.0+7$  psi, Conductivity = 70.0
- Five load cases

Conduction heat transfer due to a heat flux of  $Q_0 = 100.0$  at grid 4.

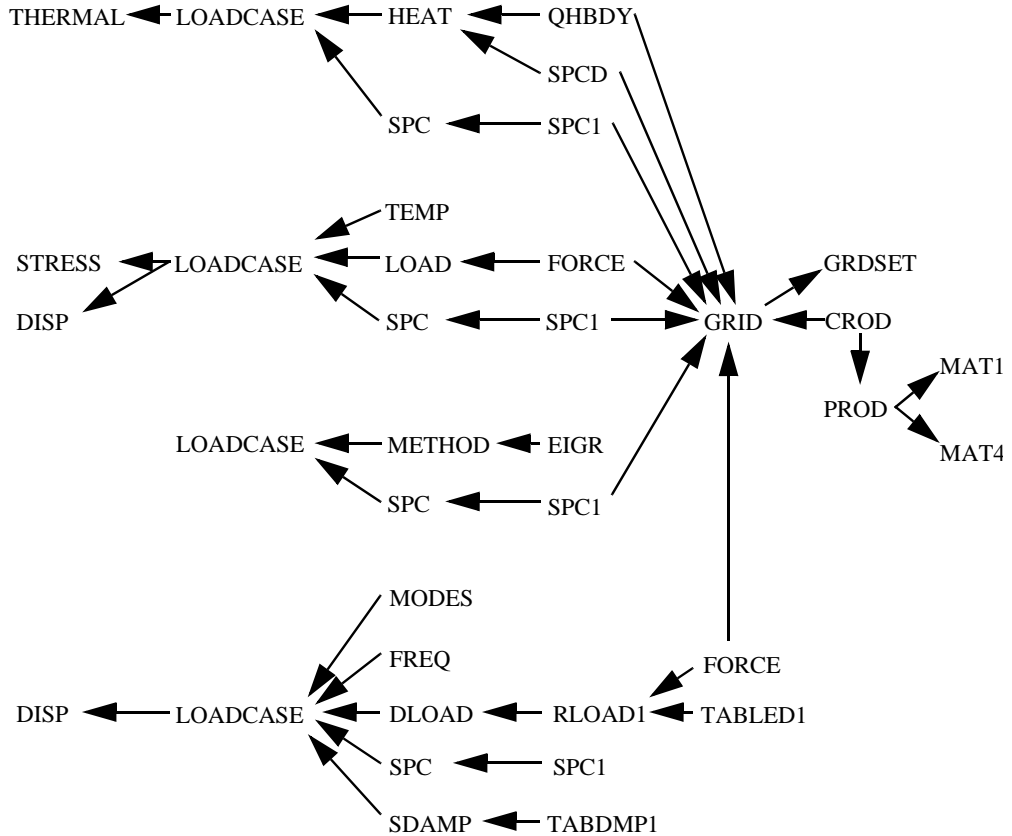
Static analysis:  $P_2=20000$  and TEMP due to heat conduction.

Natural vibrations. Distributed mass:  $m=0.1$ .

Direct dynamic response  $P_4=20000$ ,  $f=20$  hz.

Modal dynamic response  $P_5=19000$ ,  $f=20$  hz, modal damping  $ge=5\%$ .

## Analysis Data Relationships:



## Special Modeling Techniques:

NONE

## Reference Solutions:

NONE



## Calculated Solutions:

Heat transfer Analysis:

$$T4 = 29.18$$

Static analysis:

$$U4 = 0.0$$

$$V4 = -0.0073$$

$$SXX\_1 = 3,691.361$$

$$SXX\_2 = 7,389.814$$

$$SXX\_3 = 3,691.361$$

Normal modes analysis:

$$f1 = 105.5$$

Direct dynamic response (undamped solutions):

$$U4 = 0.002347$$

$$V4 = 0.004475$$

Modal dynamic response (damped solutions):

$$U4 = 0.02230$$

$$V4 = 0.00425$$

---

## 2.29.1 Input Data

```
ID A029
SOL COMPAT1
CEND
TITLE = THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS
ECHO=NONE
LINE = 64,80
DYNOUTPUT = POLAR
LOADCASE 1
LABEL = HEAT TRANSFER LOACASE
SPC=50
HEAT=50
THERMAL=ALL
LOADCASE 2
LABEL = STATIC LOADCASE: USES TEMPERATURE LOADING GENERATED BY HEAT TRANSFER
SPC=100
TEMP=1
LOAD=100
DISP = ALL
STRESS = ALL
LOADCASE 3
LABEL = NATURAL FREQUENCY LOACASE
SPC=100
METHOD=300
LOADCASE 4
LABEL = FREQUENCY RESPONSE LOADCASE USING DIRECT DYNAMIC ANALYSIS
SPC=100
FREQ=200
DLOAD=400
DISP = ALL
LOADCASE 5
LABEL = FREQUENCY RESPONSE LOADCASE USING MODAL DYNAMIC ANALYSIS AND MODAL
DAMPING
FREQ=200
DLOAD=500
MODES=3
SDAMPING=500
DISP = ALL
BEGIN BULK
$
$---- FREQUENCY DEFINITION
$
FREQ      200      20.0
$
$---- ANALYSIS DATA
$
$ GRID DATA
GRID      1          -10.    0.0    0.0
GRID      2           0.0    0.0    0.0
GRID      3          10.0    0.0    0.0
GRID      4           0.0   -10.0    0.0
$ STRUCTURAL BOUNDARY CONDITION DATA
GRDSET                                3456
SPC1      100      123456  1      2      3
```

```

SPC1      100      3456      4
$ THERMAL BOUNDARY CONDITION DATA
SPC1      50              1      2      3
SPCD      50      1              25.0
SPCD      50      2              25.0
SPCD      50      3              25.0
$ ELEMENT DATA
CROD      1      11      1      4
CROD      2      12      2      4
CROD      3      13      3      4
$ PROPERTY AND MATERIAL DATA
PROD      11      1      1.0
PROD      12      1      2.0
PROD      13      1      1.0
MAT1      1      1.0E+7              0.33      0.1      0.1E-6      20.0
MAT4      1      70.0
$ HEAT TRANSFER LOAD
QHBDY      50      POINT      100.0      1.0      4
$ EIGENVALUE METHOD
EIGR      300      SUB              2      MASS
$ STATIC LOAD
FORCE      100      4              20000.      0.0      -1.0
$ DYNAMIC LOAD
TABLED1      45
+      1.0      1.0      100.0      1.0
RLOAD1      400              45      444
FORCE      444      4              20000.      -0.8      -0.6
RLOAD1      500              45      555
FORCE      555      4              19000.      0.8      -0.6
$ MODAL DAMPING
TABDMP1      500      G
+      1.0      0.05      100.0      0.05
ENDDATA

```

---

## 2.29.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A029  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 4
NUMBER OF CROD ELEMENTS: 3
TOTAL NUMBER OF NON RIGID ELEMENTS: 3
```

|                                       |   |
|---------------------------------------|---|
| NUMBER OF ELEMENT PROPERTIES:         | 3 |
| NUMBER OF MATERIALS:                  | 1 |
| NUMBER OF DEGREES OF FREEDOM:         | 2 |
| NUMBER OF THERMAL DEGREES OF FREEDOM: | 4 |

#### LOAD CASES SUMMARY

|                                      |   |
|--------------------------------------|---|
| NUMBER OF STATIC LOAD CASES:         | 1 |
| NUMBER OF USER FREQUENCY LOAD CASES: | 1 |
| NUMBER OF HEAT TRANSFER LOAD CASES:  | 1 |
| NUMBER OF DIRECT DYNAMIC LOAD CASES: | 1 |
| NUMBER OF MODAL DYNAMIC LOAD CASES:  | 1 |
| TOTAL NUMBER OF LOAD CASES:          | 5 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

#### M A S S / V O L U M E S U M M A R Y

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 4.828427E+00 |
| SYSTEM VOLUME      | 4.828427E+01 |
| SYSTEM MASS/VOLUME | 1.000000E-01 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

#### H E A T S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -1.630235E-14 ; HEAT ENERGY 2.092094E+02  
1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

HEAT TRANSFER LOACASE LOADCASE 1

#### G R I D T E M P E R A T U R E S

| GRID ID | TEMP         |
|---------|--------------|
| 1       | 2.500000E+01 |
| 2       | 2.500000E+01 |
| 3       | 2.500000E+01 |
| 4       | 2.918419E+01 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 6  
THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

# SOLUTION RESIDUALS

LOADCASE 2 ; RESIDUAL : 0.000000E+00 ; STRAIN ENERGY : 7.405861E+01  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 7  
 THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

# LOAD RESULTANTS

| LOADCASE | T1         | T2          | T3         | R1         | R2         | R3         |
|----------|------------|-------------|------------|------------|------------|------------|
| 2        | 0.0000E+00 | -2.0000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 8  
 THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

# SPCFORCE RESULTANTS

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 2        | 0.0000E+00 | 2.0000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 9  
 THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

# MAXIMUM APPLIED FORCE

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3         |
|----------|---------|------------|------------|-------------|------------|
| 2        | 4       | 2.0024E+04 | 0.0000E+00 | -2.0024E+04 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 10  
 THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

# MAXIMUM SPC FORCE

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 2        | 2       | 1.4780E+04 | 0.0000E+00 | 1.4780E+04 | 0.0000E+00 |

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# MAXIMUM DISPLACEMENT

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3         |
|----------|---------|------------|------------|-------------|------------|
| 2        | 4       | 7.3969E-03 | 0.0000E+00 | -7.3969E-03 | 0.0000E+00 |

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 THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

STATIC LOADCASE: USES TEMPERATURE LOADING GENERATED BY HEAT LOADCASE 2

# GRID DISPLACEMENTS

| GRID ID | T1<br>R1                     | T2<br>R2                      | T3<br>R3                     |
|---------|------------------------------|-------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 2       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 4       | 0.000000E+00<br>0.000000E+00 | -7.396906E-03<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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STATIC LOADCASE: USES TEMPERATURE LOADING GENERATED BY HEAT LOADCASE 2

#### STRESSES IN ROD ELEMENTS

| ROD ID | STRESS-A     | STRESS-B     |
|--------|--------------|--------------|
| 1      | 3.691361E+03 | 3.691361E+03 |
| 2      | 7.389814E+03 | 7.389814E+03 |
| 3      | 3.691361E+03 | 3.691361E+03 |

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THREE ROD TRUSS - MULTIDICIPLINARY ANALYSIS

NATURAL FREQUENCY LOACASE LOADCASE 3

#### EIGENVALUES

| MODE | CYCLES       | EIGENVALUE   | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|--------------|--------------|---------------------|--------------------------|
| 1    | 1.054922E+02 | 4.393398E+05 | 6.628271E+02 | 1.000000E+00        | 4.393398E+05             |
| 2    | 2.064099E+02 | 1.681981E+06 | 1.296912E+03 | 1.000000E+00        | 1.681981E+06             |

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FREQUENCY RESPONSE LOADCASE USING DIRECT DYNAMIC ANALYSIS LOADCASE 4

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 2.0000E+01)

| GRID ID           | T1<br>R1           | T2<br>R2     | T3<br>R3     |
|-------------------|--------------------|--------------|--------------|
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |





---

## 2.30 Thin Axisymmetric Annulus

### Example ID:

A030

### Analysis Data:

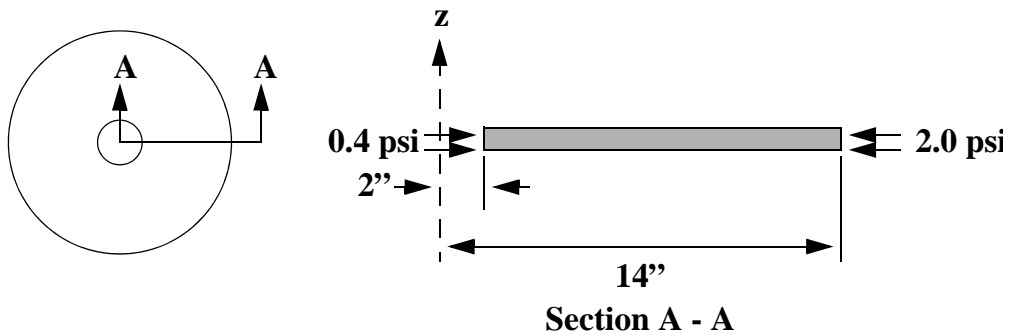
GRDSET, GRID, CTRIAX6, PAXIS, PLOADX1, RFORCE, LOADCASE, LOAD, GRDSET, SPC, SPC1, MAT1

### Special Features Used:

None

### Problem Statement:

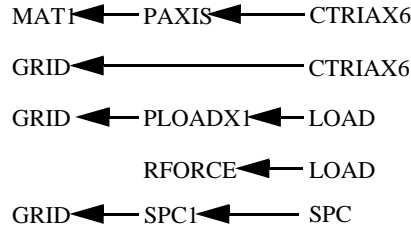
Find the circumferential stresses at the inner and outer surfaces of a thin axisymmetric annulus for separate loading conditions of centrifugal loading and combined inner and outer pressure.



### Structural Analysis Model:

1. Axisymmetric model composed of 24 CTRIAX6 elements.
2. Inner radius (a) = 2.0 Outer radius (b) = 14.0
3. Material:  $E=10.0E+6$ ,  $\nu = 0.3$ ,  $\rho = 0.1$
4. Two load cases:  
 $\omega = 0.2$  revolutions per unit time  
 $\rho_i = 0.4$ ;  $\rho_o = 2.0$

## Analysis Data Relationships:



## Special Modeling Techniques:

None.

## Reference Solutions:

For centrifugal loading, the analytical solution is:

1. At inner surface ( $r = a$ ):  $\sigma_{\theta\theta} = \frac{3+\nu}{3+4\nu} \rho \omega^2 b^2 \left[ 1 + \frac{1-\nu}{3+\nu} \left( \frac{a}{b} \right)^2 \right] = 25.6096 \text{ psi}$
2. At outer surface ( $r = b$ ):  $\sigma_{\theta\theta} = \frac{3+4\nu}{4} \rho \omega^2 b^2 \left[ \left( \frac{a}{b} \right)^2 + \frac{1-\nu}{3+\nu} \right] = 5.9376 \text{ psi}$

For pressure loading, the analytical solution is:

1. At inner surface ( $r = a$ ):  $\sigma_{\theta\theta} = \frac{p_i(a^2 + b^2) - 2p_o b^2}{2p_i a^2 - p_o(a^2 + b^2)} = -3.6667 \text{ psi}$
2. At outer surface ( $r = b$ ):  $\sigma_{\theta\theta} = \frac{2p_i a^2 - p_o(a^2 + b^2)}{b^2 - a^2} = -2.0667$

where:  $p_i$  = inner surface pressure,  $p_o$  = outer surface pressure.

## Calculated Solutions:

For the centrifugal loading loadcase:

1. At  $r = a$ :  $\sigma_{\theta\theta} = 24.26129$
2. At  $r = b$ :  $\sigma_{\theta\theta} = 5.953738$

For the pressure loadcase:

1. At  $r = a$ :  $\sigma_{\theta\theta} = -3.486024$
2. At  $r = b$ :  $\sigma_{\theta\theta} = -2.066367$

## Comparison Between Calculated Solutions and Reference Solutions:

The differences between the analytical solutions and the *GENESIS* solutions are:

For the centrifugal loading loadcase:

1. At  $r = a$ : -5.3%
2. At  $r = b$ : 0.3%

For the pressure loadcase:

1. At  $r = a$ : -4.9%
2. At  $r = b$ : -0.01%

The error at the inner surface ( $r=a$ ) is due to the high stress gradient at this point.

---

## 2.30.1 Input Data

```
ID A030
SOL COMPAT1
CEND

TITLE=Analysis of a thin axisymmetric annulus
ECHO=NONE
LINE=*,80
```

```
LOADCASE 30
  LABEL = Centrifugal loading
  CENT=300
  SPC =200
  OLOAD = ALL
  DISPLACEMENT=ALL
  STRESS = ALL
  STRAIN = ALL
  GSTRESS = ALL
```

```
LOADCASE 60
  LABEL = Inner and outer pressure loading
  LOAD=600
  SPC =200
  OLOAD = ALL
  DISPLACEMENT=ALL
  STRESS = ALL
  STRAIN = ALL
  GSTRESS = ALL
```

```
BEGIN BULK
$                                     Analysis Data
$
$   STRUCTURAL LOAD DATA
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9
RFORCE  300                                0.2    0.0    0.0    1.0
$
PLOADX1 600      306      2.0    2.0    113    313
PLOADX1 600      307      2.0    2.0    313    513
PLOADX1 600      102      0.4    0.4    101    301
PLOADX1 600      103      0.4    0.4    301    501
$
$   GRID DATA
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9
GRDSET                                         2456
GRID    101      2.0    0.0    3.0
GRID    102      3.0    0.0    3.0
GRID    103      4.0    0.0    3.0
GRID    104      5.0    0.0    3.0
GRID    105      6.0    0.0    3.0
GRID    106      7.0    0.0    3.0
GRID    107      8.0    0.0    3.0
```

|      |     |      |     |      |
|------|-----|------|-----|------|
| GRID | 108 | 9.0  | 0.0 | 3.0  |
| GRID | 109 | 10.0 | 0.0 | 3.0  |
| GRID | 110 | 11.0 | 0.0 | 3.0  |
| GRID | 111 | 12.0 | 0.0 | 3.0  |
| GRID | 112 | 13.0 | 0.0 | 3.0  |
| GRID | 113 | 14.0 | 0.0 | 3.0  |
|      |     |      |     |      |
| GRID | 201 | 2.0  | 0.0 | 3.25 |
| GRID | 202 | 3.0  | 0.0 | 3.25 |
| GRID | 203 | 4.0  | 0.0 | 3.25 |
| GRID | 204 | 5.0  | 0.0 | 3.25 |
| GRID | 205 | 6.0  | 0.0 | 3.25 |
| GRID | 206 | 7.0  | 0.0 | 3.25 |
| GRID | 207 | 8.0  | 0.0 | 3.25 |
| GRID | 208 | 9.0  | 0.0 | 3.25 |
| GRID | 209 | 10.0 | 0.0 | 3.25 |
| GRID | 210 | 11.0 | 0.0 | 3.25 |
| GRID | 211 | 12.0 | 0.0 | 3.25 |
| GRID | 212 | 13.0 | 0.0 | 3.25 |
| GRID | 213 | 14.0 | 0.0 | 3.25 |
|      |     |      |     |      |
| GRID | 301 | 2.0  | 0.0 | 3.5  |
| GRID | 302 | 3.0  | 0.0 | 3.5  |
| GRID | 303 | 4.0  | 0.0 | 3.5  |
| GRID | 304 | 5.0  | 0.0 | 3.5  |
| GRID | 305 | 6.0  | 0.0 | 3.5  |
| GRID | 306 | 7.0  | 0.0 | 3.5  |
| GRID | 307 | 8.0  | 0.0 | 3.5  |
| GRID | 308 | 9.0  | 0.0 | 3.5  |
| GRID | 309 | 10.0 | 0.0 | 3.5  |
| GRID | 310 | 11.0 | 0.0 | 3.5  |
| GRID | 311 | 12.0 | 0.0 | 3.5  |
| GRID | 312 | 13.0 | 0.0 | 3.5  |
| GRID | 313 | 14.0 | 0.0 | 3.5  |
|      |     |      |     |      |
| GRID | 401 | 2.0  | 0.0 | 3.75 |
| GRID | 402 | 3.0  | 0.0 | 3.75 |
| GRID | 403 | 4.0  | 0.0 | 3.75 |
| GRID | 404 | 5.0  | 0.0 | 3.75 |
| GRID | 405 | 6.0  | 0.0 | 3.75 |
| GRID | 406 | 7.0  | 0.0 | 3.75 |
| GRID | 407 | 8.0  | 0.0 | 3.75 |
| GRID | 408 | 9.0  | 0.0 | 3.75 |
| GRID | 409 | 10.0 | 0.0 | 3.75 |
| GRID | 410 | 11.0 | 0.0 | 3.75 |
| GRID | 411 | 12.0 | 0.0 | 3.75 |
| GRID | 412 | 13.0 | 0.0 | 3.75 |
| GRID | 413 | 14.0 | 0.0 | 3.75 |
|      |     |      |     |      |
| GRID | 501 | 2.0  | 0.0 | 4.0  |
| GRID | 502 | 3.0  | 0.0 | 4.0  |
| GRID | 503 | 4.0  | 0.0 | 4.0  |
| GRID | 504 | 5.0  | 0.0 | 4.0  |
| GRID | 505 | 6.0  | 0.0 | 4.0  |
| GRID | 506 | 7.0  | 0.0 | 4.0  |

```

GRID      507          8.0      0.0      4.0
GRID      508          9.0      0.0      4.0
GRID      509         10.0      0.0      4.0
GRID      510         11.0      0.0      4.0
GRID      511         12.0      0.0      4.0
GRID      512         13.0      0.0      4.0
GRID      513         14.0      0.0      4.0
$
$   BOUNDARY CONDITIONS
$
SPC1      200      23456      301
$
$   ELEMENT DATA
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9
CTRIAX6 101      101      101      102      103      203      303      202
CTRIAX6 102      101      101      202      303      302      301      201
CTRIAX6 103      101      301      302      303      402      501      401
CTRIAX6 104      101      501      402      303      403      503      502

CTRIAX6 105      101      103      104      105      204      303      203
CTRIAX6 106      101      105      205      305      304      303      204
CTRIAX6 107      101      303      304      305      405      505      404
CTRIAX6 108      101      303      404      505      504      503      403

CTRIAX6 201      101      105      106      107      207      307      206
CTRIAX6 202      101      105      206      307      306      305      205
CTRIAX6 203      101      305      306      307      406      505      405
CTRIAX6 204      101      505      406      307      407      507      506

CTRIAX6 205      101      107      108      109      208      307      207
CTRIAX6 206      101      109      209      309      308      307      208
CTRIAX6 207      101      307      308      309      409      509      408
CTRIAX6 208      101      307      408      509      508      507      407

CTRIAX6 301      101      109      110      111      211      311      210
CTRIAX6 302      101      109      210      311      310      309      209
CTRIAX6 303      101      309      310      311      410      509      409
CTRIAX6 304      101      509      410      311      411      511      510

CTRIAX6 305      101      111      112      113      212      311      211
CTRIAX6 306      101      113      213      313      312      311      212
CTRIAX6 307      101      311      312      313      413      513      412
CTRIAX6 308      101      311      412      513      512      511      411
$
$   PROPERTY DATA
$
PAXIS      101      11      1.0
$
$   MATERIAL DATA
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9
MAT1      11      10.0+6          0.3      0.1      0.1E-6      25.0
$
ENDDATA

```



---

## 2.30.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A030  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

### ANALYSIS PROBLEM SUMMARY

|                                     |    |
|-------------------------------------|----|
| NUMBER OF GRID POINTS:              | 65 |
| NUMBER OF TRIAX6 ELEMENTS:          | 24 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 24 |



NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 129

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 2  
 TOTAL NUMBER OF LOAD CASES: 2

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 ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

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 ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 6.031858E+01  
 SYSTEM VOLUME 6.031858E+02  
 SYSTEM MASS/VOLUME 1.000000E-01

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 ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

#### S O L U T I O N R E S I D U A L S

LOADCASE 30 ; RESIDUAL : 1.399112E-14 ; STRAIN ENERGY : 3.540702E-03  
 LOADCASE 60 ; RESIDUAL : 1.604735E-14 ; STRAIN ENERGY : 1.788327E-04  
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 ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1          | T2         | T3         | R1         | R2          | R3         |
|----------|-------------|------------|------------|------------|-------------|------------|
| 30       | 9.0489E+02  | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 3.1671E+03  | 0.0000E+00 |
| 60       | -1.7090E+02 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -5.9816E+02 | 0.0000E+00 |

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 ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1          | T2         | T3          | R1         | R2         | R3         |
|----------|-------------|------------|-------------|------------|------------|------------|
| 30       | 4.6788E-12  | 0.0000E+00 | -3.1566E-12 | 0.0000E+00 | 3.1867E-11 | 0.0000E+00 |
| 60       | -2.3712E-12 | 0.0000E+00 | -3.6669E-13 | 0.0000E+00 | 3.3130E-12 | 0.0000E+00 |

M A X I M U M A P P L I E D F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3         |
|----------|---------|------------|-------------|------------|------------|
| 30       | 312     | 5.7903E+01 | 5.7903E+01  | 0.0000E+00 | 0.0000E+00 |
| 60       | 213     | 5.8643E+01 | -5.8643E+01 | 0.0000E+00 | 0.0000E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

M A X I M U M S P C F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3          |
|----------|---------|------------|-------------|------------|-------------|
| 30       | 310     | 1.0037E-11 | -1.5667E-12 | 0.0000E+00 | -9.9145E-12 |
| 60       | 211     | 2.6432E-12 | -2.6432E-12 | 0.0000E+00 | 7.9936E-15  |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

M A X I M U M D I S P L A C E M E N T

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3          |
|----------|---------|------------|-------------|------------|-------------|
| 30       | 312     | 8.4227E-06 | 8.4227E-06  | 0.0000E+00 | 2.6039E-17  |
| 60       | 113     | 2.0542E-06 | -2.0533E-06 | 0.0000E+00 | -6.1002E-08 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

L O A D V E C T O R

| ID  | F1<br>M1                      | F2<br>M2                     | F3<br>M3                     |
|-----|-------------------------------|------------------------------|------------------------------|
| 101 | -3.368583E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 102 | 1.757788E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 103 | -6.124697E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 104 | 3.815686E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 105 | -2.327385E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |



|     |                               |                              |                              |
|-----|-------------------------------|------------------------------|------------------------------|
| 208 | 2.688742E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 209 | 3.317136E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 210 | 4.011676E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 211 | 4.772364E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 212 | 5.599198E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 213 | 3.040299E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 301 | -2.082397E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 302 | 2.633620E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 303 | -4.654769E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 304 | 9.101299E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 305 | -1.224939E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 306 | 1.527499E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 307 | -4.654769E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 308 | 2.821035E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

#### L O A D V E C T O R

| ID  | F1<br>M1                      | F2<br>M2                     | F3<br>M3                     |
|-----|-------------------------------|------------------------------|------------------------------|
| 309 | -1.224939E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |



| ID  | F1<br>M1                      | F2<br>M2                     | F3<br>M3                     |
|-----|-------------------------------|------------------------------|------------------------------|
| 413 | 3.040299E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 501 | -3.368583E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 502 | 1.757788E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 503 | -6.124697E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 504 | 3.815686E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 505 | -2.327385E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 506 | 8.666446E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 507 | -6.124697E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 508 | 1.278224E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 509 | -2.327385E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 510 | 2.086684E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 511 | -6.124697E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 512 | 2.704054E+01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 513 | 1.427054E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

#### L O A D V E C T O R

| ID | F1<br>M1 | F2<br>M2 | F3<br>M3 |
|----|----------|----------|----------|
|----|----------|----------|----------|

|     |                               |                              |                              |
|-----|-------------------------------|------------------------------|------------------------------|
| 101 | 4.188790E-01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 113 | -1.466077E+01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 201 | 1.675516E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 213 | -5.864306E+01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 301 | 8.377580E-01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 313 | -2.932153E+01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 401 | 1.675516E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 413 | -5.864306E+01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 501 | 4.188790E-01<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 513 | -1.466077E+01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

# G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 101     | 5.103899E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.956459E-07<br>0.000000E+00 |
| 102     | 4.903241E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.725951E-07<br>0.000000E+00 |
| 103     | 5.185217E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.717595E-07<br>0.000000E+00 |
| 104     | 5.658290E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.512351E-07<br>0.000000E+00 |
| 105     | 6.186207E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.366666E-07<br>0.000000E+00 |

|     |                              |                              |                              |
|-----|------------------------------|------------------------------|------------------------------|
| 106 | 6.712006E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.148896E-07<br>0.000000E+00 |
| 107 | 7.200679E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.933777E-07<br>0.000000E+00 |
| 108 | 7.630017E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.657057E-07<br>0.000000E+00 |
| 109 | 7.981138E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.375532E-07<br>0.000000E+00 |
| 110 | 8.238201E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.041323E-07<br>0.000000E+00 |
| 111 | 8.385785E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.699485E-07<br>0.000000E+00 |
| 112 | 8.412127E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.302380E-07<br>0.000000E+00 |
| 113 | 8.305752E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 8.979336E-08<br>0.000000E+00 |
| 201 | 5.125909E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.975667E-07<br>0.000000E+00 |
| 202 | 4.898145E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.863327E-07<br>0.000000E+00 |
| 203 | 5.189862E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.862237E-07<br>0.000000E+00 |
| 204 | 5.662790E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.754314E-07<br>0.000000E+00 |
| 205 | 6.189201E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.682508E-07<br>0.000000E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

# G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 206     | 6.715479E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.575187E-07<br>0.000000E+00 |
| 207     | 7.205325E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 1.467803E-07<br>0.000000E+00 |





| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                      |
|---------|------------------------------|------------------------------|-------------------------------|
| 311     | 8.395039E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.295020E-17<br>0.000000E+00  |
| 312     | 8.422692E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.603941E-17<br>0.000000E+00  |
| 313     | 8.314920E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 2.896641E-17<br>0.000000E+00  |
| 401     | 5.125909E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.975667E-07<br>0.000000E+00 |
| 402     | 4.898145E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.863327E-07<br>0.000000E+00 |
| 403     | 5.189862E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.862237E-07<br>0.000000E+00 |
| 404     | 5.662790E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.754314E-07<br>0.000000E+00 |
| 405     | 6.189201E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.682508E-07<br>0.000000E+00 |
| 406     | 6.715479E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.575187E-07<br>0.000000E+00 |
| 407     | 7.205325E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.467803E-07<br>0.000000E+00 |
| 408     | 7.635640E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.329131E-07<br>0.000000E+00 |
| 409     | 7.986976E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.187532E-07<br>0.000000E+00 |
| 410     | 8.244171E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.021088E-07<br>0.000000E+00 |
| 411     | 8.392740E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -8.505267E-08<br>0.000000E+00 |
| 412     | 8.419898E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.512112E-08<br>0.000000E+00 |
| 413     | 8.312647E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -4.489989E-08<br>0.000000E+00 |
| 501     | 5.103899E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.956459E-07<br>0.000000E+00 |
| 502     | 4.903241E-06                 | 0.000000E+00                 | -3.725951E-07                 |



|     |                               |                              |                               |
|-----|-------------------------------|------------------------------|-------------------------------|
| 101 | -7.065214E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.226232E-08<br>0.000000E+00 |
| 102 | -7.106714E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.048677E-08<br>0.000000E+00 |
| 103 | -7.810126E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.160999E-08<br>0.000000E+00 |
| 104 | -8.813021E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.090128E-08<br>0.000000E+00 |
| 105 | -9.954867E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.108051E-08<br>0.000000E+00 |
| 106 | -1.117588E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.098522E-08<br>0.000000E+00 |
| 107 | -1.244721E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.102965E-08<br>0.000000E+00 |
| 108 | -1.375270E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.099210E-08<br>0.000000E+00 |
| 109 | -1.508184E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.100628E-08<br>0.000000E+00 |
| 110 | -1.642801E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.099661E-08<br>0.000000E+00 |
| 111 | -1.778697E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.100439E-08<br>0.000000E+00 |
| 112 | -1.915589E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.099759E-08<br>0.000000E+00 |
| 113 | -2.053264E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.100215E-08<br>0.000000E+00 |
| 201 | -7.090013E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.109508E-08<br>0.000000E+00 |
| 202 | -7.098267E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.025120E-08<br>0.000000E+00 |
| 203 | -7.813261E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.083959E-08<br>0.000000E+00 |
| 204 | -8.814682E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.041591E-08<br>0.000000E+00 |
| 205 | -9.954037E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.053178E-08<br>0.000000E+00 |

## ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING

LOADCASE

60

## GRID DISPLACEMENTS

| GRID ID | T1<br>R1                      | T2<br>R2                     | T3<br>R3                      |
|---------|-------------------------------|------------------------------|-------------------------------|
| 206     | -1.117560E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.049841E-08<br>0.000000E+00 |
| 207     | -1.244729E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.051621E-08<br>0.000000E+00 |
| 208     | -1.375282E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.049503E-08<br>0.000000E+00 |
| 209     | -1.508179E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.050234E-08<br>0.000000E+00 |
| 210     | -1.642797E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.049880E-08<br>0.000000E+00 |
| 211     | -1.778698E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.050243E-08<br>0.000000E+00 |
| 212     | -1.915591E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.049843E-08<br>0.000000E+00 |
| 213     | -2.053259E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.050087E-08<br>0.000000E+00 |
| 301     | -7.103496E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  |
| 302     | -7.092190E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -6.080026E-19<br>0.000000E+00 |
| 303     | -7.819624E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.215105E-18<br>0.000000E+00 |
| 304     | -8.816004E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -1.858645E-18<br>0.000000E+00 |
| 305     | -9.953549E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -2.557270E-18<br>0.000000E+00 |
| 306     | -1.117527E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -3.314536E-18<br>0.000000E+00 |
| 307     | -1.244745E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | -4.135129E-18<br>0.000000E+00 |
| 308     | -1.375293E-06                 | 0.000000E+00                 | -5.009955E-18                 |

|     |               |              |               |
|-----|---------------|--------------|---------------|
|     | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 309 | -1.508176E-06 | 0.000000E+00 | -5.926024E-18 |
|     | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 310 | -1.642792E-06 | 0.000000E+00 | -6.862998E-18 |
|     | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

# G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1      | T2<br>R2     | T3<br>R3      |
|---------|---------------|--------------|---------------|
| 311     | -1.778701E-06 | 0.000000E+00 | -7.804032E-18 |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 312     | -1.915593E-06 | 0.000000E+00 | -8.719172E-18 |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 313     | -2.053255E-06 | 0.000000E+00 | -9.591801E-18 |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 401     | -7.090013E-07 | 0.000000E+00 | 3.109508E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 402     | -7.098267E-07 | 0.000000E+00 | 3.025120E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 403     | -7.813261E-07 | 0.000000E+00 | 3.083959E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 404     | -8.814682E-07 | 0.000000E+00 | 3.041591E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 405     | -9.954037E-07 | 0.000000E+00 | 3.053178E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 406     | -1.117560E-06 | 0.000000E+00 | 3.049841E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 407     | -1.244729E-06 | 0.000000E+00 | 3.051621E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 408     | -1.375282E-06 | 0.000000E+00 | 3.049503E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 409     | -1.508179E-06 | 0.000000E+00 | 3.050234E-08  |
|         | 0.000000E+00  | 0.000000E+00 | 0.000000E+00  |
| 410     | -1.642797E-06 | 0.000000E+00 | 3.049880E-08  |

|     |                               |                              |                              |
|-----|-------------------------------|------------------------------|------------------------------|
|     | 0.000000E+00                  | 0.000000E+00                 | 0.000000E+00                 |
| 411 | -1.778698E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.050243E-08<br>0.000000E+00 |
| 412 | -1.915591E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.049843E-08<br>0.000000E+00 |
| 413 | -2.053259E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 3.050087E-08<br>0.000000E+00 |
| 501 | -7.065214E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.226232E-08<br>0.000000E+00 |
| 502 | -7.106714E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.048677E-08<br>0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 22  
ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

#### GRID DISPLACEMENTS

| GRID ID | T1<br>R1                      | T2<br>R2                     | T3<br>R3                     |
|---------|-------------------------------|------------------------------|------------------------------|
| 503     | -7.810126E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.160999E-08<br>0.000000E+00 |
| 504     | -8.813021E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.090128E-08<br>0.000000E+00 |
| 505     | -9.954867E-07<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.108051E-08<br>0.000000E+00 |
| 506     | -1.117588E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.098522E-08<br>0.000000E+00 |
| 507     | -1.244721E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.102965E-08<br>0.000000E+00 |
| 508     | -1.375270E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.099210E-08<br>0.000000E+00 |
| 509     | -1.508184E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.100628E-08<br>0.000000E+00 |
| 510     | -1.642801E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.099661E-08<br>0.000000E+00 |
| 511     | -1.778697E-06<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 6.100439E-08<br>0.000000E+00 |
| 512     | -1.915589E-06                 | 0.000000E+00                 | 6.099759E-08                 |

|     |               |              |              |
|-----|---------------|--------------|--------------|
|     | 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
| 513 | -2.053264E-06 | 0.000000E+00 | 6.100215E-08 |
|     | 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

S T R E S S E S I N T R I A X 6 E L E M E N T S

| TRIAX6 ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE      |
|-----------|---|---------------------|----|---------------------|---|------------------------|---------------|
| 101       | X | 7.333945E+00        | XY | 0.000000E+00        | 1 | 1.750564E+01           |               |
|           | Y | 1.750564E+01        | YZ | 0.000000E+00        | 2 | 7.334185E+00           |               |
|           | Z | 6.478222E-03        | ZX | -4.199049E-02       | 3 | 6.237600E-03           |               |
|           |   | 1.522147E+01        |    | 7.175469E+00        |   | 8.749699E+00           | -8.282019E+00 |
| 102       | X | 4.771605E+00        | XY | 0.000000E+00        | 1 | 2.063059E+01           |               |
|           | Y | 2.063059E+01        | YZ | 0.000000E+00        | 2 | 4.771636E+00           |               |
|           | Z | 1.414895E-02        | ZX | 1.223711E-02        | 3 | 1.411747E-02           |               |
|           |   | 1.869732E+01        |    | 8.814002E+00        |   | 1.030824E+01           | -8.472116E+00 |
| 103       | X | 4.771605E+00        | XY | 0.000000E+00        | 1 | 2.063059E+01           |               |
|           | Y | 2.063059E+01        | YZ | 0.000000E+00        | 2 | 4.771636E+00           |               |
|           | Z | 1.414895E-02        | ZX | -1.223711E-02       | 3 | 1.411747E-02           |               |
|           |   | 1.869732E+01        |    | 8.814002E+00        |   | 1.030824E+01           | -8.472116E+00 |
| 104       | X | 7.333945E+00        | XY | 0.000000E+00        | 1 | 1.750564E+01           |               |
|           | Y | 1.750564E+01        | YZ | 0.000000E+00        | 2 | 7.334185E+00           |               |
|           | Z | 6.478222E-03        | ZX | 4.199049E-02        | 3 | 6.237600E-03           |               |
|           |   | 1.522147E+01        |    | 7.175469E+00        |   | 8.749699E+00           | -8.282019E+00 |
| 105       | X | 9.197781E+00        | XY | 0.000000E+00        | 1 | 1.461221E+01           |               |
|           | Y | 1.461221E+01        | YZ | 0.000000E+00        | 2 | 9.197833E+00           |               |
|           | Z | -2.350039E-02       | ZX | 2.185451E-02        | 3 | -2.355219E-02          |               |
|           |   | 1.281707E+01        |    | 6.042027E+00        |   | 7.317879E+00           | -7.928829E+00 |
| 106       | X | 9.306973E+00        | XY | 0.000000E+00        | 1 | 1.379188E+01           |               |
|           | Y | 1.379188E+01        | YZ | 0.000000E+00        | 2 | 9.306998E+00           |               |
|           | Z | 7.042980E-03        | ZX | 1.536937E-02        | 3 | 7.017581E-03           |               |
|           |   | 1.217839E+01        |    | 5.740947E+00        |   | 6.892432E+00           | -7.701966E+00 |
| 107       | X | 9.306973E+00        | XY | 0.000000E+00        | 1 | 1.379188E+01           |               |
|           | Y | 1.379188E+01        | YZ | 0.000000E+00        | 2 | 9.306998E+00           |               |
|           | Z | 7.042980E-03        | ZX | -1.536937E-02       | 3 | 7.017581E-03           |               |
|           |   | 1.217839E+01        |    | 5.740947E+00        |   | 6.892432E+00           | -7.701966E+00 |
| 108       | X | 9.197781E+00        | XY | 0.000000E+00        | 1 | 1.461221E+01           |               |
|           | Y | 1.461221E+01        | YZ | 0.000000E+00        | 2 | 9.197833E+00           |               |
|           | Z | -2.350039E-02       | ZX | -2.185451E-02       | 3 | -2.355219E-02          |               |
|           |   | 1.281707E+01        |    | 6.042027E+00        |   | 7.317879E+00           | -7.928829E+00 |



|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 201 | X | 8.544251E+00  | XY | 0.000000E+00  | 1 | 1.196043E+01  |               |
|     | Y | 1.196043E+01  | YZ | 0.000000E+00  | 2 | 8.544253E+00  |               |
|     | Z | -1.004880E-03 | ZX | -3.794252E-03 | 3 | -1.006565E-03 |               |
|     |   | 1.067164E+01  |    | 5.030657E+00  |   | 5.980717E+00  | -6.834558E+00 |
| 202 | X | 8.958858E+00  | XY | 0.000000E+00  | 1 | 1.252003E+01  |               |
|     | Y | 1.252003E+01  | YZ | 0.000000E+00  | 2 | 8.958869E+00  |               |
|     | Z | -1.278222E-03 | ZX | -9.901391E-03 | 3 | -1.289163E-03 |               |
|     |   | 1.117474E+01  |    | 5.267823E+00  |   | 6.260657E+00  | -7.159202E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

S T R E S S E S I N T R I A X 6 E L E M E N T S

| TRIAx6 ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE      |
|-----------|---|---------------------|----|---------------------|---|------------------------|---------------|
| 203       | X | 8.958858E+00        | XY | 0.000000E+00        | 1 | 1.252003E+01           |               |
|           | Y | 1.252003E+01        | YZ | 0.000000E+00        | 2 | 8.958869E+00           |               |
|           | Z | -1.278222E-03       | ZX | 9.901391E-03        | 3 | -1.289163E-03          |               |
|           |   | 1.117474E+01        |    | 5.267823E+00        |   | 6.260657E+00           | -7.159202E+00 |
| 204       | X | 8.544251E+00        | XY | 0.000000E+00        | 1 | 1.196043E+01           |               |
|           | Y | 1.196043E+01        | YZ | 0.000000E+00        | 2 | 8.544253E+00           |               |
|           | Z | -1.004880E-03       | ZX | 3.794252E-03        | 3 | -1.006565E-03          |               |
|           |   | 1.067164E+01        |    | 5.030657E+00        |   | 5.980717E+00           | -6.834558E+00 |
| 205       | X | 7.435289E+00        | XY | 0.000000E+00        | 1 | 1.088634E+01           |               |
|           | Y | 1.088634E+01        | YZ | 0.000000E+00        | 2 | 7.435290E+00           |               |
|           | Z | -4.989227E-04       | ZX | 2.197288E-03        | 3 | -4.995720E-04          |               |
|           |   | 9.636493E+00        |    | 4.542687E+00        |   | 5.443420E+00           | -6.107044E+00 |
| 206       | X | 6.748748E+00        | XY | 0.000000E+00        | 1 | 1.034390E+01           |               |
|           | Y | 1.034390E+01        | YZ | 0.000000E+00        | 2 | 6.748753E+00           |               |
|           | Z | -2.436511E-04       | ZX | 5.799399E-03        | 3 | -2.486345E-04          |               |
|           |   | 9.096031E+00        |    | 4.287910E+00        |   | 5.172075E+00           | -5.697469E+00 |
| 207       | X | 6.748748E+00        | XY | 0.000000E+00        | 1 | 1.034390E+01           |               |
|           | Y | 1.034390E+01        | YZ | 0.000000E+00        | 2 | 6.748753E+00           |               |
|           | Z | -2.436511E-04       | ZX | -5.799399E-03       | 3 | -2.486345E-04          |               |
|           |   | 9.096031E+00        |    | 4.287910E+00        |   | 5.172075E+00           | -5.697469E+00 |
| 208       | X | 7.435289E+00        | XY | 0.000000E+00        | 1 | 1.088634E+01           |               |
|           | Y | 1.088634E+01        | YZ | 0.000000E+00        | 2 | 7.435290E+00           |               |
|           | Z | -4.989227E-04       | ZX | -2.197288E-03       | 3 | -4.995720E-04          |               |
|           |   | 9.636493E+00        |    | 4.542687E+00        |   | 5.443420E+00           | -6.107044E+00 |
| 301       | X | 4.244436E+00        | XY | 0.000000E+00        | 1 | 8.599099E+00           |               |
|           | Y | 8.599099E+00        | YZ | 0.000000E+00        | 2 | 4.244437E+00           |               |
|           | Z | -5.568710E-04       | ZX | -2.111130E-03       | 3 | -5.579209E-04          |               |
|           |   | 7.447724E+00        |    | 3.510891E+00        |   | 4.299829E+00           | -4.280993E+00 |

|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 302 | X | 5.153632E+00  | XY | 0.000000E+00  | 1 | 9.205834E+00  |               |
|     | Y | 9.205834E+00  | YZ | 0.000000E+00  | 2 | 5.153637E+00  |               |
|     | Z | 1.250512E-03  | ZX | -4.885708E-03 | 3 | 1.245879E-03  |               |
|     |   | 7.990365E+00  |    | 3.766694E+00  |   | 4.602294E+00  | -4.786906E+00 |
|     |   |               |    |               |   |               |               |
| 303 | X | 5.153632E+00  | XY | 0.000000E+00  | 1 | 9.205834E+00  |               |
|     | Y | 9.205834E+00  | YZ | 0.000000E+00  | 2 | 5.153637E+00  |               |
|     | Z | 1.250512E-03  | ZX | 4.885708E-03  | 3 | 1.245879E-03  |               |
|     |   | 7.990365E+00  |    | 3.766694E+00  |   | 4.602294E+00  | -4.786906E+00 |
|     |   |               |    |               |   |               |               |
| 304 | X | 4.244436E+00  | XY | 0.000000E+00  | 1 | 8.599099E+00  |               |
|     | Y | 8.599099E+00  | YZ | 0.000000E+00  | 2 | 4.244437E+00  |               |
|     | Z | -5.568710E-04 | ZX | 2.111130E-03  | 3 | -5.579209E-04 |               |
|     |   | 7.447724E+00  |    | 3.510891E+00  |   | 4.299829E+00  | -4.280993E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

S T R E S S E S I N T R I A X 6 E L E M E N T S

| TRIAX6 ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE      |
|-----------|---|---------------------|----|---------------------|---|------------------------|---------------|
| 305       | X | 2.243721E+00        | XY | 0.000000E+00        | 1 | 7.320398E+00           |               |
|           | Y | 7.320398E+00        | YZ | 0.000000E+00        | 2 | 2.243722E+00           |               |
|           | Z | -2.053575E-04       | ZX | 9.419530E-04        | 3 | -2.057529E-04          |               |
|           |   | 6.496117E+00        |    | 3.062299E+00        |   | 3.660302E+00           | -3.187971E+00 |
|           |   |                     |    |                     |   |                        |               |
| 306       | X | 1.143814E+00        | XY | 0.000000E+00        | 1 | 6.640925E+00           |               |
|           | Y | 6.640925E+00        | YZ | 0.000000E+00        | 2 | 1.143814E+00           |               |
|           | Z | 2.360518E-04        | ZX | 3.130736E-04        | 3 | 2.359661E-04           |               |
|           |   | 6.149176E+00        |    | 2.898749E+00        |   | 3.320344E+00           | -2.594992E+00 |
|           |   |                     |    |                     |   |                        |               |
| 307       | X | 1.143814E+00        | XY | 0.000000E+00        | 1 | 6.640925E+00           |               |
|           | Y | 6.640925E+00        | YZ | 0.000000E+00        | 2 | 1.143814E+00           |               |
|           | Z | 2.360518E-04        | ZX | -3.130736E-04       | 3 | 2.359661E-04           |               |
|           |   | 6.149176E+00        |    | 2.898749E+00        |   | 3.320344E+00           | -2.594992E+00 |
|           |   |                     |    |                     |   |                        |               |
| 308       | X | 2.243721E+00        | XY | 0.000000E+00        | 1 | 7.320398E+00           |               |
|           | Y | 7.320398E+00        | YZ | 0.000000E+00        | 2 | 2.243722E+00           |               |
|           | Z | -2.053575E-04       | ZX | -9.419530E-04       | 3 | -2.057529E-04          |               |
|           |   | 6.496117E+00        |    | 3.062299E+00        |   | 3.660302E+00           | -3.187971E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

S T R E S S E S I N T R I A X 6 E L E M E N T S

| TRIAX6 ID |  | NORMAL<br>VON MISES |  | SHEAR<br>OCTAHEDRAL |  | PRINCIPAL<br>MAX SHEAR | PRESSURE |
|-----------|--|---------------------|--|---------------------|--|------------------------|----------|
|-----------|--|---------------------|--|---------------------|--|------------------------|----------|



S T R E S S E S   I N   T R I A X 6   E L E M E N T S

| TRIAX6 ID | NORMAL<br>VON MISES |               | SHEAR<br>OCTAHEDRAL |               | PRINCIPAL<br>MAX SHEAR |               | PRESSURE     |
|-----------|---------------------|---------------|---------------------|---------------|------------------------|---------------|--------------|
| 203       | X                   | -1.884584E+00 | XY                  | 0.000000E+00  | 1                      | 4.307163E-04  |              |
|           | Y                   | -2.182296E+00 | YZ                  | 0.000000E+00  | 2                      | -1.884584E+00 |              |
|           | Z                   | 4.304135E-04  | ZX                  | -7.555241E-04 | 3                      | -2.182296E+00 |              |
|           |                     | 2.050147E+00  |                     | 9.664487E-01  |                        | 1.091363E+00  | 1.355483E+00 |
| 204       | X                   | -1.910056E+00 | XY                  | 0.000000E+00  | 1                      | -3.408511E-05 |              |
|           | Y                   | -2.156211E+00 | YZ                  | 0.000000E+00  | 2                      | -1.910056E+00 |              |
|           | Z                   | -3.410794E-05 | ZX                  | -2.088492E-04 | 3                      | -2.156211E+00 |              |
|           |                     | 2.044245E+00  |                     | 9.636664E-01  |                        | 1.078089E+00  | 1.355434E+00 |
| 205       | X                   | -1.945715E+00 | XY                  | 0.000000E+00  | 1                      | 1.155307E-04  |              |
|           | Y                   | -2.120964E+00 | YZ                  | 0.000000E+00  | 2                      | -1.945715E+00 |              |
|           | Z                   | 1.155241E-04  | ZX                  | -1.135104E-04 | 3                      | -2.120964E+00 |              |
|           |                     | 2.039111E+00  |                     | 9.612463E-01  |                        | 1.060540E+00  | 1.355521E+00 |
| 206       | X                   | -1.957697E+00 | XY                  | 0.000000E+00  | 1                      | -5.143151E-05 |              |
|           | Y                   | -2.108838E+00 | YZ                  | 0.000000E+00  | 2                      | -1.957697E+00 |              |
|           | Z                   | -5.143746E-05 | ZX                  | -1.078948E-04 | 3                      | -2.108838E+00 |              |
|           |                     | 2.037425E+00  |                     | 9.604515E-01  |                        | 1.054393E+00  | 1.355529E+00 |
| 207       | X                   | -1.957697E+00 | XY                  | 0.000000E+00  | 1                      | -5.143151E-05 |              |
|           | Y                   | -2.108838E+00 | YZ                  | 0.000000E+00  | 2                      | -1.957697E+00 |              |
|           | Z                   | -5.143746E-05 | ZX                  | 1.078948E-04  | 3                      | -2.108838E+00 |              |
|           |                     | 2.037425E+00  |                     | 9.604515E-01  |                        | 1.054393E+00  | 1.355529E+00 |
| 208       | X                   | -1.945715E+00 | XY                  | 0.000000E+00  | 1                      | 1.155307E-04  |              |
|           | Y                   | -2.120964E+00 | YZ                  | 0.000000E+00  | 2                      | -1.945715E+00 |              |
|           | Z                   | 1.155241E-04  | ZX                  | 1.135104E-04  | 3                      | -2.120964E+00 |              |
|           |                     | 2.039111E+00  |                     | 9.612463E-01  |                        | 1.060540E+00  | 1.355521E+00 |
| 301       | X                   | -1.982186E+00 | XY                  | 0.000000E+00  | 1                      | -4.144417E-06 |              |
|           | Y                   | -2.084397E+00 | YZ                  | 0.000000E+00  | 2                      | -1.982186E+00 |              |
|           | Z                   | -4.144895E-06 | ZX                  | 3.080522E-05  | 3                      | -2.084397E+00 |              |
|           |                     | 2.035213E+00  |                     | 9.594086E-01  |                        | 1.042196E+00  | 1.355529E+00 |
| 302       | X                   | -1.975628E+00 | XY                  | 0.000000E+00  | 1                      | 1.701804E-05  |              |
|           | Y                   | -2.091016E+00 | YZ                  | 0.000000E+00  | 2                      | -1.975628E+00 |              |
|           | Z                   | 1.701467E-05  | ZX                  | 8.153424E-05  | 3                      | -2.091016E+00 |              |
|           |                     | 2.035793E+00  |                     | 9.596818E-01  |                        | 1.045516E+00  | 1.355542E+00 |
| 303       | X                   | -1.975628E+00 | XY                  | 0.000000E+00  | 1                      | 1.701804E-05  |              |
|           | Y                   | -2.091016E+00 | YZ                  | 0.000000E+00  | 2                      | -1.975628E+00 |              |
|           | Z                   | 1.701467E-05  | ZX                  | -8.153424E-05 | 3                      | -2.091016E+00 |              |
|           |                     | 2.035793E+00  |                     | 9.596818E-01  |                        | 1.045516E+00  | 1.355542E+00 |
| 304       | X                   | -1.982186E+00 | XY                  | 0.000000E+00  | 1                      | -4.144417E-06 |              |
|           | Y                   | -2.084397E+00 | YZ                  | 0.000000E+00  | 2                      | -1.982186E+00 |              |
|           | Z                   | -4.144895E-06 | ZX                  | -3.080522E-05 | 3                      | -2.084397E+00 |              |
|           |                     | 2.035213E+00  |                     | 9.594086E-01  |                        | 1.042196E+00  | 1.355529E+00 |

INNER AND OUTER PRESSURE LOADING LOADCASE 60

S T R E S S E S I N T R I A X 6 E L E M E N T S

| TRIAX6 ID | NORMAL<br>VON MISES |               | SHEAR<br>OCTAHEDRAL |               | PRINCIPAL<br>MAX SHEAR |               | PRESSURE     |
|-----------|---------------------|---------------|---------------------|---------------|------------------------|---------------|--------------|
| 305       | X                   | -1.992472E+00 | XY                  | 0.000000E+00  | 1                      | 1.667326E-05  |              |
|           | Y                   | -2.074162E+00 | YZ                  | 0.000000E+00  | 2                      | -1.992472E+00 |              |
|           | Z                   | 1.667308E-05  | ZX                  | -1.910872E-05 | 3                      | -2.074162E+00 |              |
|           |                     | 2.034564E+00  |                     | 9.591026E-01  |                        | 1.037089E+00  | 1.355539E+00 |
| 306       | X                   | -1.996426E+00 | XY                  | 0.000000E+00  | 1                      | -4.616309E-06 |              |
|           | Y                   | -2.070164E+00 | YZ                  | 0.000000E+00  | 2                      | -1.996426E+00 |              |
|           | Z                   | -4.616320E-06 | ZX                  | -4.723096E-06 | 3                      | -2.070164E+00 |              |
|           |                     | 2.034293E+00  |                     | 9.589750E-01  |                        | 1.035080E+00  | 1.355532E+00 |
| 307       | X                   | -1.996426E+00 | XY                  | 0.000000E+00  | 1                      | -4.616309E-06 |              |
|           | Y                   | -2.070164E+00 | YZ                  | 0.000000E+00  | 2                      | -1.996426E+00 |              |
|           | Z                   | -4.616320E-06 | ZX                  | 4.723096E-06  | 3                      | -2.070164E+00 |              |
|           |                     | 2.034293E+00  |                     | 9.589750E-01  |                        | 1.035080E+00  | 1.355532E+00 |
| 308       | X                   | -1.992472E+00 | XY                  | 0.000000E+00  | 1                      | 1.667326E-05  |              |
|           | Y                   | -2.074162E+00 | YZ                  | 0.000000E+00  | 2                      | -1.992472E+00 |              |
|           | Z                   | 1.667308E-05  | ZX                  | 1.910872E-05  | 3                      | -2.074162E+00 |              |
|           |                     | 2.034564E+00  |                     | 9.591026E-01  |                        | 1.037089E+00  | 1.355539E+00 |

CENTRIFUGAL LOADING LOADCASE 30

G R I D S T R E S S E S

| GRID ID | NORMAL<br>VON MISES |               | SHEAR<br>OCTAHEDRAL |               | PRINCIPAL<br>MAX SHEAR |               | PRESSURE      |
|---------|---------------------|---------------|---------------------|---------------|------------------------|---------------|---------------|
| 101     | X                   | 2.326530E+00  | XY                  | 0.000000E+00  | 1                      | 2.322940E+01  |               |
|         | Y                   | 2.322940E+01  | YZ                  | 0.000000E+00  | 2                      | 2.326639E+00  |               |
|         | Z                   | -3.283341E-02 | ZX                  | -1.609089E-02 | 3                      | -3.294314E-02 |               |
|         |                     | 2.217690E+01  |                     | 1.045429E+01  |                        | 1.163117E+01  | -8.507698E+00 |
| 102     | X                   | 5.985299E+00  | XY                  | 0.000000E+00  | 1                      | 1.864372E+01  |               |
|         | Y                   | 1.864372E+01  | YZ                  | 0.000000E+00  | 2                      | 5.988120E+00  |               |
|         | Z                   | -4.803320E-02 | ZX                  | -1.304885E-01 | 3                      | -5.085408E-02 |               |
|         |                     | 1.652454E+01  |                     | 7.789742E+00  |                        | 9.347288E+00  | -8.193663E+00 |
| 103     | X                   | 9.446508E+00  | XY                  | 0.000000E+00  | 1                      | 1.531771E+01  |               |
|         | Y                   | 1.531771E+01  | YZ                  | 0.000000E+00  | 2                      | 9.446676E+00  |               |
|         | Z                   | 2.159359E-02  | ZX                  | 3.986093E-02  | 3                      | 2.142501E-02  |               |
|         |                     | 1.336564E+01  |                     | 6.300622E+00  |                        | 7.648141E+00  | -8.261936E+00 |

|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 104 | X | 9.259262E+00  | XY | 0.000000E+00  | 1 | 1.417051E+01  |               |
|     | Y | 1.417051E+01  | YZ | 0.000000E+00  | 2 | 9.259264E+00  |               |
|     | Z | 1.053968E-02  | ZX | -4.406535E-03 | 3 | 1.053758E-02  |               |
|     |   | 1.245319E+01  |    | 5.870491E+00  |   | 7.079984E+00  | -7.813436E+00 |
| 105 | X | 9.403076E+00  | XY | 0.000000E+00  | 1 | 1.300226E+01  |               |
|     | Y | 1.300226E+01  | YZ | 0.000000E+00  | 2 | 9.403079E+00  |               |
|     | Z | 1.383007E-02  | ZX | -5.309717E-03 | 3 | 1.382707E-02  |               |
|     |   | 1.161490E+01  |    | 5.475315E+00  |   | 6.494218E+00  | -7.473056E+00 |
| 106 | X | 8.739018E+00  | XY | 0.000000E+00  | 1 | 1.222436E+01  |               |
|     | Y | 1.222436E+01  | YZ | 0.000000E+00  | 2 | 8.739022E+00  |               |
|     | Z | -2.164244E-03 | ZX | -5.940809E-03 | 3 | -2.168282E-03 |               |
|     |   | 1.090972E+01  |    | 5.142892E+00  |   | 6.113265E+00  | -6.987072E+00 |
| 107 | X | 8.120595E+00  | XY | 0.000000E+00  | 1 | 1.142315E+01  |               |
|     | Y | 1.142315E+01  | YZ | 0.000000E+00  | 2 | 8.120595E+00  |               |
|     | Z | 2.884593E-03  | ZX | 1.479288E-03  | 3 | 2.884323E-03  |               |
|     |   | 1.017906E+01  |    | 4.798455E+00  |   | 5.710133E+00  | -6.515543E+00 |
| 108 | X | 7.084049E+00  | XY | 0.000000E+00  | 1 | 1.060667E+01  |               |
|     | Y | 1.060667E+01  | YZ | 0.000000E+00  | 2 | 7.084050E+00  |               |
|     | Z | -8.322505E-04 | ZX | 3.026620E-03  | 3 | -8.335435E-04 |               |
|     |   | 9.357443E+00  |    | 4.411141E+00  |   | 5.303750E+00  | -5.896628E+00 |
| 109 | X | 6.055790E+00  | XY | 0.000000E+00  | 1 | 9.796524E+00  |               |
|     | Y | 9.796524E+00  | YZ | 0.000000E+00  | 2 | 6.055790E+00  |               |
|     | Z | -4.682539E-04 | ZX | 1.069687E-03  | 3 | -4.684428E-04 |               |
|     |   | 8.563072E+00  |    | 4.036671E+00  |   | 4.898496E+00  | -5.283948E+00 |
| 110 | X | 4.691642E+00  | XY | 0.000000E+00  | 1 | 8.896163E+00  |               |
|     | Y | 8.896163E+00  | YZ | 0.000000E+00  | 2 | 4.691643E+00  |               |
|     | Z | -1.462503E-03 | ZX | -2.351016E-03 | 3 | -1.463681E-03 |               |
|     |   | 7.709442E+00  |    | 3.634266E+00  |   | 4.448813E+00  | -4.528781E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

# G R I D S T R E S S E S

| GRID ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE      |
|---------|---|---------------------|----|---------------------|---|------------------------|---------------|
| 111     | X | 3.325677E+00        | XY | 0.000000E+00        | 1 | 7.990892E+00           |               |
|         | Y | 7.990892E+00        | YZ | 0.000000E+00        | 2 | 3.325677E+00           |               |
|         | Z | 2.276608E-03        | ZX | 6.874443E-04        | 3 | 2.276466E-03           |               |
|         |   | 6.950799E+00        |    | 3.276638E+00        |   | 3.994308E+00           | -3.772949E+00 |
| 112     | X | 1.692751E+00        | XY | 0.000000E+00        | 1 | 6.976991E+00           |               |
|         | Y | 6.976991E+00        | YZ | 0.000000E+00        | 2 | 1.692752E+00           |               |
|         | Z | -6.168913E-04       | ZX | 1.220377E-03        | 3 | -6.177708E-04          |               |
|         |   | 6.303876E+00        |    | 2.971676E+00        |   | 3.488804E+00           | -2.889708E+00 |

|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 113 | X | 5.095573E-02  | XY | 0.000000E+00  | 1 | 5.956761E+00  |               |
|     | Y | 5.956761E+00  | YZ | 0.000000E+00  | 2 | 5.099847E-02  |               |
|     | Z | -6.827165E-04 | ZX | -1.486146E-03 | 3 | -7.254521E-04 |               |
|     |   | 5.931794E+00  |    | 2.796274E+00  |   | 2.978743E+00  | -2.002345E+00 |
| 201 | X | 2.301235E+00  | XY | 0.000000E+00  | 1 | 2.428462E+01  |               |
|     | Y | 2.428462E+01  | YZ | 0.000000E+00  | 2 | 2.321514E+00  |               |
|     | Z | 6.283901E-02  | ZX | 2.140199E-01  | 3 | 4.255964E-02  |               |
|     |   | 2.318673E+01  |    | 1.093033E+01  |   | 1.212103E+01  | -8.882898E+00 |
| 202 | X | 6.099160E+00  | XY | 0.000000E+00  | 1 | 1.875270E+01  |               |
|     | Y | 1.875270E+01  | YZ | 0.000000E+00  | 2 | 6.100011E+00  |               |
|     | Z | -8.349235E-03 | ZX | -7.212615E-02 | 3 | -9.200885E-03 |               |
|     |   | 1.657441E+01  |    | 7.813250E+00  |   | 9.380950E+00  | -8.281170E+00 |
| 203 | X | 9.499767E+00  | XY | 0.000000E+00  | 1 | 1.534012E+01  |               |
|     | Y | 1.534012E+01  | YZ | 0.000000E+00  | 2 | 9.500318E+00  |               |
|     | Z | 1.677693E-02  | ZX | 7.227601E-02  | 3 | 1.622610E-02  |               |
|     |   | 1.339539E+01  |    | 6.314648E+00  |   | 7.661947E+00  | -8.285555E+00 |
| 204 | X | 9.242598E+00  | XY | 0.000000E+00  | 1 | 1.416581E+01  |               |
|     | Y | 1.416581E+01  | YZ | 0.000000E+00  | 2 | 9.242731E+00  |               |
|     | Z | -1.070969E-02 | ZX | 3.510880E-02  | 3 | -1.084290E-02 |               |
|     |   | 1.246681E+01  |    | 5.876911E+00  |   | 7.088325E+00  | -7.799232E+00 |
| 205 | X | 9.414941E+00  | XY | 0.000000E+00  | 1 | 1.307048E+01  |               |
|     | Y | 1.307048E+01  | YZ | 0.000000E+00  | 2 | 9.414944E+00  |               |
|     | Z | 1.229448E-02  | ZX | -4.965946E-03 | 3 | 1.229186E-02  |               |
|     |   | 1.166810E+01  |    | 5.500394E+00  |   | 6.529093E+00  | -7.499238E+00 |
| 206 | X | 8.752293E+00  | XY | 0.000000E+00  | 1 | 1.223195E+01  |               |
|     | Y | 1.223195E+01  | YZ | 0.000000E+00  | 2 | 8.752311E+00  |               |
|     | Z | -5.997827E-03 | ZX | -1.263272E-02 | 3 | -6.016048E-03 |               |
|     |   | 1.092209E+01  |    | 5.148722E+00  |   | 6.118983E+00  | -6.992749E+00 |
| 207 | X | 8.133419E+00  | XY | 0.000000E+00  | 1 | 1.143114E+01  |               |
|     | Y | 1.143114E+01  | YZ | 0.000000E+00  | 2 | 8.133419E+00  |               |
|     | Z | 1.812691E-03  | ZX | 9.990156E-04  | 3 | 1.812569E-03  |               |
|     |   | 1.018890E+01  |    | 4.803094E+00  |   | 5.714663E+00  | -6.522123E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

# G R I D S T R E S S E S

| GRID ID |   | NORMAL<br>VON MISES | SHEAR<br>OCTAHEDRAL | PRINCIPAL<br>MAX SHEAR | PRESSURE |               |
|---------|---|---------------------|---------------------|------------------------|----------|---------------|
| 208     | X | 7.095377E+00        | XY                  | 0.000000E+00           | 1        | 1.061397E+01  |
|         | Y | 1.061397E+01        | YZ                  | 0.000000E+00           | 2        | 7.095388E+00  |
|         | Z | -3.608767E-03       | ZX                  | 8.580428E-03           | 3        | -3.619138E-03 |
|         |   | 9.367750E+00        |                     | 4.416000E+00           |          | 5.308794E+00  |
|         |   |                     |                     |                        |          | -5.901913E+00 |

|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 209 | X | 6.056095E+00  | XY | 0.000000E+00  | 1 | 9.804091E+00  |               |
|     | Y | 9.804091E+00  | YZ | 0.000000E+00  | 2 | 6.056095E+00  |               |
|     | Z | 6.990796E-03  | ZX | 5.749489E-04  | 3 | 6.990742E-03  |               |
|     |   | 8.562193E+00  |    | 4.036257E+00  |   | 4.898550E+00  | -5.289059E+00 |
| 210 | X | 4.703051E+00  | XY | 0.000000E+00  | 1 | 8.903431E+00  |               |
|     | Y | 8.903431E+00  | YZ | 0.000000E+00  | 2 | 4.703064E+00  |               |
|     | Z | -2.739606E-03 | ZX | -7.673606E-03 | 3 | -2.752119E-03 |               |
|     |   | 7.717120E+00  |    | 3.637885E+00  |   | 4.453091E+00  | -4.534581E+00 |
| 211 | X | 3.336760E+00  | XY | 0.000000E+00  | 1 | 7.998492E+00  |               |
|     | Y | 7.998492E+00  | YZ | 0.000000E+00  | 2 | 3.336760E+00  |               |
|     | Z | 1.606330E-03  | ZX | 3.215829E-05  | 3 | 1.606329E-03  |               |
|     |   | 6.957197E+00  |    | 3.279654E+00  |   | 3.998443E+00  | -3.778953E+00 |
| 212 | X | 1.701471E+00  | XY | 0.000000E+00  | 1 | 6.984394E+00  |               |
|     | Y | 6.984394E+00  | YZ | 0.000000E+00  | 2 | 1.701477E+00  |               |
|     | Z | -5.061432E-04 | ZX | 3.215185E-03  | 3 | -5.122170E-04 |               |
|     |   | 6.308521E+00  |    | 2.973865E+00  |   | 3.492453E+00  | -2.895120E+00 |
| 213 | X | 3.656624E-02  | XY | 0.000000E+00  | 1 | 5.952966E+00  |               |
|     | Y | 5.952966E+00  | YZ | 0.000000E+00  | 2 | 3.798870E-02  |               |
|     | Z | 9.925027E-04  | ZX | -7.254353E-03 | 3 | -4.299578E-04 |               |
|     |   | 5.934280E+00  |    | 2.797446E+00  |   | 2.976698E+00  | -1.996842E+00 |
| 301 | X | 2.124507E+00  | XY | 0.000000E+00  | 1 | 2.426129E+01  |               |
|     | Y | 2.426129E+01  | YZ | 0.000000E+00  | 2 | 2.124507E+00  |               |
|     | Z | 2.331878E-02  | ZX | -2.345624E-13 | 3 | 2.331878E-02  |               |
|     |   | 2.325867E+01  |    | 1.096424E+01  |   | 1.211899E+01  | -8.803039E+00 |
| 302 | X | 5.918426E+00  | XY | 0.000000E+00  | 1 | 1.879192E+01  |               |
|     | Y | 1.879192E+01  | YZ | 0.000000E+00  | 2 | 5.918426E+00  |               |
|     | Z | -2.995620E-02 | ZX | -4.896084E-14 | 3 | -2.995620E-02 |               |
|     |   | 1.666393E+01  |    | 7.855450E+00  |   | 9.410936E+00  | -8.226795E+00 |
| 303 | X | 9.454527E+00  | XY | 0.000000E+00  | 1 | 1.483073E+01  |               |
|     | Y | 1.483073E+01  | YZ | 0.000000E+00  | 2 | 9.454527E+00  |               |
|     | Z | -1.949495E-02 | ZX | -2.902539E-14 | 3 | -1.949495E-02 |               |
|     |   | 1.302286E+01  |    | 6.139036E+00  |   | 7.425113E+00  | -8.088588E+00 |
| 304 | X | 9.223033E+00  | XY | 0.000000E+00  | 1 | 1.416277E+01  |               |
|     | Y | 1.416277E+01  | YZ | 0.000000E+00  | 2 | 9.223033E+00  |               |
|     | Z | 1.784934E-03  | ZX | -3.599725E-14 | 3 | 1.784934E-03  |               |
|     |   | 1.244921E+01  |    | 5.868615E+00  |   | 7.080491E+00  | -7.795862E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

#### GRID STRESSES

| GRID ID | NORMAL<br>VON MISES | SHEAR<br>OCTAHEDRAL | PRINCIPAL<br>MAX SHEAR | PRESSURE |
|---------|---------------------|---------------------|------------------------|----------|
|---------|---------------------|---------------------|------------------------|----------|



|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 305 | X | 9.409884E+00  | XY | 0.000000E+00  | 1 | 1.307381E+01  |               |
|     | Y | 1.307381E+01  | YZ | 0.000000E+00  | 2 | 9.409884E+00  |               |
|     | Z | 1.837404E-02  | ZX | -5.058031E-14 | 3 | 1.837404E-02  |               |
|     |   | 1.166339E+01  |    | 5.498173E+00  |   | 6.527716E+00  | -7.500688E+00 |
| 306 | X | 8.755714E+00  | XY | 0.000000E+00  | 1 | 1.223794E+01  |               |
|     | Y | 1.223794E+01  | YZ | 0.000000E+00  | 2 | 8.755714E+00  |               |
|     | Z | 6.472778E-04  | ZX | -2.858824E-15 | 3 | 6.472778E-04  |               |
|     |   | 1.092082E+01  |    | 5.148123E+00  |   | 6.118648E+00  | -6.998102E+00 |
| 307 | X | 8.141138E+00  | XY | 0.000000E+00  | 1 | 1.142196E+01  |               |
|     | Y | 1.142196E+01  | YZ | 0.000000E+00  | 2 | 8.141138E+00  |               |
|     | Z | -6.044442E-03 | ZX | 4.264812E-14  | 3 | -6.044442E-03 |               |
|     |   | 1.019165E+01  |    | 4.804391E+00  |   | 5.714001E+00  | -6.519017E+00 |
| 308 | X | 7.095101E+00  | XY | 0.000000E+00  | 1 | 1.061790E+01  |               |
|     | Y | 1.061790E+01  | YZ | 0.000000E+00  | 2 | 7.095101E+00  |               |
|     | Z | -1.686871E-03 | ZX | 4.785777E-14  | 3 | -1.686871E-03 |               |
|     |   | 9.368836E+00  |    | 4.416512E+00  |   | 5.309794E+00  | -5.903772E+00 |
| 309 | X | 6.056219E+00  | XY | 0.000000E+00  | 1 | 9.808332E+00  |               |
|     | Y | 9.808332E+00  | YZ | 0.000000E+00  | 2 | 6.056219E+00  |               |
|     | Z | 1.017564E-02  | ZX | 1.758905E-13  | 3 | 1.017564E-02  |               |
|     |   | 8.562618E+00  |    | 4.036457E+00  |   | 4.899078E+00  | -5.291576E+00 |
| 310 | X | 4.702499E+00  | XY | 0.000000E+00  | 1 | 8.906852E+00  |               |
|     | Y | 8.906852E+00  | YZ | 0.000000E+00  | 2 | 4.702499E+00  |               |
|     | Z | -6.087948E-04 | ZX | 7.912117E-14  | 3 | -6.087948E-04 |               |
|     |   | 7.718117E+00  |    | 3.638355E+00  |   | 4.453730E+00  | -4.536247E+00 |
| 311 | X | 3.350677E+00  | XY | 0.000000E+00  | 1 | 8.007695E+00  |               |
|     | Y | 8.007695E+00  | YZ | 0.000000E+00  | 2 | 3.350677E+00  |               |
|     | Z | -2.975630E-03 | ZX | 1.281948E-13  | 3 | -2.975630E-03 |               |
|     |   | 6.967986E+00  |    | 3.284740E+00  |   | 4.005335E+00  | -3.785132E+00 |
| 312 | X | 1.694696E+00  | XY | 0.000000E+00  | 1 | 6.985289E+00  |               |
|     | Y | 6.985289E+00  | YZ | 0.000000E+00  | 2 | 1.694696E+00  |               |
|     | Z | -9.771231E-04 | ZX | 1.218210E-15  | 3 | -9.771231E-04 |               |
|     |   | 6.311640E+00  |    | 2.975336E+00  |   | 3.493133E+00  | -2.893003E+00 |
| 313 | X | 3.108285E-02  | XY | 0.000000E+00  | 1 | 5.953738E+00  |               |
|     | Y | 5.953738E+00  | YZ | 0.000000E+00  | 2 | 3.108285E-02  |               |
|     | Z | -6.773962E-04 | ZX | -1.519702E-13 | 3 | -6.773962E-04 |               |
|     |   | 5.938599E+00  |    | 2.799482E+00  |   | 2.977208E+00  | -1.994714E+00 |
| 401 | X | 2.301235E+00  | XY | 0.000000E+00  | 1 | 2.428462E+01  |               |
|     | Y | 2.428462E+01  | YZ | 0.000000E+00  | 2 | 2.321514E+00  |               |
|     | Z | 6.283901E-02  | ZX | -2.140199E-01 | 3 | 4.255964E-02  |               |
|     |   | 2.318673E+01  |    | 1.093033E+01  |   | 1.212103E+01  | -8.882898E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING

LOADCASE 30

# G R I D   S T R E S S E S

| GRID ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE      |
|---------|---|---------------------|----|---------------------|---|------------------------|---------------|
| 402     | X | 6.099160E+00        | XY | 0.000000E+00        | 1 | 1.875270E+01           |               |
|         | Y | 1.875270E+01        | YZ | 0.000000E+00        | 2 | 6.100011E+00           |               |
|         | Z | -8.349235E-03       | ZX | 7.212615E-02        | 3 | -9.200885E-03          |               |
|         |   | 1.657441E+01        |    | 7.813250E+00        |   | 9.380950E+00           | -8.281170E+00 |
| 403     | X | 9.499767E+00        | XY | 0.000000E+00        | 1 | 1.534012E+01           |               |
|         | Y | 1.534012E+01        | YZ | 0.000000E+00        | 2 | 9.500318E+00           |               |
|         | Z | 1.677693E-02        | ZX | -7.227601E-02       | 3 | 1.622610E-02           |               |
|         |   | 1.339539E+01        |    | 6.314648E+00        |   | 7.661947E+00           | -8.285555E+00 |
| 404     | X | 9.242598E+00        | XY | 0.000000E+00        | 1 | 1.416581E+01           |               |
|         | Y | 1.416581E+01        | YZ | 0.000000E+00        | 2 | 9.242731E+00           |               |
|         | Z | -1.070969E-02       | ZX | -3.510880E-02       | 3 | -1.084290E-02          |               |
|         |   | 1.246681E+01        |    | 5.876911E+00        |   | 7.088325E+00           | -7.799232E+00 |
| 405     | X | 9.414941E+00        | XY | 0.000000E+00        | 1 | 1.307048E+01           |               |
|         | Y | 1.307048E+01        | YZ | 0.000000E+00        | 2 | 9.414944E+00           |               |
|         | Z | 1.229448E-02        | ZX | 4.965946E-03        | 3 | 1.229186E-02           |               |
|         |   | 1.166810E+01        |    | 5.500394E+00        |   | 6.529093E+00           | -7.499238E+00 |
| 406     | X | 8.752293E+00        | XY | 0.000000E+00        | 1 | 1.223195E+01           |               |
|         | Y | 1.223195E+01        | YZ | 0.000000E+00        | 2 | 8.752311E+00           |               |
|         | Z | -5.997827E-03       | ZX | 1.263272E-02        | 3 | -6.016048E-03          |               |
|         |   | 1.092209E+01        |    | 5.148722E+00        |   | 6.118983E+00           | -6.992749E+00 |
| 407     | X | 8.133419E+00        | XY | 0.000000E+00        | 1 | 1.143114E+01           |               |
|         | Y | 1.143114E+01        | YZ | 0.000000E+00        | 2 | 8.133419E+00           |               |
|         | Z | 1.812691E-03        | ZX | -9.990156E-04       | 3 | 1.812569E-03           |               |
|         |   | 1.018890E+01        |    | 4.803094E+00        |   | 5.714663E+00           | -6.522123E+00 |
| 408     | X | 7.095377E+00        | XY | 0.000000E+00        | 1 | 1.061397E+01           |               |
|         | Y | 1.061397E+01        | YZ | 0.000000E+00        | 2 | 7.095388E+00           |               |
|         | Z | -3.608767E-03       | ZX | -8.580428E-03       | 3 | -3.619138E-03          |               |
|         |   | 9.367750E+00        |    | 4.416000E+00        |   | 5.308794E+00           | -5.901913E+00 |
| 409     | X | 6.056095E+00        | XY | 0.000000E+00        | 1 | 9.804091E+00           |               |
|         | Y | 9.804091E+00        | YZ | 0.000000E+00        | 2 | 6.056095E+00           |               |
|         | Z | 6.990796E-03        | ZX | -5.749489E-04       | 3 | 6.990742E-03           |               |
|         |   | 8.562193E+00        |    | 4.036257E+00        |   | 4.898550E+00           | -5.289059E+00 |
| 410     | X | 4.703051E+00        | XY | 0.000000E+00        | 1 | 8.903431E+00           |               |
|         | Y | 8.903431E+00        | YZ | 0.000000E+00        | 2 | 4.703064E+00           |               |
|         | Z | -2.739606E-03       | ZX | 7.673606E-03        | 3 | -2.752119E-03          |               |
|         |   | 7.717120E+00        |    | 3.637885E+00        |   | 4.453091E+00           | -4.534581E+00 |
| 411     | X | 3.336760E+00        | XY | 0.000000E+00        | 1 | 7.998492E+00           |               |
|         | Y | 7.998492E+00        | YZ | 0.000000E+00        | 2 | 3.336760E+00           |               |
|         | Z | 1.606330E-03        | ZX | -3.215829E-05       | 3 | 1.606329E-03           |               |
|         |   | 6.957197E+00        |    | 3.279654E+00        |   | 3.998443E+00           | -3.778953E+00 |

## ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING

LOADCASE 30

## G R I D S T R E S S E S

| GRID ID | NORMAL<br>VON MISES |               | SHEAR<br>OCTAHEDRAL |               | PRINCIPAL<br>MAX SHEAR |               | PRESSURE      |
|---------|---------------------|---------------|---------------------|---------------|------------------------|---------------|---------------|
| 412     | X                   | 1.701471E+00  | XY                  | 0.000000E+00  | 1                      | 6.984394E+00  |               |
|         | Y                   | 6.984394E+00  | YZ                  | 0.000000E+00  | 2                      | 1.701477E+00  |               |
|         | Z                   | -5.061432E-04 | ZX                  | -3.215185E-03 | 3                      | -5.122170E-04 |               |
|         |                     | 6.308521E+00  |                     | 2.973865E+00  |                        | 3.492453E+00  | -2.895120E+00 |
| 413     | X                   | 3.656624E-02  | XY                  | 0.000000E+00  | 1                      | 5.952966E+00  |               |
|         | Y                   | 5.952966E+00  | YZ                  | 0.000000E+00  | 2                      | 3.798870E-02  |               |
|         | Z                   | 9.925027E-04  | ZX                  | 7.254353E-03  | 3                      | -4.299578E-04 |               |
|         |                     | 5.934280E+00  |                     | 2.797446E+00  |                        | 2.976698E+00  | -1.996842E+00 |
| 501     | X                   | 2.326530E+00  | XY                  | 0.000000E+00  | 1                      | 2.322940E+01  |               |
|         | Y                   | 2.322940E+01  | YZ                  | 0.000000E+00  | 2                      | 2.326639E+00  |               |
|         | Z                   | -3.283341E-02 | ZX                  | 1.609089E-02  | 3                      | -3.294314E-02 |               |
|         |                     | 2.217690E+01  |                     | 1.045429E+01  |                        | 1.163117E+01  | -8.507698E+00 |
| 502     | X                   | 5.985299E+00  | XY                  | 0.000000E+00  | 1                      | 1.864372E+01  |               |
|         | Y                   | 1.864372E+01  | YZ                  | 0.000000E+00  | 2                      | 5.988120E+00  |               |
|         | Z                   | -4.803320E-02 | ZX                  | 1.304885E-01  | 3                      | -5.085408E-02 |               |
|         |                     | 1.652454E+01  |                     | 7.789742E+00  |                        | 9.347288E+00  | -8.193663E+00 |
| 503     | X                   | 9.446508E+00  | XY                  | 0.000000E+00  | 1                      | 1.531771E+01  |               |
|         | Y                   | 1.531771E+01  | YZ                  | 0.000000E+00  | 2                      | 9.446676E+00  |               |
|         | Z                   | 2.159359E-02  | ZX                  | -3.986093E-02 | 3                      | 2.142501E-02  |               |
|         |                     | 1.336564E+01  |                     | 6.300622E+00  |                        | 7.648141E+00  | -8.261936E+00 |
| 504     | X                   | 9.259262E+00  | XY                  | 0.000000E+00  | 1                      | 1.417051E+01  |               |
|         | Y                   | 1.417051E+01  | YZ                  | 0.000000E+00  | 2                      | 9.259264E+00  |               |
|         | Z                   | 1.053968E-02  | ZX                  | 4.406535E-03  | 3                      | 1.053758E-02  |               |
|         |                     | 1.245319E+01  |                     | 5.870491E+00  |                        | 7.079984E+00  | -7.813436E+00 |
| 505     | X                   | 9.403076E+00  | XY                  | 0.000000E+00  | 1                      | 1.300226E+01  |               |
|         | Y                   | 1.300226E+01  | YZ                  | 0.000000E+00  | 2                      | 9.403079E+00  |               |
|         | Z                   | 1.383007E-02  | ZX                  | 5.309717E-03  | 3                      | 1.382707E-02  |               |
|         |                     | 1.161490E+01  |                     | 5.475315E+00  |                        | 6.494218E+00  | -7.473056E+00 |
| 506     | X                   | 8.739018E+00  | XY                  | 0.000000E+00  | 1                      | 1.222436E+01  |               |
|         | Y                   | 1.222436E+01  | YZ                  | 0.000000E+00  | 2                      | 8.739022E+00  |               |
|         | Z                   | -2.164244E-03 | ZX                  | 5.940809E-03  | 3                      | -2.168282E-03 |               |
|         |                     | 1.090972E+01  |                     | 5.142892E+00  |                        | 6.113265E+00  | -6.987072E+00 |
| 507     | X                   | 8.120595E+00  | XY                  | 0.000000E+00  | 1                      | 1.142315E+01  |               |
|         | Y                   | 1.142315E+01  | YZ                  | 0.000000E+00  | 2                      | 8.120595E+00  |               |
|         | Z                   | 2.884593E-03  | ZX                  | -1.479288E-03 | 3                      | 2.884323E-03  |               |
|         |                     | 1.017906E+01  |                     | 4.798455E+00  |                        | 5.710133E+00  | -6.515543E+00 |

|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 508 | X | 7.084049E+00  | XY | 0.000000E+00  | 1 | 1.060667E+01  |               |
|     | Y | 1.060667E+01  | YZ | 0.000000E+00  | 2 | 7.084050E+00  |               |
|     | Z | -8.322505E-04 | ZX | -3.026620E-03 | 3 | -8.335435E-04 |               |
|     |   | 9.357443E+00  |    | 4.411141E+00  |   | 5.303750E+00  | -5.896628E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

# G R I D S T R E S S E S

| GRID ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE      |
|---------|---|---------------------|----|---------------------|---|------------------------|---------------|
| 509     | X | 6.055790E+00        | XY | 0.000000E+00        | 1 | 9.796524E+00           |               |
|         | Y | 9.796524E+00        | YZ | 0.000000E+00        | 2 | 6.055790E+00           |               |
|         | Z | -4.682539E-04       | ZX | -1.069687E-03       | 3 | -4.684428E-04          |               |
|         |   | 8.563072E+00        |    | 4.036671E+00        |   | 4.898496E+00           | -5.283948E+00 |
| 510     | X | 4.691642E+00        | XY | 0.000000E+00        | 1 | 8.896163E+00           |               |
|         | Y | 8.896163E+00        | YZ | 0.000000E+00        | 2 | 4.691643E+00           |               |
|         | Z | -1.462503E-03       | ZX | 2.351016E-03        | 3 | -1.463681E-03          |               |
|         |   | 7.709442E+00        |    | 3.634266E+00        |   | 4.448813E+00           | -4.528781E+00 |
| 511     | X | 3.325677E+00        | XY | 0.000000E+00        | 1 | 7.990892E+00           |               |
|         | Y | 7.990892E+00        | YZ | 0.000000E+00        | 2 | 3.325677E+00           |               |
|         | Z | 2.276608E-03        | ZX | -6.874443E-04       | 3 | 2.276466E-03           |               |
|         |   | 6.950799E+00        |    | 3.276638E+00        |   | 3.994308E+00           | -3.772949E+00 |
| 512     | X | 1.692751E+00        | XY | 0.000000E+00        | 1 | 6.976991E+00           |               |
|         | Y | 6.976991E+00        | YZ | 0.000000E+00        | 2 | 1.692752E+00           |               |
|         | Z | -6.168913E-04       | ZX | -1.220377E-03       | 3 | -6.177708E-04          |               |
|         |   | 6.303876E+00        |    | 2.971676E+00        |   | 3.488804E+00           | -2.889708E+00 |
| 513     | X | 5.095573E-02        | XY | 0.000000E+00        | 1 | 5.956761E+00           |               |
|         | Y | 5.956761E+00        | YZ | 0.000000E+00        | 2 | 5.099847E-02           |               |
|         | Z | -6.827165E-04       | ZX | 1.486146E-03        | 3 | -7.254521E-04          |               |
|         |   | 5.931794E+00        |    | 2.796274E+00        |   | 2.978743E+00           | -2.002345E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

# G R I D S T R E S S E S

| GRID ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE     |
|---------|---|---------------------|----|---------------------|---|------------------------|--------------|
| 101     | X | -6.891967E-01       | XY | 0.000000E+00        | 1 | 4.570140E-03           |              |
|         | Y | -3.353569E+00       | YZ | 0.000000E+00        | 2 | -6.892069E-01          |              |
|         | Z | 4.559885E-03        | ZX | 2.667332E-03        | 3 | -3.353569E+00          |              |
|         |   | 3.070607E+00        |    | 1.447498E+00        |   | 1.679070E+00           | 1.346069E+00 |

|     |   |               |    |               |   |               |              |
|-----|---|---------------|----|---------------|---|---------------|--------------|
| 102 | X | -1.209616E+00 | XY | 0.000000E+00  | 1 | 6.380430E-03  |              |
|     | Y | -2.796682E+00 | YZ | 0.000000E+00  | 2 | -1.209846E+00 |              |
|     | Z | 6.150145E-03  | ZX | 1.673395E-02  | 3 | -2.796682E+00 |              |
|     |   | 2.434585E+00  |    | 1.147675E+00  |   | 1.401531E+00  | 1.333383E+00 |
| 103 | X | -1.705104E+00 | XY | 0.000000E+00  | 1 | -2.530962E-03 |              |
|     | Y | -2.401491E+00 | YZ | 0.000000E+00  | 2 | -1.705119E+00 |              |
|     | Z | -2.546064E-03 | ZX | -5.070633E-03 | 3 | -2.401491E+00 |              |
|     |   | 2.137610E+00  |    | 1.007679E+00  |   | 1.199480E+00  | 1.369714E+00 |
| 104 | X | -1.763368E+00 | XY | 0.000000E+00  | 1 | -1.520722E-03 |              |
|     | Y | -2.301806E+00 | YZ | 0.000000E+00  | 2 | -1.763369E+00 |              |
|     | Z | -1.521230E-03 | ZX | 9.458069E-04  | 3 | -2.301806E+00 |              |
|     |   | 2.083907E+00  |    | 9.823631E-01  |   | 1.150143E+00  | 1.355565E+00 |
| 105 | X | -1.864342E+00 | XY | 0.000000E+00  | 1 | -1.909215E-03 |              |
|     | Y | -2.199988E+00 | YZ | 0.000000E+00  | 2 | -1.864343E+00 |              |
|     | Z | -1.909730E-03 | ZX | 9.793845E-04  | 3 | -2.199988E+00 |              |
|     |   | 2.050959E+00  |    | 9.668315E-01  |   | 1.099040E+00  | 1.355413E+00 |
| 106 | X | -1.896427E+00 | XY | 0.000000E+00  | 1 | 1.783106E-04  |              |
|     | Y | -2.167703E+00 | YZ | 0.000000E+00  | 2 | -1.896427E+00 |              |
|     | Z | 1.781312E-04  | ZX | 5.832468E-04  | 3 | -2.167703E+00 |              |
|     |   | 2.045778E+00  |    | 9.643888E-01  |   | 1.083941E+00  | 1.354651E+00 |
| 107 | X | -1.935147E+00 | XY | 0.000000E+00  | 1 | -9.804263E-05 |              |
|     | Y | -2.133453E+00 | YZ | 0.000000E+00  | 2 | -1.935147E+00 |              |
|     | Z | -9.805661E-05 | ZX | -1.644725E-04 | 3 | -2.133453E+00 |              |
|     |   | 2.041438E+00  |    | 9.623433E-01  |   | 1.066677E+00  | 1.356233E+00 |
| 108 | X | -1.951673E+00 | XY | 0.000000E+00  | 1 | -2.215872E-05 |              |
|     | Y | -2.114491E+00 | YZ | 0.000000E+00  | 2 | -1.951673E+00 |              |
|     | Z | -2.216173E-05 | ZX | -7.668911E-05 | 3 | -2.114491E+00 |              |
|     |   | 2.037944E+00  |    | 9.606960E-01  |   | 1.057235E+00  | 1.355396E+00 |
| 109 | X | -1.969398E+00 | XY | 0.000000E+00  | 1 | -6.238679E-05 |              |
|     | Y | -2.096883E+00 | YZ | 0.000000E+00  | 2 | -1.969398E+00 |              |
|     | Z | -6.238707E-05 | ZX | 2.331591E-05  | 3 | -2.096883E+00 |              |
|     |   | 2.036073E+00  |    | 9.598142E-01  |   | 1.048410E+00  | 1.355448E+00 |
| 110 | X | -1.978794E+00 | XY | 0.000000E+00  | 1 | 2.595957E-05  |              |
|     | Y | -2.087462E+00 | YZ | 0.000000E+00  | 2 | -1.978794E+00 |              |
|     | Z | 2.595596E-05  | ZX | 8.444464E-05  | 3 | -2.087462E+00 |              |
|     |   | 2.035331E+00  |    | 9.594642E-01  |   | 1.043744E+00  | 1.355410E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

#### GRID STRESSES

| GRID ID | NORMAL<br>VON MISES | SHEAR<br>OCTAHEDRAL | PRINCIPAL<br>MAX SHEAR | PRESSURE |
|---------|---------------------|---------------------|------------------------|----------|
|---------|---------------------|---------------------|------------------------|----------|

|     |   |               |    |               |   |               |              |
|-----|---|---------------|----|---------------|---|---------------|--------------|
| 111 | X | -1.988663E+00 | XY | 0.000000E+00  | 1 | -1.498235E-05 |              |
|     | Y | -2.078283E+00 | YZ | 0.000000E+00  | 2 | -1.988663E+00 |              |
|     | Z | -1.498255E-05 | ZX | -1.978378E-05 | 3 | -2.078283E+00 |              |
|     |   | 2.034939E+00  |    | 9.592795E-01  |   | 1.039134E+00  | 1.355654E+00 |
| 112 | X | -1.994459E+00 | XY | 0.000000E+00  | 1 | -2.424706E-06 |              |
|     | Y | -2.072073E+00 | YZ | 0.000000E+00  | 2 | -1.994459E+00 |              |
|     | Z | -2.424894E-06 | ZX | -1.937189E-05 | 3 | -2.072073E+00 |              |
|     |   | 2.034374E+00  |    | 9.590131E-01  |   | 1.036035E+00  | 1.355511E+00 |
| 113 | X | -2.000465E+00 | XY | 0.000000E+00  | 1 | -1.917861E-05 |              |
|     | Y | -2.066106E+00 | YZ | 0.000000E+00  | 2 | -2.000465E+00 |              |
|     | Z | -1.917864E-05 | ZX | 7.340983E-06  | 3 | -2.066106E+00 |              |
|     |   | 2.034060E+00  |    | 9.588653E-01  |   | 1.033043E+00  | 1.355530E+00 |
| 201 | X | -6.881260E-01 | XY | 0.000000E+00  | 1 | -7.018215E-03 |              |
|     | Y | -3.489449E+00 | YZ | 0.000000E+00  | 2 | -6.891340E-01 |              |
|     | Z | -8.026219E-03 | ZX | -2.620228E-02 | 3 | -3.489449E+00 |              |
|     |   | 3.196434E+00  |    | 1.506813E+00  |   | 1.741216E+00  | 1.395201E+00 |
| 202 | X | -1.223527E+00 | XY | 0.000000E+00  | 1 | 1.136728E-03  |              |
|     | Y | -2.810060E+00 | YZ | 0.000000E+00  | 2 | -1.223591E+00 |              |
|     | Z | 1.072674E-03  | ZX | 8.856937E-03  | 3 | -2.810060E+00 |              |
|     |   | 2.441277E+00  |    | 1.150829E+00  |   | 1.405598E+00  | 1.344172E+00 |
| 203 | X | -1.710587E+00 | XY | 0.000000E+00  | 1 | -1.958549E-03 |              |
|     | Y | -2.403453E+00 | YZ | 0.000000E+00  | 2 | -1.710640E+00 |              |
|     | Z | -2.011994E-03 | ZX | -9.556022E-03 | 3 | -2.403453E+00 |              |
|     |   | 2.140883E+00  |    | 1.009222E+00  |   | 1.200747E+00  | 1.372017E+00 |
| 204 | X | -1.759586E+00 | XY | 0.000000E+00  | 1 | 9.760173E-04  |              |
|     | Y | -2.300142E+00 | YZ | 0.000000E+00  | 2 | -1.759594E+00 |              |
|     | Z | 9.683828E-04  | ZX | -3.666208E-03 | 3 | -2.300142E+00 |              |
|     |   | 2.084100E+00  |    | 9.824539E-01  |   | 1.150559E+00  | 1.352920E+00 |
| 205 | X | -1.866004E+00 | XY | 0.000000E+00  | 1 | -8.127722E-04 |              |
|     | Y | -2.208740E+00 | YZ | 0.000000E+00  | 2 | -1.866004E+00 |              |
|     | Z | -8.131639E-04 | ZX | 8.547957E-04  | 3 | -2.208740E+00 |              |
|     |   | 2.058076E+00  |    | 9.701862E-01  |   | 1.103964E+00  | 1.358519E+00 |
| 206 | X | -1.896868E+00 | XY | 0.000000E+00  | 1 | 4.852362E-04  |              |
|     | Y | -2.167855E+00 | YZ | 0.000000E+00  | 2 | -1.896868E+00 |              |
|     | Z | 4.849566E-04  | ZX | 7.282606E-04  | 3 | -2.167855E+00 |              |
|     |   | 2.046348E+00  |    | 9.646578E-01  |   | 1.084170E+00  | 1.354746E+00 |
| 207 | X | -1.935352E+00 | XY | 0.000000E+00  | 1 | -6.591495E-05 |              |
|     | Y | -2.133511E+00 | YZ | 0.000000E+00  | 2 | -1.935352E+00 |              |
|     | Z | -6.595096E-05 | ZX | -2.640109E-04 | 3 | -2.133511E+00 |              |
|     |   | 2.041591E+00  |    | 9.624152E-01  |   | 1.066722E+00  | 1.356310E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING

LOADCASE

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# G R I D   S T R E S S E S

| GRID ID | NORMAL<br>VON MISES |               |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR |  | PRESSURE     |
|---------|---------------------|---------------|----|---------------------|---|------------------------|--|--------------|
| 208     | X                   | -1.951638E+00 | XY | 0.000000E+00        | 1 | 6.241094E-05           |  |              |
|         | Y                   | -2.114439E+00 | YZ | 0.000000E+00        | 2 | -1.951638E+00          |  |              |
|         | Z                   | 6.238658E-05  | ZX | -2.180473E-04       | 3 | -2.114439E+00          |  |              |
|         |                     | 2.037984E+00  |    | 9.607148E-01        |   | 1.057251E+00           |  | 1.355338E+00 |
| 209     | X                   | -1.969580E+00 | XY | 0.000000E+00        | 1 | -1.155375E-04          |  |              |
|         | Y                   | -2.097890E+00 | YZ | 0.000000E+00        | 2 | -1.969580E+00          |  |              |
|         | Z                   | -1.155394E-04 | ZX | 6.024116E-05        | 3 | -2.097890E+00          |  |              |
|         |                     | 2.036653E+00  |    | 9.600875E-01        |   | 1.048887E+00           |  | 1.355862E+00 |
| 210     | X                   | -1.978866E+00 | XY | 0.000000E+00        | 1 | 4.659051E-05           |  |              |
|         | Y                   | -2.087488E+00 | YZ | 0.000000E+00        | 2 | -1.978866E+00          |  |              |
|         | Z                   | 4.658591E-05  | ZX | 9.544505E-05        | 3 | -2.087488E+00          |  |              |
|         |                     | 2.035398E+00  |    | 9.594960E-01        |   | 1.043767E+00           |  | 1.355436E+00 |
| 211     | X                   | -1.988699E+00 | XY | 0.000000E+00        | 1 | -9.860101E-06          |  |              |
|         | Y                   | -2.078293E+00 | YZ | 0.000000E+00        | 2 | -1.988699E+00          |  |              |
|         | Z                   | -9.860671E-06 | ZX | -3.365589E-05       | 3 | -2.078293E+00          |  |              |
|         |                     | 2.034966E+00  |    | 9.592921E-01        |   | 1.039142E+00           |  | 1.355667E+00 |
| 212     | X                   | -1.994453E+00 | XY | 0.000000E+00        | 1 | 4.886525E-06           |  |              |
|         | Y                   | -2.072066E+00 | YZ | 0.000000E+00        | 2 | -1.994453E+00          |  |              |
|         | Z                   | 4.885945E-06  | ZX | -3.400737E-05       | 3 | -2.072066E+00          |  |              |
|         |                     | 2.034375E+00  |    | 9.590135E-01        |   | 1.036035E+00           |  | 1.355505E+00 |
| 213     | X                   | -2.000448E+00 | XY | 0.000000E+00        | 1 | -8.438761E-06          |  |              |
|         | Y                   | -2.066390E+00 | YZ | 0.000000E+00        | 2 | -2.000448E+00          |  |              |
|         | Z                   | -8.440500E-06 | ZX | 5.897989E-05        | 3 | -2.066390E+00          |  |              |
|         |                     | 2.034212E+00  |    | 9.589368E-01        |   | 1.033191E+00           |  | 1.355615E+00 |
| 301     | X                   | -6.661161E-01 | XY | 0.000000E+00        | 1 | -3.282193E-03          |  |              |
|         | Y                   | -3.486024E+00 | YZ | 0.000000E+00        | 2 | -6.661161E-01          |  |              |
|         | Z                   | -3.282193E-03 | ZX | 3.440564E-14        | 3 | -3.486024E+00          |  |              |
|         |                     | 3.203180E+00  |    | 1.509993E+00        |   | 1.741371E+00           |  | 1.385141E+00 |
| 302     | X                   | -1.201378E+00 | XY | 0.000000E+00        | 1 | 3.751421E-03           |  |              |
|         | Y                   | -2.814799E+00 | YZ | 0.000000E+00        | 2 | -1.201378E+00          |  |              |
|         | Z                   | 3.751421E-03  | ZX | -2.427919E-14       | 3 | -2.814799E+00          |  |              |
|         |                     | 2.449458E+00  |    | 1.154686E+00        |   | 1.409275E+00           |  | 1.337475E+00 |
| 303     | X                   | -1.702979E+00 | XY | 0.000000E+00        | 1 | 2.223460E-03           |  |              |
|         | Y                   | -2.336648E+00 | YZ | 0.000000E+00        | 2 | -1.702979E+00          |  |              |
|         | Z                   | 2.223460E-03  | ZX | -2.428569E-14       | 3 | -2.336648E+00          |  |              |
|         |                     | 2.095182E+00  |    | 9.876781E-01        |   | 1.169436E+00           |  | 1.345801E+00 |
| 304     | X                   | -1.756845E+00 | XY | 0.000000E+00        | 1 | -4.909839E-04          |  |              |
|         | Y                   | -2.299116E+00 | YZ | 0.000000E+00        | 2 | -1.756845E+00          |  |              |
|         | Z                   | -4.909839E-04 | ZX | -1.494659E-14       | 3 | -2.299116E+00          |  |              |
|         |                     | 2.081167E+00  |    | 9.810717E-01        |   | 1.149313E+00           |  | 1.352151E+00 |

## ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING

LOADCASE

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## G R I D S T R E S S E S

| GRID ID | NORMAL<br>VON MISES |               | SHEAR<br>OCTAHEDRAL |               | PRINCIPAL<br>MAX SHEAR |               | PRESSURE     |
|---------|---------------------|---------------|---------------------|---------------|------------------------|---------------|--------------|
| 305     | X                   | -1.865262E+00 | XY                  | 0.000000E+00  | 1                      | -1.225990E-03 |              |
|         | Y                   | -2.208599E+00 | YZ                  | 0.000000E+00  | 2                      | -1.865262E+00 |              |
|         | Z                   | -1.225990E-03 | ZX                  | -1.104446E-14 | 3                      | -2.208599E+00 |              |
|         |                     | 2.057305E+00  |                     | 9.698227E-01  |                        | 1.103686E+00  | 1.358362E+00 |
| 306     | X                   | -1.897452E+00 | XY                  | 0.000000E+00  | 1                      | -1.087048E-04 |              |
|         | Y                   | -2.168235E+00 | YZ                  | 0.000000E+00  | 2                      | -1.897452E+00 |              |
|         | Z                   | -1.087048E-04 | ZX                  | -1.452020E-14 | 3                      | -2.168235E+00 |              |
|         |                     | 2.046217E+00  |                     | 9.645958E-01  |                        | 1.084063E+00  | 1.355265E+00 |
| 307     | X                   | -1.934774E+00 | XY                  | 0.000000E+00  | 1                      | 9.245095E-05  |              |
|         | Y                   | -2.130874E+00 | YZ                  | 0.000000E+00  | 2                      | -1.934774E+00 |              |
|         | Z                   | 9.245095E-05  | ZX                  | -1.921079E-14 | 3                      | -2.130874E+00 |              |
|         |                     | 2.039998E+00  |                     | 9.616642E-01  |                        | 1.065483E+00  | 1.355185E+00 |
| 308     | X                   | -1.951458E+00 | XY                  | 0.000000E+00  | 1                      | 2.141045E-05  |              |
|         | Y                   | -2.114368E+00 | YZ                  | 0.000000E+00  | 2                      | -1.951458E+00 |              |
|         | Z                   | 2.141045E-05  | ZX                  | -2.947732E-14 | 3                      | -2.114368E+00 |              |
|         |                     | 2.037824E+00  |                     | 9.606396E-01  |                        | 1.057195E+00  | 1.355268E+00 |
| 309     | X                   | -1.969510E+00 | XY                  | 0.000000E+00  | 1                      | -1.546200E-04 |              |
|         | Y                   | -2.097879E+00 | YZ                  | 0.000000E+00  | 2                      | -1.969510E+00 |              |
|         | Z                   | -1.546200E-04 | ZX                  | -6.230420E-14 | 3                      | -2.097879E+00 |              |
|         |                     | 2.036576E+00  |                     | 9.600512E-01  |                        | 1.048862E+00  | 1.355848E+00 |
| 310     | X                   | -1.978878E+00 | XY                  | 0.000000E+00  | 1                      | 3.414183E-06  |              |
|         | Y                   | -2.087511E+00 | YZ                  | 0.000000E+00  | 2                      | -1.978878E+00 |              |
|         | Z                   | 3.414183E-06  | ZX                  | -3.685620E-14 | 3                      | -2.087511E+00 |              |
|         |                     | 2.035374E+00  |                     | 9.594844E-01  |                        | 1.043757E+00  | 1.355462E+00 |
| 311     | X                   | -1.988615E+00 | XY                  | 0.000000E+00  | 1                      | 2.572477E-05  |              |
|         | Y                   | -2.077819E+00 | YZ                  | 0.000000E+00  | 2                      | -1.988615E+00 |              |
|         | Z                   | 2.572477E-05  | ZX                  | -3.407596E-14 | 3                      | -2.077819E+00 |              |
|         |                     | 2.034710E+00  |                     | 9.591713E-01  |                        | 1.038922E+00  | 1.355469E+00 |
| 312     | X                   | -1.994384E+00 | XY                  | 0.000000E+00  | 1                      | 2.202793E-06  |              |
|         | Y                   | -2.072040E+00 | YZ                  | 0.000000E+00  | 2                      | -1.994384E+00 |              |
|         | Z                   | 2.202793E-06  | ZX                  | -2.774243E-15 | 3                      | -2.072040E+00 |              |
|         |                     | 2.034326E+00  |                     | 9.589906E-01  |                        | 1.036021E+00  | 1.355474E+00 |
| 313     | X                   | -2.000386E+00 | XY                  | 0.000000E+00  | 1                      | 1.373546E-06  |              |
|         | Y                   | -2.066367E+00 | YZ                  | 0.000000E+00  | 2                      | -2.000386E+00 |              |
|         | Z                   | 1.373546E-06  | ZX                  | 2.739240E-14  | 3                      | -2.066367E+00 |              |
|         |                     | 2.034180E+00  |                     | 9.589218E-01  |                        | 1.033184E+00  | 1.355584E+00 |





|     |   |               |    |               |   |               |              |
|-----|---|---------------|----|---------------|---|---------------|--------------|
| 410 | X | -1.978866E+00 | XY | 0.000000E+00  | 1 | 4.659051E-05  |              |
|     | Y | -2.087488E+00 | YZ | 0.000000E+00  | 2 | -1.978866E+00 |              |
|     | Z | 4.658591E-05  | ZX | -9.544505E-05 | 3 | -2.087488E+00 |              |
|     |   | 2.035398E+00  |    | 9.594960E-01  |   | 1.043767E+00  | 1.355436E+00 |
| 411 | X | -1.988699E+00 | XY | 0.000000E+00  | 1 | -9.860101E-06 |              |
|     | Y | -2.078293E+00 | YZ | 0.000000E+00  | 2 | -1.988699E+00 |              |
|     | Z | -9.860671E-06 | ZX | 3.365589E-05  | 3 | -2.078293E+00 |              |
|     |   | 2.034966E+00  |    | 9.592921E-01  |   | 1.039142E+00  | 1.355667E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

# G R I D S T R E S S E S

| GRID ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE     |
|---------|---|---------------------|----|---------------------|---|------------------------|--------------|
| 412     | X | -1.994453E+00       | XY | 0.000000E+00        | 1 | 4.886525E-06           |              |
|         | Y | -2.072066E+00       | YZ | 0.000000E+00        | 2 | -1.994453E+00          |              |
|         | Z | 4.885945E-06        | ZX | 3.400737E-05        | 3 | -2.072066E+00          |              |
|         |   | 2.034375E+00        |    | 9.590135E-01        |   | 1.036035E+00           | 1.355505E+00 |
| 413     | X | -2.000448E+00       | XY | 0.000000E+00        | 1 | -8.438761E-06          |              |
|         | Y | -2.066390E+00       | YZ | 0.000000E+00        | 2 | -2.000448E+00          |              |
|         | Z | -8.440500E-06       | ZX | -5.897989E-05       | 3 | -2.066390E+00          |              |
|         |   | 2.034212E+00        |    | 9.589368E-01        |   | 1.033191E+00           | 1.355615E+00 |
| 501     | X | -6.891967E-01       | XY | 0.000000E+00        | 1 | 4.570140E-03           |              |
|         | Y | -3.353569E+00       | YZ | 0.000000E+00        | 2 | -6.892069E-01          |              |
|         | Z | 4.559885E-03        | ZX | -2.667332E-03       | 3 | -3.353569E+00          |              |
|         |   | 3.070607E+00        |    | 1.447498E+00        |   | 1.679070E+00           | 1.346069E+00 |
| 502     | X | -1.209616E+00       | XY | 0.000000E+00        | 1 | 6.380430E-03           |              |
|         | Y | -2.796682E+00       | YZ | 0.000000E+00        | 2 | -1.209846E+00          |              |
|         | Z | 6.150145E-03        | ZX | -1.673395E-02       | 3 | -2.796682E+00          |              |
|         |   | 2.434585E+00        |    | 1.147675E+00        |   | 1.401531E+00           | 1.333383E+00 |
| 503     | X | -1.705104E+00       | XY | 0.000000E+00        | 1 | -2.530962E-03          |              |
|         | Y | -2.401491E+00       | YZ | 0.000000E+00        | 2 | -1.705119E+00          |              |
|         | Z | -2.546064E-03       | ZX | 5.070633E-03        | 3 | -2.401491E+00          |              |
|         |   | 2.137610E+00        |    | 1.007679E+00        |   | 1.199480E+00           | 1.369714E+00 |
| 504     | X | -1.763368E+00       | XY | 0.000000E+00        | 1 | -1.520722E-03          |              |
|         | Y | -2.301806E+00       | YZ | 0.000000E+00        | 2 | -1.763369E+00          |              |
|         | Z | -1.521230E-03       | ZX | -9.458069E-04       | 3 | -2.301806E+00          |              |
|         |   | 2.083907E+00        |    | 9.823631E-01        |   | 1.150143E+00           | 1.355565E+00 |
| 505     | X | -1.864342E+00       | XY | 0.000000E+00        | 1 | -1.909215E-03          |              |
|         | Y | -2.199988E+00       | YZ | 0.000000E+00        | 2 | -1.864343E+00          |              |
|         | Z | -1.909730E-03       | ZX | -9.793845E-04       | 3 | -2.199988E+00          |              |
|         |   | 2.050959E+00        |    | 9.668315E-01        |   | 1.099040E+00           | 1.355413E+00 |

|     |   |               |    |               |   |               |              |
|-----|---|---------------|----|---------------|---|---------------|--------------|
| 506 | X | -1.896427E+00 | XY | 0.000000E+00  | 1 | 1.783106E-04  |              |
|     | Y | -2.167703E+00 | YZ | 0.000000E+00  | 2 | -1.896427E+00 |              |
|     | Z | 1.781312E-04  | ZX | -5.832468E-04 | 3 | -2.167703E+00 |              |
|     |   | 2.045778E+00  |    | 9.643888E-01  |   | 1.083941E+00  | 1.354651E+00 |
| 507 | X | -1.935147E+00 | XY | 0.000000E+00  | 1 | -9.804263E-05 |              |
|     | Y | -2.133453E+00 | YZ | 0.000000E+00  | 2 | -1.935147E+00 |              |
|     | Z | -9.805661E-05 | ZX | 1.644725E-04  | 3 | -2.133453E+00 |              |
|     |   | 2.041438E+00  |    | 9.623433E-01  |   | 1.066677E+00  | 1.356233E+00 |
| 508 | X | -1.951673E+00 | XY | 0.000000E+00  | 1 | -2.215872E-05 |              |
|     | Y | -2.114491E+00 | YZ | 0.000000E+00  | 2 | -1.951673E+00 |              |
|     | Z | -2.216173E-05 | ZX | 7.668911E-05  | 3 | -2.114491E+00 |              |
|     |   | 2.037944E+00  |    | 9.606960E-01  |   | 1.057235E+00  | 1.355396E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

#### GRID STRESSES

| GRID ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | PRESSURE     |
|---------|---|---------------------|----|---------------------|---|------------------------|--------------|
| 509     | X | -1.969398E+00       | XY | 0.000000E+00        | 1 | -6.238679E-05          |              |
|         | Y | -2.096883E+00       | YZ | 0.000000E+00        | 2 | -1.969398E+00          |              |
|         | Z | -6.238707E-05       | ZX | -2.331591E-05       | 3 | -2.096883E+00          |              |
|         |   | 2.036073E+00        |    | 9.598142E-01        |   | 1.048410E+00           | 1.355448E+00 |
| 510     | X | -1.978794E+00       | XY | 0.000000E+00        | 1 | 2.595957E-05           |              |
|         | Y | -2.087462E+00       | YZ | 0.000000E+00        | 2 | -1.978794E+00          |              |
|         | Z | 2.595596E-05        | ZX | -8.444464E-05       | 3 | -2.087462E+00          |              |
|         |   | 2.035331E+00        |    | 9.594642E-01        |   | 1.043744E+00           | 1.355410E+00 |
| 511     | X | -1.988663E+00       | XY | 0.000000E+00        | 1 | -1.498235E-05          |              |
|         | Y | -2.078283E+00       | YZ | 0.000000E+00        | 2 | -1.988663E+00          |              |
|         | Z | -1.498255E-05       | ZX | 1.978378E-05        | 3 | -2.078283E+00          |              |
|         |   | 2.034939E+00        |    | 9.592795E-01        |   | 1.039134E+00           | 1.355654E+00 |
| 512     | X | -1.994459E+00       | XY | 0.000000E+00        | 1 | -2.424706E-06          |              |
|         | Y | -2.072073E+00       | YZ | 0.000000E+00        | 2 | -1.994459E+00          |              |
|         | Z | -2.424894E-06       | ZX | 1.937189E-05        | 3 | -2.072073E+00          |              |
|         |   | 2.034374E+00        |    | 9.590131E-01        |   | 1.036035E+00           | 1.355511E+00 |
| 513     | X | -2.000465E+00       | XY | 0.000000E+00        | 1 | -1.917861E-05          |              |
|         | Y | -2.066106E+00       | YZ | 0.000000E+00        | 2 | -2.000465E+00          |              |
|         | Z | -1.917864E-05       | ZX | -7.340983E-06       | 3 | -2.066106E+00          |              |
|         |   | 2.034060E+00        |    | 9.588653E-01        |   | 1.033043E+00           | 1.355530E+00 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

S T R A I N S   I N   T R I A X 6   E L E M E N T S

| TRIAX6 ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | DELTA V / V  |
|-----------|---|---------------------|----|---------------------|---|------------------------|--------------|
| 101       | X | 2.080310E-07        | XY | 0.000000E+00        | 1 | 1.530351E-06           |              |
|           | Y | 1.530351E-06        | YZ | 0.000000E+00        | 2 | 2.080623E-07           |              |
|           | Z | -7.445396E-07       | ZX | -1.091753E-08       | 3 | -7.445709E-07          |              |
|           |   | 1.319194E-06        |    | 9.328109E-07        |   | 1.137461E-06           | 9.938423E-07 |
| 102       | X | -1.421818E-07       | XY | 0.000000E+00        | 1 | 1.919487E-06           |              |
|           | Y | 1.919487E-06        | YZ | 0.000000E+00        | 2 | -1.421777E-07          |              |
|           | Z | -7.606511E-07       | ZX | 3.181650E-09        | 3 | -7.606551E-07          |              |
|           |   | 1.620434E-06        |    | 1.145820E-06        |   | 1.340071E-06           | 1.016654E-06 |
| 103       | X | -1.421818E-07       | XY | 0.000000E+00        | 1 | 1.919487E-06           |              |
|           | Y | 1.919487E-06        | YZ | 0.000000E+00        | 2 | -1.421777E-07          |              |
|           | Z | -7.606511E-07       | ZX | -3.181650E-09       | 3 | -7.606551E-07          |              |
|           |   | 1.620434E-06        |    | 1.145820E-06        |   | 1.340071E-06           | 1.016654E-06 |
| 104       | X | 2.080310E-07        | XY | 0.000000E+00        | 1 | 1.530351E-06           |              |
|           | Y | 1.530351E-06        | YZ | 0.000000E+00        | 2 | 2.080623E-07           |              |
|           | Z | -7.445396E-07       | ZX | 1.091753E-08        | 3 | -7.445709E-07          |              |
|           |   | 1.319194E-06        |    | 9.328109E-07        |   | 1.137461E-06           | 9.938423E-07 |
| 105       | X | 4.821169E-07        | XY | 0.000000E+00        | 1 | 1.185992E-06           |              |
|           | Y | 1.185992E-06        | YZ | 0.000000E+00        | 2 | 4.821237E-07           |              |
|           | Z | -7.166496E-07       | ZX | 5.682172E-09        | 3 | -7.166564E-07          |              |
|           |   | 1.110813E-06        |    | 7.854635E-07        |   | 9.513243E-07           | 9.514595E-07 |
| 106       | X | 5.167296E-07        | XY | 0.000000E+00        | 1 | 1.099768E-06           |              |
|           | Y | 1.099768E-06        | YZ | 0.000000E+00        | 2 | 5.167329E-07           |              |
|           | Z | -6.922613E-07       | ZX | 3.996037E-09        | 3 | -6.922646E-07          |              |
|           |   | 1.055460E-06        |    | 7.463231E-07        |   | 8.960161E-07           | 9.242359E-07 |
| 107       | X | 5.167296E-07        | XY | 0.000000E+00        | 1 | 1.099768E-06           |              |
|           | Y | 1.099768E-06        | YZ | 0.000000E+00        | 2 | 5.167329E-07           |              |
|           | Z | -6.922613E-07       | ZX | -3.996037E-09       | 3 | -6.922646E-07          |              |
|           |   | 1.055460E-06        |    | 7.463231E-07        |   | 8.960161E-07           | 9.242359E-07 |
| 108       | X | 4.821169E-07        | XY | 0.000000E+00        | 1 | 1.185992E-06           |              |
|           | Y | 1.185992E-06        | YZ | 0.000000E+00        | 2 | 4.821237E-07           |              |
|           | Z | -7.166496E-07       | ZX | -5.682172E-09       | 3 | -7.166564E-07          |              |
|           |   | 1.110813E-06        |    | 7.854635E-07        |   | 9.513243E-07           | 9.514595E-07 |
| 201       | X | 4.956424E-07        | XY | 0.000000E+00        | 1 | 9.397454E-07           |              |
|           | Y | 9.397454E-07        | YZ | 0.000000E+00        | 2 | 4.956426E-07           |              |
|           | Z | -6.152409E-07       | ZX | -9.865055E-10       | 3 | -6.152411E-07          |              |
|           |   | 9.248751E-07        |    | 6.539855E-07        |   | 7.774933E-07           | 8.201470E-07 |
| 202       | X | 5.203234E-07        | XY | 0.000000E+00        | 1 | 9.832752E-07           |              |
|           | Y | 9.832752E-07        | YZ | 0.000000E+00        | 2 | 5.203248E-07           |              |
|           | Z | -6.444943E-07       | ZX | -2.574362E-09       | 3 | -6.444957E-07          |              |
|           |   | 9.684774E-07        |    | 6.848169E-07        |   | 8.138855E-07           | 8.591042E-07 |

CENTRIFUGAL LOADING LOADCASE 30

| S T R A I N S I N T R I A X 6 E L E M E N T S |   |                     |    |                     |   |                        |  |              |  |
|---|---|---------------------|----|---------------------|---|------------------------|--|--------------|--|
| TRIAX6 ID                                     |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR |  | DELTA V / V  |  |
| 203   | X | 5.203234E-07        | XY | 0.000000E+00        | 1 | 9.832752E-07           |  |              |  |
|   | Y | 9.832752E-07        | YZ | 0.000000E+00        | 2 | 5.203248E-07           |  |              |  |
|   | Z | -6.444943E-07       | ZX | 2.574362E-09        | 3 | -6.444957E-07          |  |              |  |
|   |   | 9.684774E-07        |    | 6.848169E-07        |   | 8.138855E-07           |  | 8.591042E-07 |  |
| 204   | X | 4.956424E-07        | XY | 0.000000E+00        | 1 | 9.397454E-07           |  |              |  |
|   | Y | 9.397454E-07        | YZ | 0.000000E+00        | 2 | 4.956426E-07           |  |              |  |
|   | Z | -6.152409E-07       | ZX | 9.865055E-10        | 3 | -6.152411E-07          |  |              |  |
|   |   | 9.248751E-07        |    | 6.539855E-07        |   | 7.774933E-07           |  | 8.201470E-07 |  |
| 205   | X | 4.169536E-07        | XY | 0.000000E+00        | 1 | 8.655904E-07           |  |              |  |
|   | Y | 8.655904E-07        | YZ | 0.000000E+00        | 2 | 4.169537E-07           |  |              |  |
|   | Z | -5.496988E-07       | ZX | 5.712949E-10        | 3 | -5.496989E-07          |  |              |  |
|   |   | 8.351628E-07        |    | 5.905492E-07        |   | 7.076446E-07           |  | 7.328452E-07 |  |
| 206   | X | 3.645651E-07        | XY | 0.000000E+00        | 1 | 8.319350E-07           |  |              |  |
|   | Y | 8.319350E-07        | YZ | 0.000000E+00        | 2 | 3.645657E-07           |  |              |  |
|   | Z | -5.128039E-07       | ZX | 1.507844E-09        | 3 | -5.128045E-07          |  |              |  |
|   |   | 7.883227E-07        |    | 5.574283E-07        |   | 6.723698E-07           |  | 6.836962E-07 |  |
| 207   | X | 3.645651E-07        | XY | 0.000000E+00        | 1 | 8.319350E-07           |  |              |  |
|   | Y | 8.319350E-07        | YZ | 0.000000E+00        | 2 | 3.645657E-07           |  |              |  |
|   | Z | -5.128039E-07       | ZX | -1.507844E-09       | 3 | -5.128045E-07          |  |              |  |
|   |   | 7.883227E-07        |    | 5.574283E-07        |   | 6.723698E-07           |  | 6.836962E-07 |  |
| 208   | X | 4.169536E-07        | XY | 0.000000E+00        | 1 | 8.655904E-07           |  |              |  |
|   | Y | 8.655904E-07        | YZ | 0.000000E+00        | 2 | 4.169537E-07           |  |              |  |
|   | Z | -5.496988E-07       | ZX | -5.712949E-10       | 3 | -5.496989E-07          |  |              |  |
|   |   | 8.351628E-07        |    | 5.905492E-07        |   | 7.076446E-07           |  | 7.328452E-07 |  |
| 301   | X | 1.664873E-07        | XY | 0.000000E+00        | 1 | 7.325936E-07           |  |              |  |
|   | Y | 7.325936E-07        | YZ | 0.000000E+00        | 2 | 1.664875E-07           |  |              |  |
|   | Z | -3.853617E-07       | ZX | -5.488939E-10       | 3 | -3.853619E-07          |  |              |  |
|   |   | 6.454694E-07        |    | 4.564158E-07        |   | 5.589777E-07           |  | 5.137191E-07 |  |
| 302   | X | 2.391507E-07        | XY | 0.000000E+00        | 1 | 7.659369E-07           |  |              |  |
|   | Y | 7.659369E-07        | YZ | 0.000000E+00        | 2 | 2.391513E-07           |  |              |  |
|   | Z | -4.306589E-07       | ZX | -1.270284E-09       | 3 | -4.306595E-07          |  |              |  |
|   |   | 6.924983E-07        |    | 4.896703E-07        |   | 5.982982E-07           |  | 5.744287E-07 |  |
| 303   | X | 2.391507E-07        | XY | 0.000000E+00        | 1 | 7.659369E-07           |  |              |  |
|   | Y | 7.659369E-07        | YZ | 0.000000E+00        | 2 | 2.391513E-07           |  |              |  |
|   | Z | -4.306589E-07       | ZX | 1.270284E-09        | 3 | -4.306595E-07          |  |              |  |
|   |   | 6.924983E-07        |    | 4.896703E-07        |   | 5.982982E-07           |  | 5.744287E-07 |  |

|     |   |               |    |              |   |               |              |
|-----|---|---------------|----|--------------|---|---------------|--------------|
| 304 | X | 1.664873E-07  | XY | 0.000000E+00 | 1 | 7.325936E-07  |              |
|     | Y | 7.325936E-07  | YZ | 0.000000E+00 | 2 | 1.664875E-07  |              |
|     | Z | -3.853617E-07 | ZX | 5.488939E-10 | 3 | -3.853619E-07 |              |
|     |   | 6.454694E-07  |    | 4.564158E-07 |   | 5.589777E-07  | 5.137191E-07 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

CENTRIFUGAL LOADING LOADCASE 30

S T R A I N S I N T R I A X 6 E L E M E N T S

| TRIAX6 ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | DELTA V / V  |
|-----------|---|---------------------|----|---------------------|---|------------------------|--------------|
| 305       | X | 4.766362E-09        | XY | 0.000000E+00        | 1 | 6.647343E-07           |              |
|           | Y | 6.647343E-07        | YZ | 0.000000E+00        | 2 | 4.766414E-09           |              |
|           | Z | -2.869441E-07       | ZX | 2.449078E-10        | 3 | -2.869442E-07          |              |
|           |   | 5.629968E-07        |    | 3.980989E-07        |   | 4.758392E-07           | 3.825565E-07 |
| 306       | X | -8.485340E-08       | XY | 0.000000E+00        | 1 | 6.297709E-07           |              |
|           | Y | 6.297709E-07        | YZ | 0.000000E+00        | 2 | -8.485339E-08          |              |
|           | Z | -2.335186E-07       | ZX | 8.139913E-11        | 3 | -2.335186E-07          |              |
|           |   | 5.329286E-07        |    | 3.768374E-07        |   | 4.316448E-07           | 3.113990E-07 |
| 307       | X | -8.485340E-08       | XY | 0.000000E+00        | 1 | 6.297709E-07           |              |
|           | Y | 6.297709E-07        | YZ | 0.000000E+00        | 2 | -8.485339E-08          |              |
|           | Z | -2.335186E-07       | ZX | -8.139913E-11       | 3 | -2.335186E-07          |              |
|           |   | 5.329286E-07        |    | 3.768374E-07        |   | 4.316448E-07           | 3.113990E-07 |
| 308       | X | 4.766362E-09        | XY | 0.000000E+00        | 1 | 6.647343E-07           |              |
|           | Y | 6.647343E-07        | YZ | 0.000000E+00        | 2 | 4.766414E-09           |              |
|           | Z | -2.869441E-07       | ZX | -2.449078E-10       | 3 | -2.869442E-07          |              |
|           |   | 5.629968E-07        |    | 3.980989E-07        |   | 4.758392E-07           | 3.825565E-07 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

S T R A I N S I N T R I A X 6 E L E M E N T S

| TRIAX6 ID |   | NORMAL<br>VON MISES |    | SHEAR<br>OCTAHEDRAL |   | PRINCIPAL<br>MAX SHEAR | DELTA V / V   |
|-----------|---|---------------------|----|---------------------|---|------------------------|---------------|
| 101       | X | -6.008153E-08       | XY | 0.000000E+00        | 1 | 1.217040E-07           |               |
|           | Y | -2.240802E-07       | YZ | 0.000000E+00        | 2 | -6.008399E-08          |               |
|           | Z | 1.217015E-07        | ZX | 1.336035E-09        | 3 | -2.240802E-07          |               |
|           |   | 1.997267E-07        |    | 1.412281E-07        |   | 1.728921E-07           | -1.624602E-07 |
| 102       | X | -1.248568E-08       | XY | 0.000000E+00        | 1 | 1.221783E-07           |               |
|           | Y | -2.729008E-07       | YZ | 0.000000E+00        | 2 | -1.248594E-08          |               |
|           | Z | 1.221781E-07        | ZX | -3.691448E-10       | 3 | -2.729008E-07          |               |
|           |   | 2.319185E-07        |    | 1.639912E-07        |   | 1.975396E-07           | -1.632084E-07 |

|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 103 | X | -1.248568E-08 | XY | 0.000000E+00  | 1 | 1.221783E-07  |               |
|     | Y | -2.729008E-07 | YZ | 0.000000E+00  | 2 | -1.248594E-08 |               |
|     | Z | 1.221781E-07  | ZX | 3.691448E-10  | 3 | -2.729008E-07 |               |
|     |   | 2.319185E-07  |    | 1.639912E-07  |   | 1.975396E-07  | -1.632084E-07 |
| 104 | X | -6.008153E-08 | XY | 0.000000E+00  | 1 | 1.217040E-07  |               |
|     | Y | -2.240802E-07 | YZ | 0.000000E+00  | 2 | -6.008399E-08 |               |
|     | Z | 1.217015E-07  | ZX | -1.336035E-09 | 3 | -2.240802E-07 |               |
|     |   | 1.997267E-07  |    | 1.412281E-07  |   | 1.728921E-07  | -1.624602E-07 |
| 105 | X | -1.025070E-07 | XY | 0.000000E+00  | 1 | 1.223682E-07  |               |
|     | Y | -1.824619E-07 | YZ | 0.000000E+00  | 2 | -1.025076E-07 |               |
|     | Z | 1.223676E-07  | ZX | -7.053884E-10 | 3 | -1.824619E-07 |               |
|     |   | 1.825031E-07  |    | 1.290492E-07  |   | 1.524150E-07  | -1.626013E-07 |
| 106 | X | -1.115224E-07 | XY | 0.000000E+00  | 1 | 1.218295E-07  |               |
|     | Y | -1.729364E-07 | YZ | 0.000000E+00  | 2 | -1.115225E-07 |               |
|     | Z | 1.218294E-07  | ZX | -3.419918E-10 | 3 | -1.729364E-07 |               |
|     |   | 1.795747E-07  |    | 1.269785E-07  |   | 1.473830E-07  | -1.626294E-07 |
| 107 | X | -1.115224E-07 | XY | 0.000000E+00  | 1 | 1.218295E-07  |               |
|     | Y | -1.729364E-07 | YZ | 0.000000E+00  | 2 | -1.115225E-07 |               |
|     | Z | 1.218294E-07  | ZX | 3.419918E-10  | 3 | -1.729364E-07 |               |
|     |   | 1.795747E-07  |    | 1.269785E-07  |   | 1.473830E-07  | -1.626294E-07 |
| 108 | X | -1.025070E-07 | XY | 0.000000E+00  | 1 | 1.223682E-07  |               |
|     | Y | -1.824619E-07 | YZ | 0.000000E+00  | 2 | -1.025076E-07 |               |
|     | Z | 1.223676E-07  | ZX | 7.053884E-10  | 3 | -1.824619E-07 |               |
|     |   | 1.825031E-07  |    | 1.290492E-07  |   | 1.524150E-07  | -1.626013E-07 |
| 201 | X | -1.263183E-07 | XY | 0.000000E+00  | 1 | 1.219846E-07  |               |
|     | Y | -1.583184E-07 | YZ | 0.000000E+00  | 2 | -1.263183E-07 |               |
|     | Z | 1.219846E-07  | ZX | 5.430080E-11  | 3 | -1.583184E-07 |               |
|     |   | 1.771679E-07  |    | 1.252766E-07  |   | 1.401515E-07  | -1.626521E-07 |
| 202 | X | -1.230024E-07 | XY | 0.000000E+00  | 1 | 1.220495E-07  |               |
|     | Y | -1.617049E-07 | YZ | 0.000000E+00  | 2 | -1.230025E-07 |               |
|     | Z | 1.220494E-07  | ZX | 1.964363E-10  | 3 | -1.617049E-07 |               |
|     |   | 1.776794E-07  |    | 1.256383E-07  |   | 1.418772E-07  | -1.626580E-07 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

# S T R A I N S I N T R I A X 6 E L E M E N T S

| TRIAX6 ID | NORMAL<br>VON MISES | SHEAR<br>OCTAHEDRAL | PRINCIPAL<br>MAX SHEAR | DELTA V / V   |
|-----------|---------------------|---------------------|------------------------|---------------|
| 203       | X -1.230024E-07     | XY 0.000000E+00     | 1 1.220495E-07         |               |
|           | Y -1.617049E-07     | YZ 0.000000E+00     | 2 -1.230025E-07        |               |
|           | Z 1.220494E-07      | ZX -1.964363E-10    | 3 -1.617049E-07        |               |
|           | 1.776794E-07        | 1.256383E-07        | 1.418772E-07           | -1.626580E-07 |

|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 204 | X | -1.263183E-07 | XY | 0.000000E+00  | 1 | 1.219846E-07  |               |
|     | Y | -1.583184E-07 | YZ | 0.000000E+00  | 2 | -1.263183E-07 |               |
|     | Z | 1.219846E-07  | ZX | -5.430080E-11 | 3 | -1.583184E-07 |               |
|     |   | 1.771679E-07  |    | 1.252766E-07  |   | 1.401515E-07  | -1.626521E-07 |
| 205 | X | -1.309461E-07 | XY | 0.000000E+00  | 1 | 1.220119E-07  |               |
|     | Y | -1.537285E-07 | YZ | 0.000000E+00  | 2 | -1.309461E-07 |               |
|     | Z | 1.220119E-07  | ZX | -2.951270E-11 | 3 | -1.537285E-07 |               |
|     |   | 1.767230E-07  |    | 1.249620E-07  |   | 1.378702E-07  | -1.626626E-07 |
| 206 | X | -1.325031E-07 | XY | 0.000000E+00  | 1 | 1.219909E-07  |               |
|     | Y | -1.521514E-07 | YZ | 0.000000E+00  | 2 | -1.325031E-07 |               |
|     | Z | 1.219909E-07  | ZX | -2.805264E-11 | 3 | -1.521514E-07 |               |
|     |   | 1.765769E-07  |    | 1.248587E-07  |   | 1.370711E-07  | -1.626635E-07 |
| 207 | X | -1.325031E-07 | XY | 0.000000E+00  | 1 | 1.219909E-07  |               |
|     | Y | -1.521514E-07 | YZ | 0.000000E+00  | 2 | -1.325031E-07 |               |
|     | Z | 1.219909E-07  | ZX | 2.805264E-11  | 3 | -1.521514E-07 |               |
|     |   | 1.765769E-07  |    | 1.248587E-07  |   | 1.370711E-07  | -1.626635E-07 |
| 208 | X | -1.309461E-07 | XY | 0.000000E+00  | 1 | 1.220119E-07  |               |
|     | Y | -1.537285E-07 | YZ | 0.000000E+00  | 2 | -1.309461E-07 |               |
|     | Z | 1.220119E-07  | ZX | 2.951270E-11  | 3 | -1.537285E-07 |               |
|     |   | 1.767230E-07  |    | 1.249620E-07  |   | 1.378702E-07  | -1.626626E-07 |
| 301 | X | -1.356866E-07 | XY | 0.000000E+00  | 1 | 1.219971E-07  |               |
|     | Y | -1.489740E-07 | YZ | 0.000000E+00  | 2 | -1.356866E-07 |               |
|     | Z | 1.219971E-07  | ZX | 8.009358E-12  | 3 | -1.489740E-07 |               |
|     |   | 1.763851E-07  |    | 1.247231E-07  |   | 1.354855E-07  | -1.626635E-07 |
| 302 | X | -1.348328E-07 | XY | 0.000000E+00  | 1 | 1.220010E-07  |               |
|     | Y | -1.498332E-07 | YZ | 0.000000E+00  | 2 | -1.348328E-07 |               |
|     | Z | 1.220010E-07  | ZX | 2.119890E-11  | 3 | -1.498332E-07 |               |
|     |   | 1.764354E-07  |    | 1.247586E-07  |   | 1.359171E-07  | -1.626650E-07 |
| 303 | X | -1.348328E-07 | XY | 0.000000E+00  | 1 | 1.220010E-07  |               |
|     | Y | -1.498332E-07 | YZ | 0.000000E+00  | 2 | -1.348328E-07 |               |
|     | Z | 1.220010E-07  | ZX | -2.119890E-11 | 3 | -1.498332E-07 |               |
|     |   | 1.764354E-07  |    | 1.247586E-07  |   | 1.359171E-07  | -1.626650E-07 |
| 304 | X | -1.356866E-07 | XY | 0.000000E+00  | 1 | 1.219971E-07  |               |
|     | Y | -1.489740E-07 | YZ | 0.000000E+00  | 2 | -1.356866E-07 |               |
|     | Z | 1.219971E-07  | ZX | -8.009358E-12 | 3 | -1.489740E-07 |               |
|     |   | 1.763851E-07  |    | 1.247231E-07  |   | 1.354855E-07  | -1.626635E-07 |

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ANALYSIS OF A THIN AXISYMMETRIC ANNULUS

INNER AND OUTER PRESSURE LOADING LOADCASE 60

# S T R A I N S I N T R I A X 6 E L E M E N T S

| TRIAX6 ID | NORMAL<br>VON MISES | SHEAR<br>OCTAHEDRAL | PRINCIPAL<br>MAX SHEAR | DELTA V / V |
|-----------|---------------------|---------------------|------------------------|-------------|
|-----------|---------------------|---------------------|------------------------|-------------|



|     |   |               |    |               |   |               |               |
|-----|---|---------------|----|---------------|---|---------------|---------------|
| 305 | X | -1.370228E-07 | XY | 0.000000E+00  | 1 | 1.220007E-07  |               |
|     | Y | -1.476425E-07 | YZ | 0.000000E+00  | 2 | -1.370228E-07 |               |
|     | Z | 1.220007E-07  | ZX | -4.968268E-12 | 3 | -1.476425E-07 |               |
|     |   | 1.763289E-07  |    | 1.246833E-07  |   | 1.348216E-07  | -1.626647E-07 |
| 306 | X | -1.375375E-07 | XY | 0.000000E+00  | 1 | 1.219973E-07  |               |
|     | Y | -1.471235E-07 | YZ | 0.000000E+00  | 2 | -1.375375E-07 |               |
|     | Z | 1.219973E-07  | ZX | -1.228005E-12 | 3 | -1.471235E-07 |               |
|     |   | 1.763054E-07  |    | 1.246668E-07  |   | 1.345604E-07  | -1.626638E-07 |
| 307 | X | -1.375375E-07 | XY | 0.000000E+00  | 1 | 1.219973E-07  |               |
|     | Y | -1.471235E-07 | YZ | 0.000000E+00  | 2 | -1.375375E-07 |               |
|     | Z | 1.219973E-07  | ZX | 1.228005E-12  | 3 | -1.471235E-07 |               |
|     |   | 1.763054E-07  |    | 1.246668E-07  |   | 1.345604E-07  | -1.626638E-07 |
| 308 | X | -1.370228E-07 | XY | 0.000000E+00  | 1 | 1.220007E-07  |               |
|     | Y | -1.476425E-07 | YZ | 0.000000E+00  | 2 | -1.370228E-07 |               |
|     | Z | 1.220007E-07  | ZX | 4.968268E-12  | 3 | -1.476425E-07 |               |
|     |   | 1.763289E-07  |    | 1.246833E-07  |   | 1.348216E-07  | -1.626647E-07 |

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.31 Three Independent Beams

### Example ID:

A031

### Analysis Data:

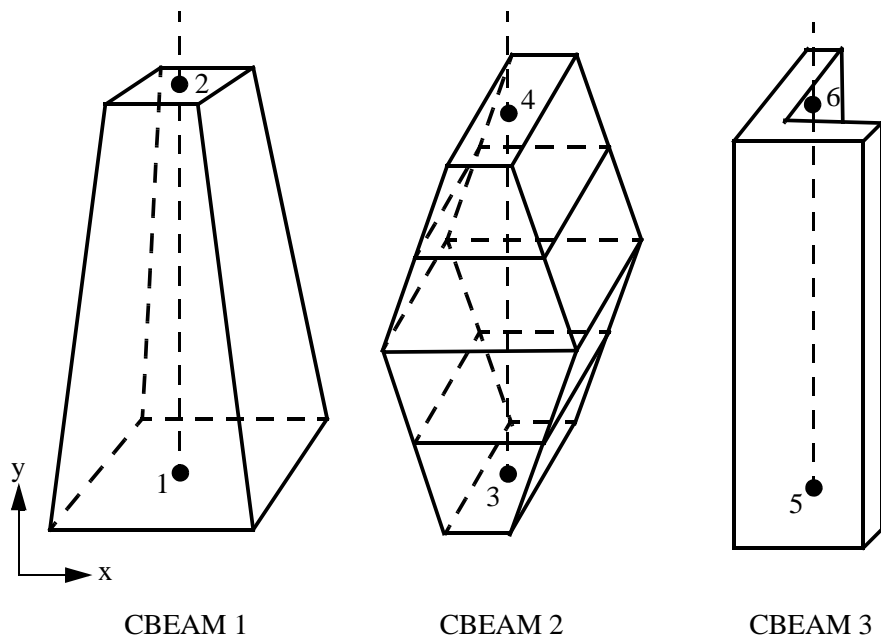
GRID, CBEAM, PBEAM, MAT1, FORCE, MOMENT, SPC1

### Special Features Used:

PBEAM statement.

### Problem Statement:

Find the nodal displacements of the beams shown in the figure below.



### Structural Analysis Model:

1. Three CBEAM elements in the x-y plane.
2. The relevant sections properties are:

Beam 1 (tapered):

$$I_1 = I_2 = \frac{A^4}{12}$$

$$J = 0.1406A^2$$

then:

| X/X <sub>B</sub> | Area (mm <sup>2</sup> ) | I <sub>1</sub> = I <sub>2</sub> (mm <sup>4</sup> ) | J (mm <sup>4</sup> )  |
|------------------|-------------------------|--|-----------------------|
| 0.0 (Grid 1)     | 10,000                  | 8.333X10 <sup>6</sup>                              | 1.406X10 <sup>7</sup> |
| 0.2              | 8,100                   | 5.467X10 <sup>6</sup>                              | 9.225X10 <sup>6</sup> |
| 0.4              | 6,400                   | 3.413X10 <sup>6</sup>                              | 5.759X10 <sup>6</sup> |
| 0.6              | 4,900                   | 2.000X10 <sup>6</sup>                              | 3.375X10 <sup>6</sup> |
| 0.8              | 3,600                   | 1.080X10 <sup>6</sup>                              | 1.822X10 <sup>6</sup> |
| 1.0 (Grid 2)     | 2,500                   | 5.208X10 <sup>5</sup>                              | 8.787X10 <sup>5</sup> |

Beam 2:

$$I_1 = I_{zz} = \frac{BH^3}{12} \quad I_2 = I_{yy} = \frac{HB^3}{12}$$

$$J = BH^3 \left[ 0.1406 + \frac{0.1148462}{\tan(H/B - 1)} \right] \quad B \geq H$$

$$J = HB^3 \left[ 0.1406 + \frac{0.1148462}{\tan(H/B - 1)} \right] \quad B < H$$

then:

| X/X <sub>B</sub> | Area (mm <sup>2</sup> ) | I <sub>1</sub> (mm <sup>4</sup> ) | I <sub>2</sub> (mm <sup>4</sup> ) | J (mm <sup>4</sup> ) |
|------------------|-------------------------|-----------------------------------|-----------------------------------|----------------------|
| 0.0 (Grid 3)     | 600                     | 45,000                            | 20,000                            | 84,200               |
| 0.25             | 800                     | 2.667X10 <sup>4</sup>             | 1.066X10 <sup>5</sup>             | 68,590               |
| 0.5              | 500                     | 4,166.67                          | 1.041X10 <sup>5</sup>             | 11,9906              |
| 0.75             | 800                     | 2.667X10 <sup>4</sup>             | 1.066X10 <sup>5</sup>             | 68,590               |
| 1.0 (Grid 4)     | 600                     | 45,000                            | 20,000                            | 84,200               |

### Beam 3 (L beam):

Area=275 mm<sup>2</sup>,  $I_1 = I_2 = 22,200 \text{ mm}^4$ ,  $I_{12} = -12,784 \text{ mm}^4$ ,  $J = 2,291 \text{ mm}^4$

Material:  $E=7.06\text{E}4 \text{ N/mm}^2$ , Poisson's ratio=0.33, Mass density=0.1

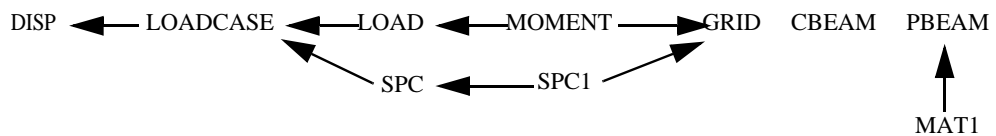
Two load cases:

Load case 1: force  $F=5,000 \text{ N}$

Load case 2: moment  $M=20\text{E}6 \text{ N.mm}$

Degrees of freedom are constrained with SPC1 data entries.

### **Analysis Data Relationships:**



### **Special Modeling Techniques:**

None.

### **Reference Solution:**

None.

---

## 2.31.1 Input Data

```
ID A031
SOL COMPAT1
CEND
TITLE = BEAM WITH AND WITHOUT WARPING
ECHO=NONE
LINE=64,80
DISP = ALL
FORCE = ALL
SPC = 100
SUBCASE 1
  LABEL = FORCE
  LOAD = 110
SUBCASE 2
  LABEL = MOMENT
  LOAD = 120
$
$
BEGIN BULK
$
$ BOUNDARY CONDITIONS
$
SPC1          100 123456 1          3          5
$
$ GRID POINTS
$
GRID   1          0.0      0.0      0.0
GRID   2          0.0     1000.      0.0
GRID   3         1000.      0.0      0.0
GRID   4         1000.     1000.0      0.0
GRID   5         2000.      0.0      0.0
GRID   6         2000.     1000.0      0.0
$
$ ELEMENT DEFINITION
$
CBEAM   1          10      1          2          0.0      0.0      -1.0
CBEAM   2          20      3          4          0.0      0.0      -1.0
CBEAM   3          30      5          6          0.0      0.0      -1.0
$
$ ELEMENTS PROPERTIES
$
PBEAM   10          51     10000.0  8.333E+68.333E+6      0.01.406E+7
+       NO          0.2     8100.0  5.467E+65.467E+6      0.09.225E+6
+       NO          0.4     6400.0  3.413E+63.413E+6      0.05.759E+6
+       NO          0.6     4900.0  2.0E+6  2.0E+6      0.03.375E+6
+       NO          0.8     3600.0  1.08E+6  1.08E+6      0.01.822E+6
+       NO          1.0     2500.0  5.208E+55.208E+5      0.08.787E+5
+       0.833      0.833          0.0          0.0      0.0
$
$
PBEAM   20          51      600.0  20000.0  45000.0      0.084200.0
+       NO          0.25     800.0  1.066E+526666.66      0.068590.0
+       NO          0.50     500.0  1.041E+54166.666      0.011990.0
```

```

+      NO      0.75    800.0    1.066E+526666.66    0.068590.0
+      NO      1.0     600.0    20000.0 45000.0    0.084200.0
+      0.833    0.833                0.0
$
$
PBEAM   30      51      275.    22200.0 22200.0 -12784.02291.0
+      NO      1.0     275.    22200.0 22200.0 -12784.02291.0
+      0.4641   0.4641                0.0
+      0.0      0.0     0.0     0.0     6.8181  6.8181  6.8181  6.8181
$
$
$
$ MATERIAL, LOADS, BC  DEFINITIONS
$
MAT1    51      7.06E4          0.33    0.1
FORCE   110     2          5000.0 0.0     1.0     0.0
FORCE   110     4          5000.0 0.0     1.0     0.0
FORCE   110     6          5000.0 0.0     1.0     0.0
MOMENT  120     2          20.E6  0.0     1.0     0.0
MOMENT  120     4          20.E6  0.0     1.0     0.0
MOMENT  120     6          20.E6  0.0     1.0     0.0
ENDDATA

```

---

## 2.31.2 Output

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A031  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
BEAM WITH AND WITHOUT WARPING

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 6
NUMBER OF CBEAM ELEMENTS: 3
TOTAL NUMBER OF NON RIGID ELEMENTS: 3
```

NUMBER OF ELEMENT PROPERTIES: 3  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 18

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 2  
 TOTAL NUMBER OF LOAD CASES: 2

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 BEAM WITH AND WITHOUT WARPING

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 BEAM WITH AND WITHOUT WARPING

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 6.800000E+05  
 SYSTEM VOLUME 6.800000E+06  
 SYSTEM MASS/VOLUME 1.000000E-01

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 BEAM WITH AND WITHOUT WARPING

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -1.510450E-15 ; STRAIN ENERGY : 2.693356E+03  
 LOADCASE 2 ; RESIDUAL : -2.626878E-19 ; STRAIN ENERGY : 3.456426E+09  
 1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 BEAM WITH AND WITHOUT WARPING

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 0.0000E+00 | 1.5000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 1.5000E+07 |
| 2        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 6.0000E+07 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 6  
 BEAM WITH AND WITHOUT WARPING

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1         | T2          | T3         | R1          | R2          | R3          |
|----------|------------|-------------|------------|-------------|-------------|-------------|
| 1        | 0.0000E+00 | -1.5000E+04 | 0.0000E+00 | -8.7311E-11 | 0.0000E+00  | -1.5000E+07 |
| 2        | 0.0000E+00 | 0.0000E+00  | 0.0000E+00 | 0.0000E+00  | -6.0000E+07 | 0.0000E+00  |



M A X I M U M A P P L I E D F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        | 2       | 5.0000E+03 | 0.0000E+00 | 5.0000E+03 | 0.0000E+00 |
| 2        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 8  
 BEAM WITH AND WITHOUT WARPING

M A X I M U M S P C F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3         |
|----------|---------|------------|------------|-------------|------------|
| 1        | 1       | 5.0000E+03 | 0.0000E+00 | -5.0000E+03 | 0.0000E+00 |
| 2        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00  | 0.0000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 9  
 BEAM WITH AND WITHOUT WARPING

M A X I M U M D I S P L A C E M E N T

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2         | T3          |
|----------|---------|------------|-------------|------------|-------------|
| 1        | 6       | 3.6274E+01 | -2.5641E+01 | 9.5682E-01 | -2.5641E+01 |
| 2        |         | 0.0000E+00 | 0.0000E+00  | 0.0000E+00 | 0.0000E+00  |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 10  
 BEAM WITH AND WITHOUT WARPING

FORCE LOADCASE 1

G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 0.000000E+00<br>0.000000E+00 | 1.410998E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 4       | 0.000000E+00<br>0.000000E+00 | 1.064125E-01<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 5       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

|   |               |              |               |
|---|---------------|--------------|---------------|
| 6 | -2.564084E+01 | 9.568200E-01 | -2.564084E+01 |
|   | -5.128168E-02 | 0.000000E+00 | 5.128168E-02  |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 11  
BEAM WITH AND WITHOUT WARPING

MOMENT LOADCASE 2

# G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.437289E-01 | 0.000000E+00<br>0.000000E+00 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 4       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>1.648525E+01 | 0.000000E+00<br>0.000000E+00 |
| 5       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 6       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>3.289136E+02 | 0.000000E+00<br>0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 12  
BEAM WITH AND WITHOUT WARPING

FORCE LOADCASE 1

# F O R C E S I N B E A M E L E M E N T S

| BEAM ID | X/XB  | MOMENT 1<br>SHEAR FORCE 1    | MOMENT 2<br>SHEAR FORCE 2     | AXIAL FORCE<br>TORQUE        | WARPING      |
|---------|-------|------------------------------|-------------------------------|------------------------------|--------------|
| 1       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 5.000000E+03<br>0.000000E+00 | 0.000000E+00 |
| 2       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 5.000000E+03<br>0.000000E+00 | 0.000000E+00 |
| 3       | 0.000 | 3.409050E+04<br>0.000000E+00 | 3.409050E+04<br>-5.684342E-14 | 5.000000E+03<br>0.000000E+00 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 13  
BEAM WITH AND WITHOUT WARPING

MOMENT LOADCASE 2

# FORCES IN BEAM ELEMENTS

| BEAM ID | X/XB  | MOMENT 1<br>SHEAR FORCE 1    | MOMENT 2<br>SHEAR FORCE 2    | AXIAL FORCE<br>TORQUE        | WARPING      |
|---------|-------|------------------------------|------------------------------|------------------------------|--------------|
| 1       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | 0.000000E+00 |
| 2       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | 0.000000E+00 |
| 3       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | 0.000000E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.32 Two Pairs of Beams with and without Warping Coefficients

### Example ID:

A032

### Analysis Data:

GRID, CBEAM, PBEAM, MAT1, FORCE, MOMENT, SPC1

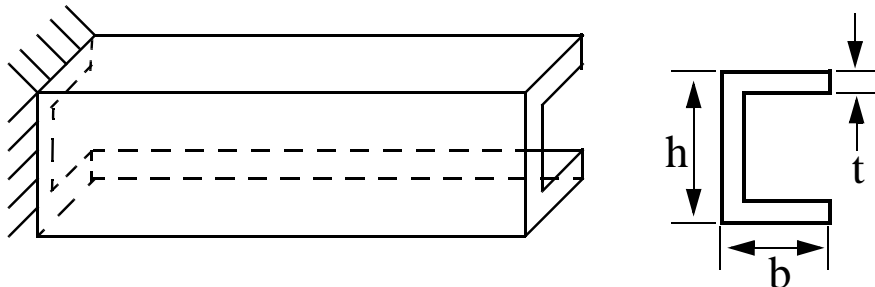
### Special Features Used:

Use of warping coefficients in the PBEAM statement.

### Problem Statement:

Compare the displacements between two pairs of beams. For each pair of beams, one beam is modeled with warping coefficient and the other one is modeled without warping coefficient.

### Structural Analysis Model:



Four CBEAM elements in the x-y plane. For two of the elements,  $t = 1$  mm and for the other two,  $t = 10$  mm. For each pair,  $C_w = 0$  for the first one and  $C_w \neq 0$  for the second one.

The relevant section properties are:

$$J = \frac{t^3}{3}(2b + h)$$

$$C_w = \frac{tb^3h^2}{12} \left( \frac{3b + 2h}{6b + h} \right)$$

For beam 1:  $t = 10 \text{ mm}$ ,  $C_w = 0.0$

For beam 2:  $t = 10 \text{ mm}$ ,  $C_w = 9.114\text{E}7 \text{ mm}^6$

For beam 3:  $t = 1 \text{ mm}$ ,  $C_w = 0.0 \text{ mm}^6$

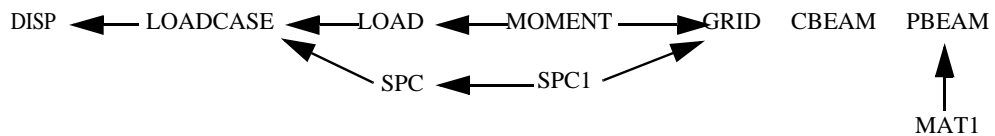
For beam 4:  $t = 1 \text{ mm}$ ,  $C_w = 9.114\text{E}7 \text{ mm}^6$

Material:  $E = 7.06\text{E}4 \text{ N/mm}^2$

One moment load case:  $M = 20\text{E}6 \text{ N.mm}$

Degrees of freedom are constrained with SPC1 data entries.

### Analysis Data Relationships:



### Special Modeling Techniques:

None.

### Reference Solution:

None.

### Calculated Solution:

For the thick-walled sections (beams 1 and 2), the calculated end rotations are 11.3 (without  $C_w$ ) and 9.14 (with  $C_w$ ). Thus the effect of including the warping reduced the rotation by 23 %.

For the thin-walled sections (beams 3 & 4), the calculated end rotations are 11303 (without  $C_w$ ) and 933 (with  $C_w$ ). Thus the effect of including the warping reduced the rotation by 91 %.

Therefore, as expected, including the effects of warping is more important for thin walled sections than for thick walled sections.

---

## 2.32.1 Input Data

```
ID A032
SOL COMPAT1
CEND
TITLE = TAPERED BEAMS
ECHO=NONE
LINE=64,80
DISP = ALL
FORCE = ALL
SPC = 100
SUBCASE 1
  LABEL = MOMENT
  LOAD = 110
$
$
BEGIN BULK
$
$ BOUNDARY CONDITIONS
$
SPC1      100  123456  1      3      5      7
SPC1      100      9      11
$
$ GRID POINTS
$
GRID      1      0.0      0.0      0.0
GRID      2      1000.0  0.0      0.0
GRID      3      0.0      1000.0  0.0
GRID      4      1000.0  1000.0  0.0
GRID      5      0.0      2000.0  0.0
GRID      6      1000.0  2000.0  0.0
GRID      7      0.0      3000.0  0.0
GRID      8      1000.0  3000.0  0.0
$
$ ELEMENT DEFINITION
$
SPOINT    9      10
CBEAM     1      10      1      2      0.0      0.0      -1.0
CBEAM     2      20      3      4      0.0      0.0      -1.0
+
+      9      10
$
SPOINT    11     12
CBEAM     3      30      5      6      0.0      0.0      -1.0
CBEAM     4      40      7      8      0.0      0.0      -1.0
+
+      11     12
$
$ ELEMENTS PROPERTIES
$
PBEAM     10      51      1800.    2460000.392780.    0.066666.
+      NO      1.0      1800.    2460000.392780.    0.066666.
+      0.4727  0.4727      0.0      0.0      0.0      0.0
+      0.0      0.0      0.0      0.0      29.87  0.0      29.87  0.0
```

```

$
$
PBEAM 20 51 1800. 2460000.392780. 0.066666.
+ NO 1.0 1800. 2460000.392780. 0.066666.
+ 0.4727 0.4727 0.0 9.114E8 9.114E8
+ 0.0 0.0 0.0 0.0 29.87 0.0 29.87 0.0
$
$
PBEAM 30 51 198. 323466. 50526. 0.066.666
+ NO 1.0 198. 323466. 50526. 0.066.666
+ 0.4297 0.4297 0.0 0.0 0.0 0.0
+ 0.0 0.0 0.0 0.0 31.12 0.0 31.12 0.0
$
$
PBEAM 40 51 198. 323466. 50526. 0.066.666
+ NO 1.0 198. 323466. 50526. 0.066.666
+ 0.4297 0.4297 0.0 9.114E7 9.114E7
+ 0.0 0.0 0.0 0.0 31.12 0.0 31.12 0.0
$
$ MATERIAL, LOADS, BC DEFINITIONS
$
MAT1 51 7.06E4 0.33 0.1
MOMENT 110 2 20.E6 1.0 0.0 0.0
MOMENT 110 4 20.E6 1.0 0.0 0.0
MOMENT 110 6 20.E6 1.0 0.0 0.0
MOMENT 110 8 20.E6 1.0 0.0 0.0
ENDDATA

```

---

## 2.32.2 Output

1

```
  GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N  E      S      I      S
G  GG  EEEE   N N N  EEEE   SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
```

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A032

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2

DATE 12-13-2002 TIME 12:50 PAGE 1

TAPERED BEAMS

### ANALYSIS PROBLEM SUMMARY

|                           |   |
|---------------------------|---|
| NUMBER OF GRID POINTS:    | 8 |
| NUMBER OF SPOINTS:        | 4 |
| NUMBER OF CBEAM ELEMENTS: | 4 |



TOTAL NUMBER OF NON RIGID ELEMENTS: 4  
 NUMBER OF ELEMENT PROPERTIES: 4  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 26

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 1

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 TAPERED BEAMS

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

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 TAPERED BEAMS

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 3.996000E+05  
 SYSTEM VOLUME 3.996000E+06  
 SYSTEM MASS/VOLUME 1.000000E-01

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 TAPERED BEAMS

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -5.665803E-17 ; STRAIN ENERGY : 1.225721E+11  
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 TAPERED BEAMS

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2         | R3         |
|----------|------------|------------|------------|------------|------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 8.0000E+07 | 0.0000E+00 | 0.0000E+00 |

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 TAPERED BEAMS

#### S P C F O R C E R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1          | R2         | R3         |
|----------|------------|------------|------------|-------------|------------|------------|
| 1        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | -8.0000E+07 | 0.0000E+00 | 0.0000E+00 |

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 TAPERED BEAMS

# M A X I M U M   A P P L I E D   F O R C E

| LOADCASE      | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|---------------|-------------|------------|-----------------|------------|------------|---|
| 1             |             | 0.0000E+00 | 0.0000E+00      | 0.0000E+00 | 0.0000E+00 |   |
| 1GENESIS      | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:50 | PAGE       | 8 |
| TAPERED BEAMS |             |            |                 |            |            |   |

# M A X I M U M   S P C   F O R C E

| LOADCASE      | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |   |
|---------------|-------------|------------|-----------------|------------|------------|---|
| 1             |             | 0.0000E+00 | 0.0000E+00      | 0.0000E+00 | 0.0000E+00 |   |
| 1GENESIS      | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:50 | PAGE       | 9 |
| TAPERED BEAMS |             |            |                 |            |            |   |

# M A X I M U M   D I S P L A C E M E N T

| LOADCASE      | GRID ID     | MAGNITUDE  | T1              | T2         | T3         |    |
|---------------|-------------|------------|-----------------|------------|------------|----|
| 1             |             | 0.0000E+00 | 0.0000E+00      | 0.0000E+00 | 0.0000E+00 |    |
| 1GENESIS      | VERSION 7.2 |            | DATE 12-13-2002 | TIME 12:50 | PAGE       | 10 |
| TAPERED BEAMS |             |            |                 |            |            |    |

|        |  |          |   |
|--------|--|----------|---|
| MOMENT |  | LOADCASE | 1 |
|--------|--|----------|---|

# G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 0.000000E+00<br>1.130323E+01 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 4       | 0.000000E+00<br>9.147861E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 5       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 6       | 0.000000E+00<br>1.130323E+04 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 7       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

|   |              |              |              |
|---|--------------|--------------|--------------|
| 8 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|   | 9.335328E+02 | 0.000000E+00 | 0.000000E+00 |

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TAPERED BEAMS

MOMENT LOADCASE 1

# S P O I N T D I S P L A C E M E N T S

SPOINT ID T

|    |               |
|----|---------------|
| 9  | 0.000000E+00  |
| 10 | -1.118388E-02 |
| 11 | 0.000000E+00  |
| 12 | -1.393973E+00 |

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TAPERED BEAMS

MOMENT LOADCASE 1

# F O R C E S I N B E A M E L E M E N T S

| BEAM ID | X/XB  | MOMENT 1<br>SHEAR FORCE 1    | MOMENT 2<br>SHEAR FORCE 2    | AXIAL FORCE<br>TORQUE        | WARPING       |
|---------|-------|------------------------------|------------------------------|------------------------------|---------------|
| 1       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | 0.000000E+00  |
| 2       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | -3.813721E+09 |
| 3       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | 0.000000E+00  |
| 4       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | -1.834820E+10 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.33 Two Independant Beams, with and without Offset

### Example ID:

A033

### Analysis Data:

GRID, CBEAM, PBEAM, MAT1, FORCE, MOMENT, SPC1

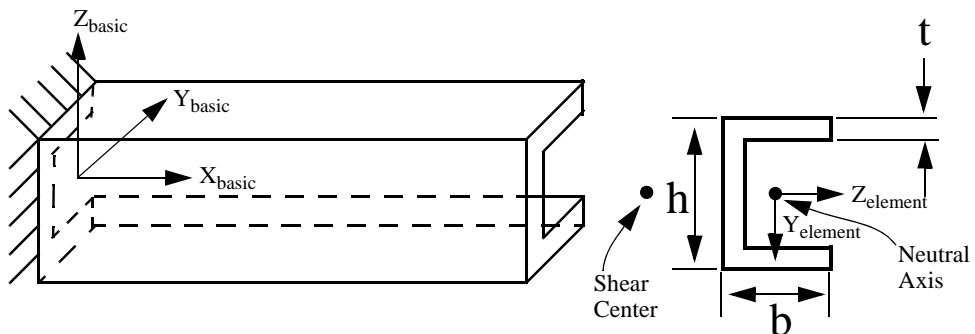
### Special Features Used:

Use of offset in the CBEAM statement.

### Problem Statement:

Compare the displacements between two beams. One beam is modeled without offset (beam 1) and the other one is modeled with offset (beam 2).

### Structural Analysis Model:



### Two CBEAM elements in the x-y plane.

The relevant section properties are:

$b = 50 \text{ mm}$ ,  $h = 100 \text{ mm}$   $t = 10 \text{ mm}$

Beam 1: No offset (grids are on the shear center axis).

Beam 2: Beam is offset such that the grids are on the neutral axis.

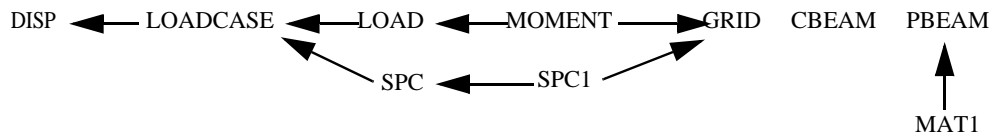
Material:  $E = 7.06E4 \text{ N/mm}^2$

Three force load cases:  $F = 5,000 \text{ N}$

Three moment load cases:  $M = 20E6 \text{ N-mm}$

Degrees of freedom are constrained with SPC1 data entries.

## Analysis Data Relationships:



## Special Modeling Techniques:

None.

## Reference Solution:

None.

## Calculated Solution:

Loadcase 1:

For extensional loading applied at the neutral axis (Beam 2), only extensional displacement is produced. For extensional loading applied at the neutral axis, the results show that, as expected, the beam bends as well as extends.

Loadcase 2:

For shear loading in the y-direction, the beam bends in the x-y plane. Because the beam is symmetric about this plane, and the load is applied in this plane for both beams 1 and 2, the bending displacements are the same for both beams. When the grid is at the shear center (Beam 1), there is extensional displacement caused by the rotation of the end about the neutral axis.

Loadcase 3:

For shear loading in the z-direction, applied at the neutral axis (Beam 2), there is an effective torsion about the shear center and, therefore, a rotation about the x-axis.

Loadcase 4:

For torsional loading, the beam twists about the shear center axis. If the grid is at the neutral axis, then a z-displacement is caused by this rotation.

Loadcase 5:

For moment loading about the y-axis, there are no differences in the displacements measured at the shear center and the neutral axis.

Loadcase 6:

For moment loading about the z-axis, the bending is in the x-y plane about the neutral axis. The rotation of the end causes an extensional displacement at the shear center.

---

## 2.33.1 Input Data

```
ID A033
SOL COMPAT1
CEND
TITLE = BEAMS WITH AND WITHOUT OFFSET
ECHO=NONE
LINE=64,80
DISP = ALL
FORCE = ALL
SPC = 100
SUBCASE 1
  LABEL = FORCE X
  LOAD = 110
SUBCASE 2
  LABEL = FORCE Y
  LOAD = 120
SUBCASE 3
  LABEL = FORCE Z
  LOAD = 130
SUBCASE 4
  LABEL = MOMENT X
  LOAD = 140
SUBCASE 5
  LABEL = MOMENT Y
  LOAD = 150
SUBCASE 6
  LABEL = MOMENT Z
  LOAD = 160
$
$
BEGIN BULK
$
$ BOUNDARY CONDITIONS
$
SPC1          100 123456 1          3
$
$ GRID POINTS
$
GRID    1          0.0      0.0      0.0
GRID    2          1000.0    0.0      0.0
GRID    3          0.0      1000.0    0.0
GRID    4          1000.0    1000.0    0.0
$
$ ELEMENT DEFINITION
$
CBEAM    1          10      1          2          0.0      0.0      -1.0
CBEAM    2          20      3          4          0.0      0.0      -1.0
+
          0.0      -29.87    0.0      0.0      -29.87    0.0
$
$ ELEMENTS PROPERTIES
$
PBEAM    10          51      1800.    2460000.392780.    0.066666.
+
  NO      1.0      1800.    2460000.392780.    0.066666.
```

```

+      0.4727  0.4727      0.0      0.0      0.0
+      0.0      0.0      0.0      0.0      0.0      29.87  0.0      29.87
$
$
PBEAM  20      51      1800.  2460000.392780.  0.066666.
+      NO      1.0      1800.  2460000.392780.  0.066666.
+      0.4727  0.4727      0.0      0.0      0.0
+      0.0      0.0      0.0      0.0      0.0      29.87  0.0      29.87
$
$
$
$ MATERIAL, LOADS, BC  DEFINITIONS
$
MAT1    51      7.06E4      0.33  0.1
FORCE   110      2      5000.0  1.0  0.0  0.0
FORCE   110      4      5000.0  1.0  0.0  0.0
FORCE   120      2      5000.0  0.0  1.0  0.0
FORCE   120      4      5000.0  0.0  1.0  0.0
FORCE   130      2      5000.0  0.0  0.0  1.0
FORCE   130      4      5000.0  0.0  0.0  1.0
MOMENT  140      2      20.E6  1.0  0.0  0.0
MOMENT  140      4      20.E6  1.0  0.0  0.0
MOMENT  150      2      20.E6  0.0  1.0  0.0
MOMENT  150      4      20.E6  0.0  1.0  0.0
MOMENT  160      2      20.E6  0.0  0.0  1.0
MOMENT  160      4      20.E6  0.0  0.0  1.0
ENDDATA

```

---

## 2.33.2 Output

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A033

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

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BEAMS WITH AND WITHOUT OFFSET

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 4
NUMBER OF CBEAM ELEMENTS: 2
TOTAL NUMBER OF NON RIGID ELEMENTS: 2
```



NUMBER OF ELEMENT PROPERTIES: 2  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 12

#### LOAD CASES SUMMARY

NUMBER OF STATIC LOAD CASES: 6  
 TOTAL NUMBER OF LOAD CASES: 6

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 BEAMS WITH AND WITHOUT OFFSET

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

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 BEAMS WITH AND WITHOUT OFFSET

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 3.600000E+05  
 SYSTEM VOLUME 3.600000E+06  
 SYSTEM MASS/VOLUME 1.000000E-01

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 BEAMS WITH AND WITHOUT OFFSET

#### S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : -8.775710E-16 ; STRAIN ENERGY : 5.989120E+02  
 LOADCASE 2 ; RESIDUAL : 7.275958E-16 ; STRAIN ENERGY : 3.016210E+05  
 LOADCASE 3 ; RESIDUAL : -4.012475E-16 ; STRAIN ENERGY : 5.539218E+04  
 LOADCASE 4 ; RESIDUAL : 1.862645E-16 ; STRAIN ENERGY : 2.260646E+08  
 LOADCASE 5 ; RESIDUAL : -3.637979E-16 ; STRAIN ENERGY : 2.303139E+06  
 LOADCASE 6 ; RESIDUAL : 3.725290E-16 ; STRAIN ENERGY : 1.442467E+07  
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 BEAMS WITH AND WITHOUT OFFSET

#### O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3         | R1         | R2          | R3          |
|----------|------------|------------|------------|------------|-------------|-------------|
| 1        | 1.0000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00  | -5.0000E+06 |
| 2        | 0.0000E+00 | 1.0000E+04 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00  | 1.0000E+07  |
| 3        | 0.0000E+00 | 0.0000E+00 | 1.0000E+04 | 5.0000E+06 | -1.0000E+07 | 0.0000E+00  |
| 4        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 4.0000E+07 | 0.0000E+00  | 0.0000E+00  |
| 5        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 4.0000E+07  | 0.0000E+00  |
| 6        | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00  | 4.0000E+07  |

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 BEAMS WITH AND WITHOUT OFFSET

| LOADCASE                      | T1          | T2          | T3              | R1          | R2          | R3          |
|-------------------------------|-------------|-------------|-----------------|-------------|-------------|-------------|
| 1                             | -1.0000E+04 | 0.0000E+00  | 0.0000E+00      | 0.0000E+00  | 0.0000E+00  | 5.0000E+06  |
| 2                             | 0.0000E+00  | -1.0000E+04 | 0.0000E+00      | 0.0000E+00  | 0.0000E+00  | -1.0000E+07 |
| 3                             | 0.0000E+00  | 0.0000E+00  | -1.0000E+04     | -5.0000E+06 | 1.0000E+07  | 0.0000E+00  |
| 4                             | 0.0000E+00  | 0.0000E+00  | 0.0000E+00      | -4.0000E+07 | 0.0000E+00  | 0.0000E+00  |
| 5                             | 0.0000E+00  | 0.0000E+00  | 0.0000E+00      | 0.0000E+00  | -4.0000E+07 | 0.0000E+00  |
| 6                             | 0.0000E+00  | 0.0000E+00  | 0.0000E+00      | 0.0000E+00  | 0.0000E+00  | -4.0000E+07 |
| 1GENESIS VERSION 7.2          |             |             | DATE 12-13-2002 |             | TIME 12:50  | PAGE 7      |
| BEAMS WITH AND WITHOUT OFFSET |             |             |                 |             |             |             |

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2         | T3         |
|----------|---------|------------|------------|------------|------------|
| 1        | 2       | 5.0000E+03 | 5.0000E+03 | 0.0000E+00 | 0.0000E+00 |
| 2        | 2       | 5.0000E+03 | 0.0000E+00 | 5.0000E+03 | 0.0000E+00 |
| 3        | 2       | 5.0000E+03 | 0.0000E+00 | 0.0000E+00 | 5.0000E+03 |
| 4        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 5        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 6        |         | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

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BEAMS WITH AND WITHOUT OFFSET

| LOADCASE | GRID ID | MAGNITUDE  | T1          | T2          | T3          |
|----------|---------|------------|-------------|-------------|-------------|
| 1        | 1       | 5.0000E+03 | -5.0000E+03 | 0.0000E+00  | 0.0000E+00  |
| 2        | 1       | 5.0000E+03 | 0.0000E+00  | -5.0000E+03 | 0.0000E+00  |
| 3        | 3       | 5.0000E+03 | 0.0000E+00  | 0.0000E+00  | -5.0000E+03 |
| 4        |         | 0.0000E+00 | 0.0000E+00  | 0.0000E+00  | 0.0000E+00  |
| 5        | 1       | 1.4552E-11 | 0.0000E+00  | 0.0000E+00  | -1.4552E-11 |
| 6        |         | 0.0000E+00 | 0.0000E+00  | 0.0000E+00  | 0.0000E+00  |

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BEAMS WITH AND WITHOUT OFFSET

| LOADCASE | GRID    | ID         | MAGNITUDE  | T1         | T2          | T3    |         |
|----------|---------|------------|------------|------------|-------------|-------|---------|
| 1        | 2       | 2.7003E+00 | 2.0022E-01 | 2.6929E+00 | 0.0000E+00  |       |         |
| 2        | 2       | 6.0384E+01 | 2.6929E+00 | 6.0324E+01 | 0.0000E+00  |       |         |
| 3        | 4       | 1.2339E+01 | 0.0000E+00 | 0.0000E+00 | 1.2339E+01  |       |         |
| 4        | 4       | 3.3763E+02 | 0.0000E+00 | 0.0000E+00 | 3.3763E+02  |       |         |
| 5        | 4       | 5.7578E+01 | 0.0000E+00 | 0.0000E+00 | -5.7578E+01 |       |         |
| 6        | 2       | 3.6126E+02 | 2.1543E+01 | 3.6062E+02 | 0.0000E+00  |       |         |
| 1GENESIS | VERSION | 7.2        | DATE       | 12-13-2002 | TIME        | 12:50 | PAGE 10 |

BEAMS WITH AND WITHOUT OFFSET

FORCE X LOADCASE 1

G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 2.002195E-01<br>0.000000E+00 | 2.692906E+00<br>0.000000E+00 | 0.000000E+00<br>5.385812E-03 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 4       | 3.934529E-02<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

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 BEAMS WITH AND WITHOUT OFFSET

FORCE Y LOADCASE 2

G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 2.692906E+00<br>0.000000E+00 | 6.032420E+01<br>0.000000E+00 | 0.000000E+00<br>9.015420E-02 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 4       | 0.000000E+00<br>0.000000E+00 | 6.032420E+01<br>0.000000E+00 | 0.000000E+00<br>9.015420E-02 |

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 BEAMS WITH AND WITHOUT OFFSET

FORCE Z LOADCASE 3

G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |

|   |                              |                               |                              |
|---|------------------------------|-------------------------------|------------------------------|
| 2 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>-1.439462E-02 | 9.817819E+00<br>0.000000E+00 |
| 3 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 |
| 4 | 0.000000E+00<br>8.440686E-02 | 0.000000E+00<br>-1.439462E-02 | 1.233905E+01<br>0.000000E+00 |

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BEAMS WITH AND WITHOUT OFFSET

MOMENT X LOADCASE 4

# G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 0.000000E+00<br>1.130323E+01 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 4       | 0.000000E+00<br>1.130323E+01 | 0.000000E+00<br>0.000000E+00 | 3.376275E+02<br>0.000000E+00 |

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BEAMS WITH AND WITHOUT OFFSET

MOMENT Y LOADCASE 5

# G R I D D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                      |
|---------|------------------------------|------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  |
| 2       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>1.151570E-01 | -5.757848E+01<br>0.000000E+00 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00  |
| 4       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>1.151570E-01 | -5.757848E+01<br>0.000000E+00 |

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BEAMS WITH AND WITHOUT OFFSET

MOMENT Z

LOADCASE

6

G R I D   D I S P L A C E M E N T S

| GRID ID | T1<br>R1                     | T2<br>R2                     | T3<br>R3                     |
|---------|------------------------------|------------------------------|------------------------------|
| 1       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 2       | 2.154325E+01<br>0.000000E+00 | 3.606168E+02<br>0.000000E+00 | 0.000000E+00<br>7.212336E-01 |
| 3       | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 |
| 4       | 0.000000E+00<br>0.000000E+00 | 3.606168E+02<br>0.000000E+00 | 0.000000E+00<br>7.212336E-01 |

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BEAMS WITH AND WITHOUT OFFSET

FORCE X

LOADCASE

1

F O R C E S   I N   B E A M   E L E M E N T S

| BEAM ID | X/XB  | MOMENT 1<br>SHEAR FORCE 1    | MOMENT 2<br>SHEAR FORCE 2    | AXIAL FORCE<br>TORQUE        | WARPING      |
|---------|-------|------------------------------|------------------------------|------------------------------|--------------|
| 1       | 0.000 | 0.000000E+00<br>0.000000E+00 | 1.493500E+05<br>0.000000E+00 | 5.000000E+03<br>0.000000E+00 | 0.000000E+00 |
| 2       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 5.000000E+03<br>0.000000E+00 | 0.000000E+00 |

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BEAMS WITH AND WITHOUT OFFSET

FORCE Y

LOADCASE

2

F O R C E S   I N   B E A M   E L E M E N T S

| BEAM ID | X/XB  | MOMENT 1<br>SHEAR FORCE 1    | MOMENT 2<br>SHEAR FORCE 2    | AXIAL FORCE<br>TORQUE        | WARPING      |
|---------|-------|------------------------------|------------------------------|------------------------------|--------------|
| 1       | 0.000 | 0.000000E+00<br>0.000000E+00 | 5.000000E+06<br>5.000000E+03 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00 |
| 2       | 0.000 | 0.000000E+00<br>0.000000E+00 | 5.000000E+06<br>5.000000E+03 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00 |

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BEAMS WITH AND WITHOUT OFFSET

FORCE Z LOADCASE 3

F O R C E S   I N   B E A M   E L E M E N T S

| BEAM ID | X/XB  | MOMENT 1<br>SHEAR FORCE 1      | MOMENT 2<br>SHEAR FORCE 2    | AXIAL FORCE<br>TORQUE        | WARPING      |
|---------|-------|--------------------------------|------------------------------|------------------------------|--------------|
| 1       | 0.000 | -5.000000E+06<br>-5.000000E+03 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00 |
| 2       | 0.000 | -5.000000E+06<br>-5.000000E+03 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>1.493500E+05 | 0.000000E+00 |

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 BEAMS WITH AND WITHOUT OFFSET

MOMENT X LOADCASE 4

F O R C E S   I N   B E A M   E L E M E N T S

| BEAM ID | X/XB  | MOMENT 1<br>SHEAR FORCE 1    | MOMENT 2<br>SHEAR FORCE 2    | AXIAL FORCE<br>TORQUE        | WARPING      |
|---------|-------|------------------------------|------------------------------|------------------------------|--------------|
| 1       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | 0.000000E+00 |
| 2       | 0.000 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>2.000000E+07 | 0.000000E+00 |

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 BEAMS WITH AND WITHOUT OFFSET

MOMENT Y LOADCASE 5

F O R C E S   I N   B E A M   E L E M E N T S

| BEAM ID | X/XB  | MOMENT 1<br>SHEAR FORCE 1     | MOMENT 2<br>SHEAR FORCE 2    | AXIAL FORCE<br>TORQUE        | WARPING      |
|---------|-------|-------------------------------|------------------------------|------------------------------|--------------|
| 1       | 0.000 | 2.000000E+07<br>-1.455192E-11 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00 |
| 2       | 0.000 | 2.000000E+07<br>0.000000E+00  | 0.000000E+00<br>0.000000E+00 | 0.000000E+00<br>0.000000E+00 | 0.000000E+00 |

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 BEAMS WITH AND WITHOUT OFFSET

MOMENT Z LOADCASE 6

F O R C E S   I N   B E A M   E L E M E N T S

| BEAM ID | X/XB | MOMENT 1<br>SHEAR FORCE 1 | MOMENT 2<br>SHEAR FORCE 2 | AXIAL FORCE<br>TORQUE | WARPING |
|---------|------|---------------------------|---------------------------|-----------------------|---------|
|---------|------|---------------------------|---------------------------|-----------------------|---------|

```

1 0.000  0.000000E+00  2.000000E+07  0.000000E+00  0.000000E+00
        0.000000E+00  0.000000E+00  0.000000E+00
2 0.000  0.000000E+00  2.000000E+07  0.000000E+00  0.000000E+00
        0.000000E+00  0.000000E+00  0.000000E+00
1
        ***** END OF OUTPUT *****

```

---

## 2.34 Buckling Analysis of a Tripod

### Example ID:

A034

### Analysis Data:

GRID, CBAR, PBARL, MAT1, SPC, GRAV, CONM2, EIGR, GRAV, CONM2

### Special Features Used:

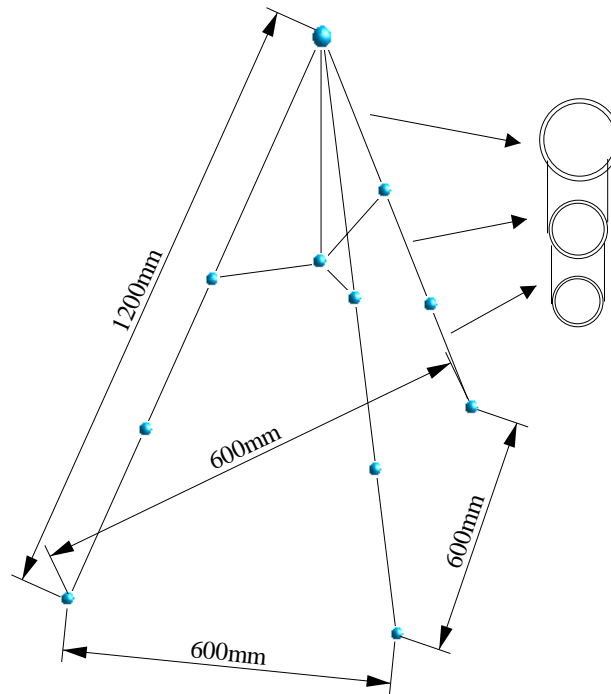
STATSUB in solution control for buckling analysis

### Problem Statement:

A tripod was modeled using CBAR elements together with PBARL. The dimensions of the each members of tripod and bar cross-sectional areas are shown in the picture.



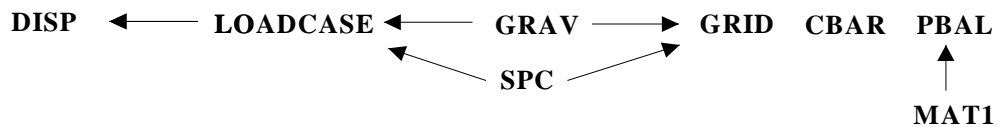
## Structural Analysis Model:



## Analysis Model Description:

1. The unit used here is the N, mm, Mg and material is Aluminum ( $E=110.0E3$  MPa,  $\rho=2.79$  g/cm<sup>3</sup>,  $\nu=0.35$ ).
2. Concentrated mass was used in the connections in each bars.
3. As a load, 1000g of concentrated mass was applied at the tripod top.

## Analysis Data Relationships:



## Special Modeling Techniques:

None.

## Reference Solution:

None.

## Calculated Solution:

| MODE | BUCKLING LOAD FACTOR |
|------|----------------------|
| 1    | 9.683800E+00         |
| 2    | 9.684140E+00         |
| 3    | 1.401889E+01         |
| 4    | 1.642446E+01         |

---

## 2.34.1 Input Data

```
ID A034
SOL COMPAT1
POST=PUNCH
CEND
TITLE=TRIPOD BUCKLING ANALYSIS PROBLEM
ECHO = NONE
LINE = *,80
TIMES=SCREEN
$
LOADCASE      1
  LABEL=CASE1
  SPC      =      1
  GRAV     =      1
  DISP     =BOTH,ALL
  STRESS   =BOTH,ALL
LOADCASE      2
  LABEL=BACKLING
  METHOD    =      1
  STATLOAD =      1
  DISP     =BOTH,ALL
  MASS     = YES
BEGIN BULK
$
$ --- ANALYSIS DATA ---
$
EIGR1      LAN      1.0E-8      4
GRAV       1      -980.0      0.0      0.0      1.0
$
CBAR       1      1      4      6      5
CBAR       2      1     10      4      5
CBAR       3      1      8      4      5
CBAR       4      2      6      7      5
CBAR       5      2     10     11      5
CBAR       6      2      8      9      5
CBAR       7      3      7      1      5
CBAR       8      3     11      3      5
CBAR       9      3      9      2      5
CBAR      10      4     10      5      4
CBAR      11      4      6      5      4
CBAR      12      4      8      5      4
CBAR      13      5      4      5      8
$
CONM2      21      1      0.00001
CONM2      22      2      0.00001
CONM2      23      3      0.00001
CONM2      24      4      0.01010
CONM2      25      5      0.00001
CONM2      26      6      0.00001
CONM2      27      7      0.00001
CONM2      28      8      0.00001
CONM2      29      9      0.00001
CONM2      30     10      0.00001
```

```

CONM2      31      11      0.00001
$
PBARL      1      1  CSLIB1  TUBE
+          15.    0.20
PBARL      2      1  CSLIB1  TUBE
+          14.    0.20
PBARL      3      1  CSLIB1  TUBE
+          13.    0.20
PBARL      4      1  CSLIB1  TUBE
+          12.    0.20
PBARL      5      1  CSLIB1  TUBE
+          12.    0.20
$
MAT1       1  7.30E4      0.35  2.79E-9
$
GRID       1      0.0      0.0      0.0
GRID       2     600.0000      0.0      0.0
GRID       3     300.0000519.6000      0.0
GRID       4     300.0000173.20001187.400
GRID       5     300.0000173.2000791.6000
GRID       6     200.0000115.4667791.6000
GRID       7     100.000057.73334395.8000
GRID       8     400.0000115.4667791.6000
GRID       9     500.000057.73334395.8000
GRID      10     300.0000288.6666791.6000
GRID      11     300.0000404.1333395.8000
SPC        1      1     1236      0.0
SPC        1      2       3      0.0
SPC        1      3       3      0.0
ENDDATA

```

---

## 2.34.2 Output Data

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A034

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
TRIPOD BUCKLING ANALYSIS PROBLEM

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 11
NUMBER OF CBAR ELEMENTS: 13
NUMBER OF CONM2 ELEMENTS: 11
```

|                                     |    |
|-------------------------------------|----|
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 24 |
| NUMBER OF ELEMENT PROPERTIES:       | 5  |
| NUMBER OF MATERIALS:                | 1  |
| NUMBER OF DEGREES OF FREEDOM:       | 60 |

#### LOAD CASES SUMMARY

|                                |   |
|--------------------------------|---|
| NUMBER OF STATIC LOAD CASES:   | 1 |
| NUMBER OF BUCKLING LOAD CASES: | 1 |
| TOTAL NUMBER OF LOAD CASES:    | 2 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
TRIPOD BUCKLING ANALYSIS PROBLEM

#### LANCZOS EIGENSOLUTION STATISTICS

| SHIFT<br>COUNT | SHIFT<br>VALUE | EVALUES<br>BELOW | NEW<br>FOUND | TERM.<br>CODE | CPU<br>TIME |
|----------------|----------------|------------------|--------------|---------------|-------------|
| 1              | 1.000000E+00   | 0                | 4            | 2             | 0.03        |
| 2              | 2.494797E+01   | 4                | 0            | 10            | 0.00        |

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TRIPOD BUCKLING ANALYSIS PROBLEM

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
TRIPOD BUCKLING ANALYSIS PROBLEM

#### M A S S / V O L U M E S U M M A R Y

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 1.030824E-02 |
| SYSTEM VOLUME      | 3.879662E+04 |
| SYSTEM MASS/VOLUME | 2.656995E-07 |

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TRIPOD BUCKLING ANALYSIS PROBLEM

#### M A S S T A B L E

| ITEM   | NUMBER | MASS         |
|--------|--------|--------------|
| SYSTEM |        | 1.030824E-02 |
| PBAR   | 1      | 3.295804E-05 |
| PBAR   | 2      | 3.078975E-05 |
| PBAR   | 3      | 2.862145E-05 |
| PBAR   | 4      | 7.408462E-06 |
| PBAR   | 5      | 8.464858E-06 |

MAT1 1 1.082426E-04

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TRIPOD BUCKLING ANALYSIS PROBLEM

S O L U T I O N R E S I D U A L S

LOADCASE 1 ; RESIDUAL : 8.857431E-11 ; STRAIN ENERGY : 1.737482E+01  
1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 7  
TRIPOD BUCKLING ANALYSIS PROBLEM

O L O A D R E S U L T A N T S

| LOADCASE | T1         | T2         | T3          | R1          | R2         | R3         |
|----------|------------|------------|-------------|-------------|------------|------------|
| 1        | 0.0000E+00 | 5.2612E-19 | -1.0102E+01 | -1.7497E+03 | 3.0306E+03 | 1.3615E-16 |

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TRIPOD BUCKLING ANALYSIS PROBLEM

S P C F O R C E R E S U L T A N T S

| LOADCASE | T1         | T2          | T3         | R1         | R2          | R3          |
|----------|------------|-------------|------------|------------|-------------|-------------|
| 1        | 2.2598E-13 | -3.4449E-13 | 1.0102E+01 | 1.7497E+03 | -3.0306E+03 | -2.0367E-10 |

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TRIPOD BUCKLING ANALYSIS PROBLEM

M A X I M U M A P P L I E D F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3          |
|----------|---------|------------|------------|-------------|-------------|
| 1        | 4       | 9.9183E+00 | 0.0000E+00 | -2.1684E-19 | -9.9183E+00 |

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TRIPOD BUCKLING ANALYSIS PROBLEM

M A X I M U M S P C F O R C E

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3         |
|----------|---------|------------|------------|-------------|------------|
| 1        | 1       | 3.3674E+00 | 5.5937E-12 | -1.9121E-10 | 3.3674E+00 |

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TRIPOD BUCKLING ANALYSIS PROBLEM

M A X I M U M D I S P L A C E M E N T

| LOADCASE | GRID ID | MAGNITUDE  | T1         | T2          | T3         |
|----------|---------|------------|------------|-------------|------------|
| 1        | 2       | 2.0485E+01 | 2.0485E+01 | -4.5048E-08 | 0.0000E+00 |

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# TRIPOD BUCKLING ANALYSIS PROBLEM

CASE1

LOADCASE

1

## GRID DISPLACEMENTS

| GRID ID | T1<br>R1                      | T2<br>R2                       | T3<br>R3                       |
|---------|-------------------------------|--------------------------------|--------------------------------|
| 1       | 0.000000E+00<br>-1.110259E-02 | 0.000000E+00<br>1.923134E-02   | 0.000000E+00<br>0.000000E+00   |
| 2       | 2.048533E+01<br>-1.110259E-02 | -4.504767E-08<br>-1.923134E-02 | 0.000000E+00<br>-1.616128E-10  |
| 3       | 1.024267E+01<br>2.220605E-02  | 1.774027E+01<br>-5.632925E-13  | 0.000000E+00<br>-7.909560E-11  |
| 4       | 1.024267E+01<br>3.071618E-07  | 5.913071E+00<br>-5.451122E-13  | -3.460630E+00<br>-7.922978E-11 |
| 5       | 1.024267E+01<br>3.090593E-07  | 5.913192E+00<br>-6.289193E-13  | -3.472425E+00<br>-7.904107E-11 |
| 6       | 1.024200E+01<br>-7.976579E-04 | 5.912805E+00<br>1.382153E-03   | -3.454534E+00<br>-1.338904E-10 |
| 7       | 6.922066E+00<br>-8.075706E-03 | 3.996219E+00<br>1.398847E-02   | -2.334071E+00<br>2.116870E-12  |
| 8       | 1.024333E+01<br>-7.976579E-04 | 5.912805E+00<br>-1.382153E-03  | -3.454534E+00<br>-2.439540E-11 |
| 9       | 1.356326E+01<br>-8.075706E-03 | 3.996219E+00<br>-1.398847E-02  | -2.334071E+00<br>-1.616128E-10 |
| 10      | 1.024267E+01<br>1.596218E-03  | 5.913965E+00<br>-5.632925E-13  | -3.454481E+00<br>-7.909560E-11 |
| 11      | 1.024267E+01<br>1.615229E-02  | 9.747488E+00<br>-5.632925E-13  | -2.334017E+00<br>-7.909560E-11 |

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# TRIPOD BUCKLING ANALYSIS PROBLEM

CASE1

LOADCASE

1

## STRESSES IN BAR ELEMENTS

| BAR ID | END | STRESS-C      | STRESS-D      | STRESS-E      | STRESS-F      |
|--------|-----|---------------|---------------|---------------|---------------|
| 1      | A   | -1.000364E+00 | -5.239680E+00 | -1.000355E+00 | 3.238961E+00  |
|        | B   | -1.001443E+00 | 7.583741E+00  | -1.001440E+00 | -9.586624E+00 |
| 2      | A   | -1.001440E+00 | 7.583500E+00  | -1.001440E+00 | -9.586379E+00 |



|    |   |               |               |               |               |
|----|---|---------------|---------------|---------------|---------------|
|    | B | -1.000357E+00 | -5.239566E+00 | -1.000357E+00 | 3.238851E+00  |
| 3  | A | -1.001443E+00 | 7.583741E+00  | -1.001440E+00 | -9.586624E+00 |
|    | B | -1.000364E+00 | -5.239679E+00 | -1.000355E+00 | 3.238961E+00  |
| 4  | A | -3.581261E-01 | 2.435578E+01  | -3.581065E-01 | -2.507201E+01 |
|    | B | -3.592025E-01 | 1.203379E+01  | -3.591945E-01 | -1.275219E+01 |
| 5  | A | -3.581169E-01 | 2.435524E+01  | -3.581169E-01 | -2.507148E+01 |
|    | B | -3.591991E-01 | 1.203352E+01  | -3.591991E-01 | -1.275192E+01 |
| 6  | A | -3.581065E-01 | 2.435578E+01  | -3.581261E-01 | -2.507201E+01 |
|    | B | -3.591945E-01 | 1.203379E+01  | -3.592025E-01 | -1.275219E+01 |
| 7  | A | -3.875471E-01 | 1.396894E+01  | -3.875426E-01 | -1.474403E+01 |
|    | B | -3.886270E-01 | -3.886270E-01 | -3.886270E-01 | -3.886270E-01 |
| 8  | A | -3.875455E-01 | 1.396863E+01  | -3.875455E-01 | -1.474372E+01 |
|    | B | -3.886277E-01 | -3.886277E-01 | -3.886277E-01 | -3.886277E-01 |
| 9  | A | -3.875426E-01 | 1.396894E+01  | -3.875471E-01 | -1.474403E+01 |
|    | B | -3.886270E-01 | -3.886270E-01 | -3.886270E-01 | -3.886270E-01 |
| 10 | A | 4.887905E-01  | -1.969791E+01 | 4.887905E-01  | 2.067549E+01  |
|    | B | 4.887905E-01  | 8.166385E+00  | 4.887905E-01  | -7.188804E+00 |
| 11 | A | 4.888013E-01  | -1.969825E+01 | 4.888023E-01  | 2.067585E+01  |
|    | B | 4.888027E-01  | 8.166606E+00  | 4.888009E-01  | -7.189002E+00 |
| 12 | A | 4.888023E-01  | -1.969825E+01 | 4.888013E-01  | 2.067585E+01  |
|    | B | 4.888009E-01  | 8.166606E+00  | 4.888027E-01  | -7.189002E+00 |
| 13 | A | 2.175858E+00  | 2.175863E+00  | 2.175882E+00  | 2.175877E+00  |
|    | B | 2.174804E+00  | 2.174797E+00  | 2.174772E+00  | 2.174779E+00  |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 14  
TRIPOD BUCKLING ANALYSIS PROBLEM

BACKLING LOADCASE 2

#### BUCKLING LOAD FACTORS

MODE BUCKLING LOAD FACTOR

|   |              |
|---|--------------|
| 1 | 9.685081E+00 |
| 2 | 9.694449E+00 |
| 3 | 1.428663E+01 |
| 4 | 1.642446E+01 |

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TRIPOD BUCKLING ANALYSIS PROBLEM

BACKLING LOADCASE 2

#### BUCKLING MODES

MODE NUMBER 1 BUCKLING LOAD FACTOR = 9.685081E+00

| GRID ID | T1<br>R1                      | T2<br>R2                       | T3<br>R3                      |
|---------|-------------------------------|--------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>-9.913269E-04 | 0.000000E+00<br>1.826365E-03   | 0.000000E+00<br>0.000000E+00  |
| 2       | 1.861754E+00<br>4.806850E-03  | 3.158330E+00<br>-1.595634E-03  | 0.000000E+00<br>6.468675E-04  |
| 3       | 3.660510E+00<br>-1.055707E-03 | -3.312731E-02<br>-4.907358E-03 | 0.000000E+00<br>-6.279140E-04 |
| 4       | 1.602126E-02<br>7.912720E-04  | -3.501411E-02<br>-1.340903E-03 | -7.886030E-06<br>6.192928E-06 |
| 5       | 5.675375E-01<br>9.044078E-04  | 2.904379E-01<br>-1.532627E-03  | -7.906696E-06<br>6.023956E-06 |
| 6       | 5.677976E-01<br>6.990331E-04  | 2.898581E-01<br>-1.168926E-03  | -1.862615E-01<br>5.701595E-06 |
| 7       | 5.949646E-01<br>-4.497668E-04 | 3.202626E-01<br>8.669080E-04   | -1.973070E-01<br>2.292304E-06 |
| 8       | 5.709710E-01<br>1.310649E-03  | 2.964487E-01<br>-1.530000E-03  | 9.158662E-02<br>4.417984E-05  |
| 9       | 1.199917E+00<br>3.695865E-03  | 1.347729E+00<br>-1.579791E-03  | 9.702210E-02<br>3.914153E-04  |
| 10      | 5.730023E-01<br>6.921434E-04  | 2.904051E-01<br>-1.879629E-03  | 9.465124E-02<br>-3.171819E-05 |
| 11      | 1.783360E+00<br>-4.958066E-04 | 3.101018E-01<br>-3.947719E-03  | 1.002687E-01<br>-3.752137E-04 |

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TRIPOD BUCKLING ANALYSIS PROBLEM

BACKLING LOADCASE 2

#### B U C K L I N G M O D E S

MODE NUMBER 2 BUCKLING LOAD FACTOR = 9.694449E+00

| GRID ID | T1<br>R1                     | T2<br>R2                       | T3<br>R3                     |
|---------|------------------------------|--------------------------------|------------------------------|
| 1       | 0.000000E+00<br>4.991001E-03 | 0.000000E+00<br>2.858797E-03   | 0.000000E+00<br>0.000000E+00 |
| 2       | 3.152130E+00<br>1.487777E-03 | -1.519026E+00<br>-2.949054E-03 | 0.000000E+00<br>9.913259E-04 |
| 3       | -3.300739E-01                | -3.489280E+00                  | 0.000000E+00                 |

|    |                               |                                |                               |
|----|-------------------------------|--------------------------------|-------------------------------|
|    | -1.790238E-03                 | 2.856641E-03                   | 1.003423E-03                  |
| 4  | 2.016418E+00<br>1.341704E-03  | -3.492479E+00<br>7.916807E-04  | -1.338589E-05<br>6.521111E-04 |
| 5  | 1.690789E+00<br>1.533576E-03  | -2.940618E+00<br>9.048972E-04  | -1.342097E-05<br>6.341314E-04 |
| 6  | 1.724067E+00<br>1.899318E-03  | -2.998258E+00<br>1.111477E-03  | 1.755272E-03<br>6.003482E-04  |
| 7  | 1.053218E+00<br>4.008560E-03  | -1.837044E+00<br>2.303650E-03  | 1.864336E-03<br>2.414897E-04  |
| 8  | 1.729157E+00<br>1.508500E-03  | -2.874272E+00<br>4.992143E-04  | -1.622932E-01<br>6.555148E-04 |
| 9  | 2.077265E+00<br>1.499423E-03  | -2.203794E+00<br>-1.847243E-03 | -1.719215E-01<br>8.489274E-04 |
| 10 | 1.614032E+00<br>1.173676E-03  | -2.940673E+00<br>1.092644E-03  | 1.604979E-01<br>6.562334E-04  |
| 11 | 8.362217E-01<br>-8.407061E-04 | -2.907252E+00<br>2.297496E-03  | 1.700296E-01<br>8.561998E-04  |

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TRIPOD BUCKLING ANALYSIS PROBLEM

BACKLING LOADCASE 2

# B U C K L I N G M O D E S

MODE NUMBER 3 BUCKLING LOAD FACTOR = 1.428663E+01

| GRID ID | T1<br>R1                       | T2<br>R2                       | T3<br>R3                      |
|---------|--------------------------------|--------------------------------|-------------------------------|
| 1       | 0.000000E+00<br>3.307093E-03   | 0.000000E+00<br>1.909304E-03   | 0.000000E+00<br>0.000000E+00  |
| 2       | -2.072654E-02<br>-3.452128E-03 | -3.166899E+00<br>2.021998E-03  | 0.000000E+00<br>5.271223E-04  |
| 3       | 2.752918E+00<br>2.507114E-05   | -1.565504E+00<br>-4.000571E-03 | 0.000000E+00<br>5.271377E-04  |
| 4       | 9.037939E-01<br>-1.811651E-05  | -1.565466E+00<br>-1.046856E-05 | 4.802701E-07<br>1.364071E-03  |
| 5       | 9.081340E-01<br>-2.082361E-05  | -1.572977E+00<br>-1.203309E-05 | 4.816373E-07<br>9.128722E-04  |
| 6       | 9.618464E-01<br>4.420599E-04   | -1.666012E+00<br>2.552079E-04  | -4.569113E-07<br>9.987487E-04 |

|    |                               |                                |                               |
|----|-------------------------------|--------------------------------|-------------------------------|
| 7  | 6.709104E-01<br>2.374785E-03  | -1.162080E+00<br>1.371046E-03  | -6.924335E-07<br>5.516395E-04 |
| 8  | 9.632701E-01<br>-4.887116E-04 | -1.477474E+00<br>2.635102E-04  | 2.186816E-03<br>1.049718E-03  |
| 9  | 6.656339E-01<br>-2.473214E-03 | -1.994215E+00<br>1.440919E-03  | 2.360139E-03<br>8.116848E-04  |
| 10 | 7.978609E-01<br>-1.613621E-05 | -1.572976E+00<br>-5.549920E-04 | -2.184920E-03<br>1.049715E-03 |
| 11 | 1.394184E+00<br>1.129268E-05  | -1.573580E+00<br>-2.862286E-03 | -2.358447E-03<br>8.116895E-04 |

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TRIPOD BUCKLING ANALYSIS PROBLEM

BACKLING LOADCASE 2

# B U C K L I N G M O D E S

MODE NUMBER 4 BUCKLING LOAD FACTOR = 1.642446E+01

| GRID ID | T1<br>R1                      | T2<br>R2                       | T3<br>R3                       |
|---------|-------------------------------|--------------------------------|--------------------------------|
| 1       | 0.000000E+00<br>-1.971522E-03 | 0.000000E+00<br>3.414695E-03   | 0.000000E+00<br>0.000000E+00   |
| 2       | 3.450666E+00<br>-1.971525E-03 | -6.546675E-06<br>-3.414693E-03 | 0.000000E+00<br>-1.821950E-08  |
| 3       | 1.725339E+00<br>3.943038E-03  | 2.988435E+00<br>-2.302179E-09  | 0.000000E+00<br>-7.867541E-09  |
| 4       | 1.725335E+00<br>1.252881E-08  | 9.961387E-01<br>-3.977412E-11  | -5.817608E-01<br>-7.723314E-09 |
| 5       | 1.725335E+00<br>6.997061E-09  | 9.961429E-01<br>-4.293790E-11  | -5.834814E-01<br>-7.660474E-09 |
| 6       | 1.725263E+00<br>-1.137479E-04 | 9.961025E-01<br>1.970162E-04   | -5.811891E-01<br>-8.975730E-09 |
| 7       | 1.203823E+00<br>-1.337276E-03 | 6.950436E-01<br>2.316177E-03   | -4.055318E-01<br>-5.798110E-09 |
| 8       | 1.725406E+00<br>-1.137480E-04 | 9.961010E-01<br>-1.970162E-04  | -5.811891E-01<br>-6.376498E-09 |
| 9       | 2.246844E+00<br>-1.337279E-03 | 6.950399E-01<br>-2.316176E-03  | -4.055318E-01<br>-1.076072E-08 |
| 10      | 1.725336E+00<br>2.275128E-04  | 9.962252E-01<br>-7.909535E-11  | -5.811872E-01<br>-7.551336E-09 |

|    |              |               |               |
|----|--------------|---------------|---------------|
| 11 | 1.725337E+00 | 1.598349E+00  | -4.055298E-01 |
|    | 2.674555E-03 | -1.572732E-09 | -7.656940E-09 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.35 Enforced Dynamic Displacement with the Large Mass Method

### Example ID:

A035

### Analysis Data:

GRID, CBAR, PBARL, MAT1, SPC, CONM2, EIGR, FREQ1, RLOAD1, DAREA, TABLED4

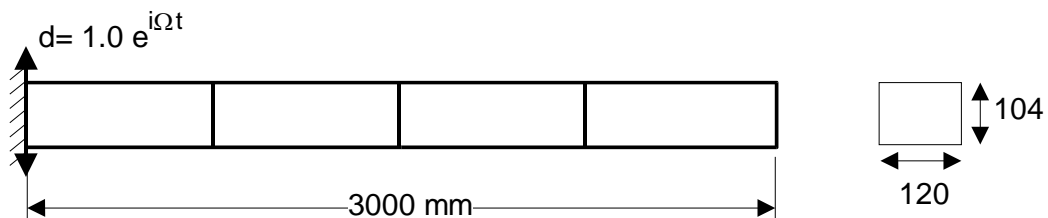
### Special Features Used:

Large Mass Method

### Problem Statement:

A cantilevered beam is modeled using CBAR elements together with PBARL. Calculate the response of the tip when the root is subjected to enforced dynamic displacement.

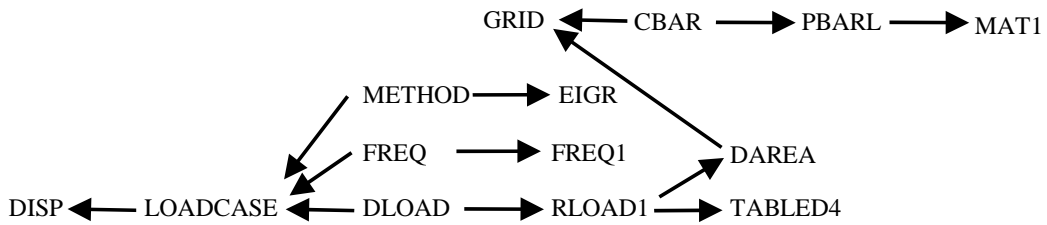
### Structural Analysis Model:



### Analysis Model Description:

1. The unit used here is the N, mm, Mg and material is Steel ( $E=207.0E3$  MPa,  $\rho=8.0$  g/cm<sup>3</sup>,  $\nu=0.3$ ).

## Analysis Data Relationships:



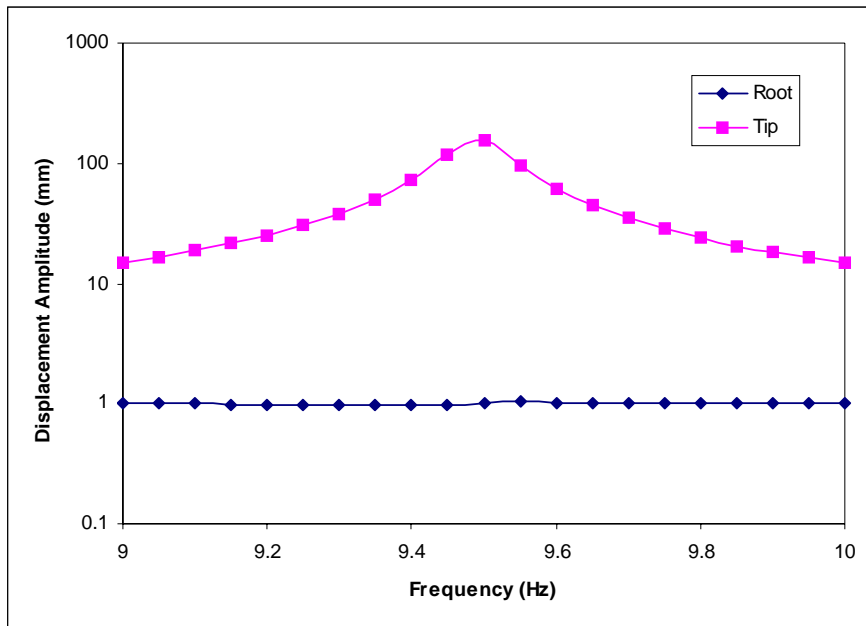
## Special Modeling Techniques:

The Large Mass Method is used to apply an enforced dynamic displacement.

## Reference Solution:

None.

## Calculated Solution:



---

## 2.35.1 Input Data

```
SOL COMPAT0
CEND
TITLE=ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM
SUBTITLE=EXAMPLE USING LARGE MASS METHOD
LINE=64,80
ECHO=NONE
$
DYNOUTPUT = MAGNITUDE
$
SET 5 = 1,5
SPC = 1
FREQ = 30
$
LOADCASE 2
LABEL = SOLUTION USING: MODAL FREQUENCY RESPONSE
DLOAD = 22
DISP=5
METHOD=4
$
LOADCASE 3
LABEL = SOLUTION USING: DIRECT FREQUENCY RESPONSE
DLOAD = 22
DISP=5
$
BEGIN BULK
PARAM,G,.01
PARAM,MODACC,1
$
$ The dynamical system of equations is:
$
$ [ -W^2 M + i W B + (1+ig) K + i KE ] U = P
$
$ where W is the loading frequency
$       M is the system mass matrix
$       B is the system viscous damping matrix
$       K is the system stiffness matrix
$       KE is the system structural damping matrix
$       g is the global structural damping coefficient
$       U is the complex displacement vector
$       P is the complex load vector
$       i is sqrt(-1)
$
$       Note, the real loads and displacements are usually taken as
$       p(t) = Re( P e^(i W t) )
$       u(t) = Re( U e^(i W t) )
$
$ to enforce U = D, a penalty mass, Mp is added to the left side, and
$ a corresponding penalty load is added to the right side of the
$ dynamical equation:
$
$ [ -W^2 M + i W B + (1+iG) K + i KE -W^2 Mp ] U = P - W^2 Mp D
$
```



```

$ Here, a penalty mass of about 1000.0 times the total system mass
$ is added to the grid where displacements are to be enforced.
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
CONM2      1001      1      300.0
$
$   LOADING FREQUENCIES
$
FREQ1      30      9.0      .05      20
$
$   LOAD DEFINITION
$
$   A load = - W^2 Mp D is applied to the grid with the penalty mass
$   Here, D = 1.0 is used to enforce U = 1.0
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
RLOAD1      22      2      4
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
DAREA      2      1      3      1.0
$
$   C(W) = - W^2 Mp
$
$   TABLED4 makes a polynomial: y(x) = sum Ai ((x-X1)/X2)^i {X3 < x < X4}
$
$   Here, X2 = 1/(2 PI) to convert W from cycles/sec to radians/sec
$   A2 comes from the value of the penalty mass that was used.
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
$TABLED4      ID      X1      X2      X3      X4
$+            A0      A1      A2      A3      ...
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
TABLED4      4      0.0 .159155      0.0 1.0E+30
+            0.0      0.0 -300.0
$
$   GRID DATA
$
GRID      1      0.00      0.      0.0
GRID      2      750.0      0.      0.0
GRID      3      1500.0      0.      0.0
GRID      4      2250.0      0.      0.0
GRID      5      3000.0      0.      0.0
SPC      1      1      12456
$
$   ELEMENT DEFINITIONS
$
CBAR      1      1      1      2      0.      1.0      0.
CBAR      2      1      2      3      0.      1.0      0.
CBAR      3      1      3      4      0.      1.0      0.
CBAR      4      1      4      5      0.      1.0      0.
$
$   PROPERTY DEFINITIONS

```

```

$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
PBARL    1        1        CSLIB1    RECT
+        120.0    104.0
$
$    MATERIAL DATA
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
MAT1      1        2.07+5        0.3        8.0-9
$
$    EIGENVALUES DATA
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
EIGR      4        SMS        200.0        10
$
ENDDATA

```

---

## 2.35.2 Output Data

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A035

RUN STARTED: Dec 13, 2002 12:50

SYSTEM TYPE: ix86 (or compatible) Windows NT

LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.

LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 5
NUMBER OF CBAR ELEMENTS: 4
NUMBER OF CONM2 ELEMENTS: 1
```

TOTAL NUMBER OF NON RIGID ELEMENTS: 5  
 NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 25

#### LOAD CASES SUMMARY

NUMBER OF USER FREQUENCY LOAD CASES: 0  
 NUMBER OF INTERNAL FREQUENCY LOAD CASES: 1  
 NUMBER OF DIRECT DYNAMIC LOAD CASES: 1  
 NUMBER OF MODAL DYNAMIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 3

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LARGE MASS METHOD

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 3  
 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LARGE MASS METHOD

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 3.002995E+02  
 SYSTEM VOLUME 3.744000E+07  
 SYSTEM MASS/VOLUME 8.020821E-06

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LARGE MASS METHOD  
 MODES FOR METHOD OF LOADCASE 2 LOADCASE 4

#### E I G E N V A L U E S

| MODE | CYCLES       | EIGENVALUE   | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|--------------|--------------|---------------------|--------------------------|
| 1    | 2.742409E-06 | 2.969096E-10 | 1.723107E-05 | 1.000000E+00        | 2.969096E-10             |
| 2    | 9.489702E+00 | 3.555207E+03 | 5.962556E+01 | 1.000000E+00        | 3.555207E+03             |
| 3    | 1.094293E+01 | 4.727447E+03 | 6.875643E+01 | 1.000000E+00        | 4.727447E+03             |
| 4    | 5.921225E+01 | 1.384149E+05 | 3.720416E+02 | 1.000000E+00        | 1.384149E+05             |
| 5    | 6.817606E+01 | 1.834947E+05 | 4.283628E+02 | 1.000000E+00        | 1.834947E+05             |
| 6    | 1.656957E+02 | 1.083882E+06 | 1.041097E+03 | 1.000000E+00        | 1.083882E+06             |
| 7    | 1.903513E+02 | 1.430447E+06 | 1.196013E+03 | 1.000000E+00        | 1.430447E+06             |

NOTE FROM ROUTINE GN42PE:  
 ALTHOUGH 10 EIGENVALUES HAVE BEEN REQUESTED,  
 ONLY 7 EIGENVALUES EXISTED IN THE RANGE  
 SPECIFIED.

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.935827E-01 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.105782E-02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 7.256227E-13  | 5.137384E-11 | 1.489824E+01 |
|                   | MAGN 2.358203E-13  | 6.355027E-03 | 4.370223E-14 |
|                   | PHASE 3.594199E+02 | 3.538246E+02 | 3.546938E+02 |
|                   | PHASE 3.594194E+02 | 1.742966E+02 | 3.563845E+02 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 6

ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.929302E-01 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.867848E-02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 7.338189E-13  | 5.820605E-11 | 1.659279E+01 |
|                   | MAGN 2.384863E-13  | 7.132347E-03 | 4.706776E-14 |
|                   | PHASE 3.594182E+02 | 3.531636E+02 | 3.540682E+02 |
|                   | PHASE 3.594176E+02 | 1.736723E+02 | 3.558497E+02 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 7

ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1000E+00)

| GRID ID           | T1 | T2 | T3 |
|-------------------|----|----|----|
|                   | R1 | R2 | R3 |
| (MAGNITUDE/PHASE) |    |    |    |

---

|     |                          |               |                |
|-----|--------------------------|---------------|----------------|
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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.878062E-01 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.301634E-01 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 7.675130E-13  | 1.140778E-10 | 3.017060E+01 |
|                   | MAGN 2.494559E-13  | 1.336377E-02 | 7.357382E-14 |
|                   | PHASE 3.593978E+02 | 3.479592E+02 | 3.490294E+02 |
|                   | PHASE 3.593959E+02 | 1.686388E+02 | 3.512326E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.3000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.850612E-01 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 2.048816E-01 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 7.763178E-13  | 1.456794E-10 | 3.774392E+01 |
|                   | MAGN 2.523281E-13  | 1.684143E-02 | 8.820748E-14 |
|                   | PHASE 3.593813E+02 | 3.450665E+02 | 3.461853E+02 |
|                   | PHASE 3.593783E+02 | 1.657960E+02 | 3.485102E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.3500E+00)

| GRID ID           | T1 | T2 | T3 |
|-------------------|----|----|----|
|                   | R1 | R2 | R3 |
| (MAGNITUDE/PHASE) |    |    |    |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.5000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.013390E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.309351E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 8.052868E-13  | 6.467721E-10 | 1.529688E+02 |
|                   | MAGN 2.616416E-13  | 7.024379E-02 | 2.949121E-13 |
|                   | PHASE 3.586665E+02 | 2.567555E+02 | 2.581064E+02 |
|                   | PHASE 3.586154E+02 | 7.772238E+01 | 2.609874E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.5500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.029329E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.312704E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 8.105259E-13  | 4.206316E-10 | 9.699376E+01 |
|                   | MAGN 2.632713E-13  | 4.485388E-02 | 1.771364E-13 |
|                   | PHASE 3.591191E+02 | 2.170468E+02 | 2.184673E+02 |
|                   | PHASE 3.590983E+02 | 3.808454E+01 | 2.215029E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.6000E+00)

| GRID ID           | T1 | T2 | T3 |
|-------------------|----|----|----|
|                   | R1 | R2 | R3 |
| (MAGNITUDE/PHASE) |    |    |    |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.7500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.010238E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.146280E-01 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 8.484581E-13  | 1.373837E-10 | 2.825931E+01 |
|                   | MAGN 2.756710E-13  | 1.343269E-02 | 4.049751E-14 |
|                   | PHASE 3.594005E+02 | 1.887812E+02 | 1.905438E+02 |
|                   | PHASE 3.593987E+02 | 1.016627E+01 | 1.941994E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.8000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.008588E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 8.197415E-02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 8.575078E-13  | 1.197656E-10 | 2.385129E+01 |
|                   | MAGN 2.786183E-13  | 1.141395E-02 | 3.197867E-14 |
|                   | PHASE 3.594080E+02 | 1.870361E+02 | 1.889045E+02 |
|                   | PHASE 3.594067E+02 | 8.528186E+00 | 1.926864E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.8500E+00)

| GRID ID           | T1 | T2 | T3 |
|-------------------|----|----|----|
|                   | R1 | R2 | R3 |
| (MAGNITUDE/PHASE) |    |    |    |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 1.0000E+01)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.005092E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.146934E-02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 8.935936E-13  | 8.563992E-11 | 1.469098E+01 |
|                   | MAGN 2.903586E-13  | 7.218065E-03 | 1.497749E-14 |
|                   | PHASE 3.594196E+02 | 1.831045E+02 | 1.855159E+02 |
|                   | PHASE 3.594191E+02 | 5.144744E+00 | 1.892077E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.935827E-01 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.105782E-02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.489824E+01 |
|                   | MAGN 0.000000E+00  | 6.355043E-03 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.546938E+02 |
|                   | PHASE 0.000000E+00 | 1.742966E+02 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0500E+00)

| GRID ID           | T1 | T2 | T3 |
|-------------------|----|----|----|
|                   | R1 | R2 | R3 |
| (MAGNITUDE/PHASE) |    |    |    |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.897001E-01 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 8.953178E-02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 2.508418E+01 |
|                   | MAGN 0.000000E+00  | 1.102887E-02 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.509242E+02 |
|                   | PHASE 0.000000E+00 | 1.705323E+02 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.878062E-01 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.301634E-01 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 3.017061E+01 |
|                   | MAGN 0.000000E+00  | 1.336381E-02 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.490294E+02 |
|                   | PHASE 0.000000E+00 | 1.686388E+02 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.3000E+00)

| GRID ID           | T1 | T2 | T3 |
|-------------------|----|----|----|
|                   | R1 | R2 | R3 |
| (MAGNITUDE/PHASE) |    |    |    |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.4500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 9.698332E-01 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 2.110417E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.196808E+02 |
|                   | MAGN 0.000000E+00  | 5.456991E-02 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.102211E+02 |
|                   | PHASE 0.000000E+00 | 1.298357E+02 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.5000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.013390E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.309351E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.529688E+02 |
|                   | MAGN 0.000000E+00  | 7.024404E-02 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 2.581064E+02 |
|                   | PHASE 0.000000E+00 | 7.772237E+01 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.5500E+00)

| GRID ID           | T1 | T2 | T3 |
|-------------------|----|----|----|
|                   | R1 | R2 | R3 |
| (MAGNITUDE/PHASE) |    |    |    |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.7000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.012577E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.714210E-01 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 3.463800E+01 |
|                   | MAGN 0.000000E+00  | 1.635339E-02 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.929311E+02 |
|                   | PHASE 0.000000E+00 | 1.255221E+01 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.7500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.010238E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.146280E-01 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 2.825932E+01 |
|                   | MAGN 0.000000E+00  | 1.343274E-02 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.905438E+02 |
|                   | PHASE 0.000000E+00 | 1.016627E+01 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.8000E+00)

| GRID ID           | T1 | T2 | T3 |
|-------------------|----|----|----|
|                   | R1 | R2 | R3 |
| (MAGNITUDE/PHASE) |    |    |    |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.9500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.005692E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.839161E-02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.624752E+01 |
|                   | MAGN 0.000000E+00  | 7.931107E-03 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.860906E+02 |
|                   | PHASE 0.000000E+00 | 5.718131E+00 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LARGE MASS METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 1.0000E+01)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.005092E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.146934E-02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.469098E+01 |
|                   | MAGN 0.000000E+00  | 7.218098E-03 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 1.855159E+02 |
|                   | PHASE 0.000000E+00 | 5.144738E+00 | 0.000000E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.36 Enforced Dynamic Displacement with the Lagrange Multiplier Method

### Example ID:

A036

### Analysis Data:

GRID, CBAR, PBARL, MAT1, SPC, CONM2, EIGR, FREQ1, RLOAD1, DAREA, TABLED1, DMIG

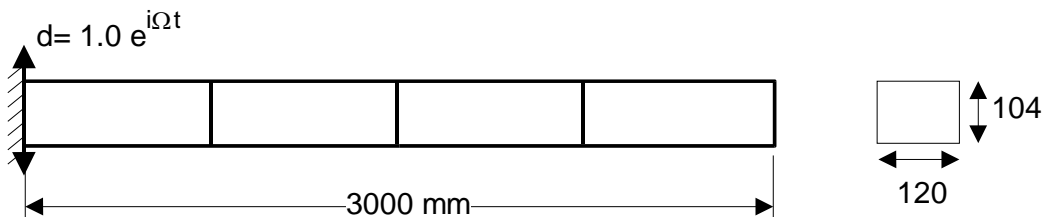
### Special Features Used:

Lagrange Multiplier constraint equations via DMIG

### Problem Statement:

A cantilevered beam is modeled using CBAR elements together with PBARL. Calculate the response of the tip when the root is subjected to enforced dynamic displacement.

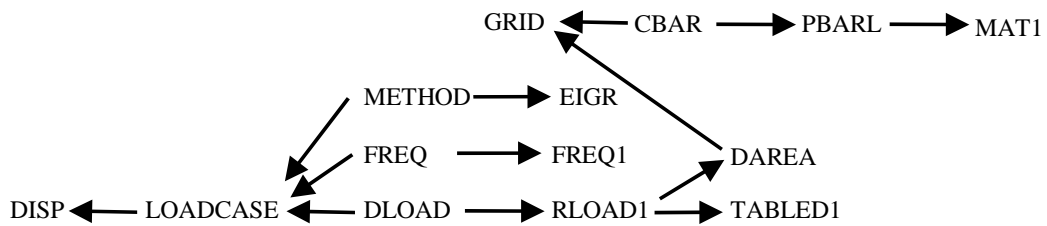
### Structural Analysis Model:



### Analysis Model Description:

1. The unit used here is the N, mm, Mg and material is Steel ( $E=207.0E3$  MPa,  $\rho=8.0$  g/cm<sup>3</sup>,  $\nu=0.3$ ).

## Analysis Data Relationships:



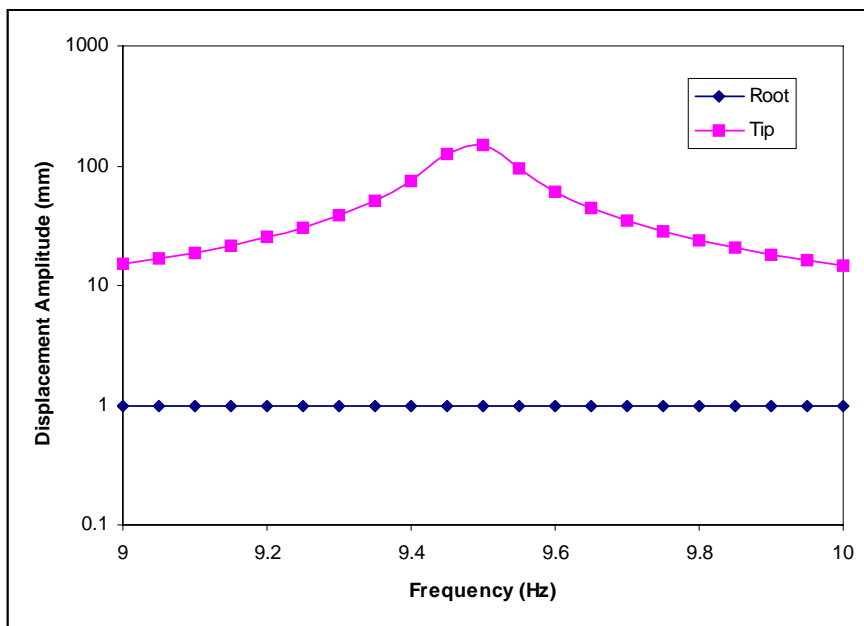
## Special Modeling Techniques:

The Lagrange Multiplier Method is used to apply an enforced dynamic displacement.

## Reference Solution:

None.

## Calculated Solution:



---

## 2.36.1 Input Data

```
SOL COMPAT0
CEND
TITLE=ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM
SUBTITLE=EXAMPLE USING LAGRANGE MULTIPLIER METHOD
LINE=64,80
ECHO=NONE
$
DYNOUTPUT = MAGNITUDE
$
K2PP=ENFORCEP
K2GG=ENFORCE
$
SET 5 = 1,5
SPC = 1
FREQ = 30
$
LOADCASE 2
LABEL = SOLUTION USING:  MODAL FREQUENCY RESPONSE
DLOAD = 22
DISP=5
METHOD=4
$
LOADCASE 3
LABEL = SOLUTION USING:  DIRECT FREQUENCY RESPONSE
DLOAD = 22
DISP=5
$
BEGIN BULK
PARAM,G,.01
PARAM,MODACC,1
$
$ The dynamical system of equations is:
$
$ [ -W^2 M + i W B + (1+ig) K + i KE ] U = P
$
$ where W is the loading frequency
$       M is the system mass matrix
$       B is the system viscous damping matrix
$       K is the system stiffness matrix
$       KE is the system structural damping matrix
$       g is the global structural damping coefficient
$       U is the complex displacement vector
$       P is the complex load vector
$       i is sqrt(-1)
$
$       Note, the real loads and displacements are usually taken as
$       p(t) = Re( P e^(i W t) )
$       u(t) = Re( U e^(i W t) )
$
$ To enforce the constraint: C U = d, an extra degree of freedom, q,
$ (the lagrange multiplier) is introduced, and the dynamical system
$ is augmented as follows:
```



```

$
$ [ -W^2 M + i W B + (1+ig) K + i KE | C^t ] {U} {P}
$ [-----+-----] {-} = {-}
$ [ C | 0 ] {q} {d}
$
$ Here, an SPOINT is added to be the lagrange multiplier dof.
$ DMIG and K2PP is used to enter the C matrix (which has only one non-zero
$ term - a 1.0 in the column corresponding to the displacement we want to
$ enforce). We would like to use K2PP here instead of K2GG so that
$ PARAM,G will not affect our constraint equation. However, if we
$ want to use modal frequency response, we must also use K2GG, because
$ K2PP does not apply to the modal solution, and the modal solution
$ needs to have the constraint equation. Also, for modal frequency
$ response, we MUST have PARAM,MODACC,1 (the default) to augment the modes
$ with an extra basis vector that moves the enforced dof (all the
$ eigenvectors have 0.0 for the enforced dof because of the constraint
$ equation).
$ Note that when we use K2GG to add a constraint equation, the augmented
$ stiffness matrix is NOT positive definite. Therefore, the SMS method
$ cannot be used for the modal solution.
$
SPOINT 10000001
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
DMIG ENFORCEP 0 6 2
DMIG ENFORCEP 1 3 10000001 .999
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
DMIG ENFORCE 0 6 2
DMIG ENFORCE 1 3 10000001 .001
$
$ LOADING FREQUENCIES
$
FREQ1 30 9.0 .05 20
$
$ LOAD DEFINITION
$
$ A load = d is applied to the lagrange multiplier degree of freedom
$ Here, d = 1.0 is used to enforce U = 1.0
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
RLOAD1 22 2 4
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
DAREA 210000001 1.0
$
$ C(W) = 1.0
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
TABLED1 4
+ 0.0 1.0 1.0 1.0
$
$ GRID DATA

```

```

$
GRID    1          0.00  0.    0.0
GRID    2          750.0  0.    0.0
GRID    3          1500.0 0.    0.0
GRID    4          2250.0 0.    0.0
GRID    5          3000.0 0.    0.0
SPC     1      1      12456
$
$      ELEMENT DEFINITIONS
$
CBAR     1      1      1      2      0.    1.0    0.
CBAR     2      1      2      3      0.    1.0    0.
CBAR     3      1      3      4      0.    1.0    0.
CBAR     4      1      4      5      0.    1.0    0.
$
$      PROPERTY DEFINITIONS
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
PBARL    1      1      CSLIB1  RECT
+      120.0    104.0
$
$      MATERIAL DATA
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
MAT1     1      2.07+5      0.3    8.0-9
$
$      EIGENVALUES DATA
$
$-----12-----23-----34-----45-----56-----67-----78-----89-----9X-----
-X
EIGR     4      LAN      200.0      10
$
ENDDATA

```

---

## 2.36.2 Output Data

1

```

      GGGG      EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N   E      S      I      S
G  GG  EEEEE  N  N  N   EEEEE  SSSS   I      SSSS
G    G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N   EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 7.2

BUILD CODE 200212131048

PROJECT NAME: A036  
RUN STARTED: Dec 13, 2002 12:50  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 40000000

LICENSED TO: Vanderplaats R&D Inc.  
LICENSE EXPIRES: Dec 31, 2020

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 1  
ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LAGRANGE MULTIPLIER METHOD

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS: 5
NUMBER OF SPOINTS: 1
NUMBER OF CBAR ELEMENTS: 4
```

TOTAL NUMBER OF NON RIGID ELEMENTS: 4  
 NUMBER OF ELEMENT PROPERTIES: 1  
 NUMBER OF MATERIALS: 1  
 NUMBER OF DEGREES OF FREEDOM: 26

#### LOAD CASES SUMMARY

NUMBER OF USER FREQUENCY LOAD CASES: 0  
 NUMBER OF INTERNAL FREQUENCY LOAD CASES: 1  
 NUMBER OF DIRECT DYNAMIC LOAD CASES: 1  
 NUMBER OF MODAL DYNAMIC LOAD CASES: 1  
 TOTAL NUMBER OF LOAD CASES: 3

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 2  
 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

#### LANCZOS EIGENSOLUTION STATISTICS

| SHIFT<br>COUNT | SHIFT<br>VALUE | FREQUENCY<br>(CYC./TIME) | EVALUES<br>BELOW | NEW<br>FOUND | TERM.<br>CODE | CPU<br>TIME |
|----------------|----------------|--------------------------|------------------|--------------|---------------|-------------|
| 1              | 1.579137E+06   | 2.000000E+02             | 7                | 0            | 10            | 0.02        |
| 2              | -1.000000E+01  | -5.032921E-01            | 1                | 6            | 4             | 0.01        |

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 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

\*\*\*\*\*  
 \* D E S I G N C Y C L E 0 (ANALYSIS)\*  
 \*\*\*\*\*

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 4  
 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

#### M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 2.995200E-01  
 SYSTEM VOLUME 3.744000E+07  
 SYSTEM MASS/VOLUME 8.000000E-09

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 5  
 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 MODES FOR METHOD OF LOADCASE 2 LOADCASE 4

#### E I G E N V A L U E S

| MODE | CYCLES       | EIGENVALUE   | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|--------------|--------------|---------------------|--------------------------|
| 1    | 9.486800E+00 | 3.553033E+03 | 5.960733E+01 | 1.000000E+00        | 3.553033E+03             |
| 2    | 1.094293E+01 | 4.727447E+03 | 6.875643E+01 | 1.000000E+00        | 4.727447E+03             |
| 3    | 5.920663E+01 | 1.383886E+05 | 3.720062E+02 | 1.000000E+00        | 1.383886E+05             |

|   |              |              |              |              |              |
|---|--------------|--------------|--------------|--------------|--------------|
| 4 | 6.817606E+01 | 1.834947E+05 | 4.283628E+02 | 1.000000E+00 | 1.834947E+05 |
| 5 | 1.656853E+02 | 1.083746E+06 | 1.041031E+03 | 1.000000E+00 | 1.083746E+06 |
| 6 | 1.903507E+02 | 1.430437E+06 | 1.196009E+03 | 1.000000E+00 | 1.430437E+06 |

NOTE FROM ROUTINE GN42PE:

ALTHOUGH 10 EIGENVALUES HAVE BEEN REQUESTED,  
ONLY 6 EIGENVALUES EXISTED IN THE RANGE  
SPECIFIED.

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0000E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000171E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599987E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 2.084616E-18 | 2.391205E-12 | 1.240110E+01 |
|                   | MAGN  | 3.571670E-18 | 5.190590E-03 | 3.853423E-16 |
|                   | PHASE | 3.553694E+02 | 1.796798E+02 | 3.556038E+02 |
|                   | PHASE | 1.753862E+02 | 1.752016E+02 | 1.819248E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0500E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000187E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599986E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 2.287980E-18 | 2.403062E-12 | 1.354994E+01 |
|                   | MAGN  | 3.918823E-18 | 5.715675E-03 | 3.683198E-16 |
|                   | PHASE | 3.549477E+02 | 1.798528E+02 | 3.551784E+02 |
|                   | PHASE | 1.749647E+02 | 1.747775E+02 | 1.831110E+02 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 8  
ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM

EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1000E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000207E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599984E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 2.531985E-18 | 2.405679E-12 | 1.492929E+01 |
|                   | MAGN  | 4.335319E-18 | 6.346190E-03 | 3.429887E-16 |
|                   | PHASE | 3.544405E+02 | 1.801027E+02 | 3.546676E+02 |
|                   | PHASE | 1.744579E+02 | 1.742680E+02 | 1.849749E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1500E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000231E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599981E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 2.830130E-18 | 2.395024E-12 | 1.661567E+01 |
|                   | MAGN  | 4.844189E-18 | 7.117163E-03 | 3.066585E-16 |
|                   | PHASE | 3.538195E+02 | 1.804731E+02 | 3.540429E+02 |
|                   | PHASE | 1.738371E+02 | 1.736445E+02 | 1.881628E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2000E+00)

| GRID ID           |      | T1           | T2           | T3           |
|-------------------|------|--------------|--------------|--------------|
|                   |      | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |      |              |              |              |
| 1                 | MAGN | 0.000000E+00 | 0.000000E+00 | 1.000260E+00 |

|   |       | MAGN         | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|---|-------|--------------|--------------|--------------|--------------|
|   |       | PHASE        | 0.000000E+00 | 0.000000E+00 | 3.599978E+02 |
|   |       | PHASE        | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5 | MAGN  | 3.202583E-18 | 2.365071E-12 | 1.872348E+01 |              |
|   | MAGN  | 5.479845E-18 | 8.080936E-03 | 2.563714E-16 |              |
|   | PHASE | 3.530417E+02 | 1.810425E+02 | 3.532614E+02 |              |
|   | PHASE | 1.730597E+02 | 1.728643E+02 | 1.943657E+02 |              |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM

EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2500E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.000298E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.599972E+02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 3.680839E-18  | 2.306571E-12 | 2.143127E+01 |
|                   | MAGN 6.296018E-18  | 9.319251E-03 | 1.927591E-16 |
|                   | PHASE 3.520401E+02 | 1.819641E+02 | 3.522560E+02 |
|                   | PHASE 1.720583E+02 | 1.718602E+02 | 2.091419E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM

EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.3000E+00)

| GRID ID           | T1                 | T2           | T3           |
|-------------------|--------------------|--------------|--------------|
|                   | R1                 | R2           | R3           |
| (MAGNITUDE/PHASE) |                    |              |              |
| 1                 | MAGN 0.000000E+00  | 0.000000E+00 | 1.000348E+00 |
|                   | MAGN 0.000000E+00  | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 3.599964E+02 |
|                   | PHASE 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN 4.316830E-18  | 2.205017E-12 | 2.503349E+01 |
|                   | MAGN 7.381313E-18  | 1.096691E-02 | 1.506285E-16 |
|                   | PHASE 3.507030E+02 | 1.835687E+02 | 3.509152E+02 |
|                   | PHASE 1.707216E+02 | 1.705207E+02 | 2.502203E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM

EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.3500E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000417E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599950E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 5.202301E-18 | 2.037885E-12 | 3.005020E+01 |
|                   | MAGN  | 8.892253E-18 | 1.326211E-02 | 2.554424E-16 |
|                   | PHASE | 3.488315E+02 | 1.866819E+02 | 3.490398E+02 |
|                   | PHASE | 1.688504E+02 | 1.686466E+02 | 2.993928E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.4000E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000516E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599924E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 6.514387E-18 | 1.776906E-12 | 3.748527E+01 |
|                   | MAGN  | 1.113104E-17 | 1.666482E-02 | 5.461065E-16 |
|                   | PHASE | 3.460337E+02 | 1.938061E+02 | 3.462382E+02 |
|                   | PHASE | 1.660529E+02 | 1.658462E+02 | 3.170638E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.4500E+00)

| GRID ID           |      | T1           | T2           | T3           |
|-------------------|------|--------------|--------------|--------------|
|                   |      | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |      |              |              |              |
| 1                 | MAGN | 0.000000E+00 | 0.000000E+00 | 1.000667E+00 |



|       |              |              |              |
|-------|--------------|--------------|--------------|
| MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| PHASE | 0.000000E+00 | 0.000000E+00 | 3.599871E+02 |
| PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 8.636840E-18 | 1.464286E-12 | 4.951273E+01 |
|   | MAGN  | 1.475234E-17 | 2.217198E-02 | 1.090097E-15 |
|   | PHASE | 3.414300E+02 | 2.141457E+02 | 3.416307E+02 |
|   | PHASE | 1.614496E+02 | 1.612400E+02 | 3.209043E+02 |

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 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.5000E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 1 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000907E+00 |
|   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599733E+02 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5 | MAGN  | 1.252135E-17 | 1.879657E-12 | 7.152003E+01 |
|   | MAGN  | 2.137959E-17 | 3.225791E-02 | 2.175195E-15 |
|   | PHASE | 3.326403E+02 | 2.596244E+02 | 3.328371E+02 |
|   | PHASE | 1.526602E+02 | 1.524477E+02 | 3.164951E+02 |

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 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: MODAL FREQUENCY RESPONSE LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.5500E+00)

|         |    |    |    |
|---------|----|----|----|
| GRID ID | T1 | T2 | T3 |
|         | R1 | R2 | R3 |

(MAGNITUDE/PHASE)

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 1 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.001115E+00 |
|   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599293E+02 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5 | MAGN  | 2.045363E-17 | 5.140901E-12 | 1.164128E+02 |
|   | MAGN  | 3.491070E-17 | 5.288149E-02 | 4.622604E-15 |
|   | PHASE | 3.116343E+02 | 2.740626E+02 | 3.118272E+02 |
|   | PHASE | 1.316546E+02 | 1.314390E+02 | 2.981188E+02 |

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 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM

EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.6000E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 9.996932E-01 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.598748E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 2.716310E-17 | 1.085628E-11 | 1.540644E+02 |
|                   | MAGN  | 4.634528E-17 | 7.048123E-02 | 7.694672E-15 |
|                   | PHASE | 2.612499E+02 | 2.370057E+02 | 2.614389E+02 |
|                   | PHASE | 8.127051E+01 | 8.105192E+01 | 2.494634E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.6500E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 9.989147E-01 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599461E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 1.770382E-17 | 1.006799E-11 | 1.000741E+02 |
|                   | MAGN  | 3.019460E-17 | 4.610363E-02 | 6.124923E-15 |
|                   | PHASE | 2.199078E+02 | 2.018454E+02 | 2.200928E+02 |
|                   | PHASE | 3.992882E+01 | 3.970717E+01 | 2.093275E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.7000E+00)

| GRID ID           |      | T1           | T2           | T3           |
|-------------------|------|--------------|--------------|--------------|
|                   |      | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |      |              |              |              |
| 1                 | MAGN | 0.000000E+00 | 0.000000E+00 | 9.991722E-01 |

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|                |               |                          |     |
|----------------|---------------|--------------------------|-----|
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EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.8500E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 9.995949E-01 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599948E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 5.148736E-18 | 7.252611E-12 | 2.873870E+01 |
|                   | MAGN  | 8.767722E-18 | 1.360809E-02 | 3.421148E-15 |
|                   | PHASE | 1.905965E+02 | 1.806234E+02 | 1.907654E+02 |
|                   | PHASE | 1.061903E+01 | 1.038475E+01 | 1.823990E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.9000E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 9.996563E-01 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599961E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 4.351611E-18 | 7.251721E-12 | 2.421810E+01 |
|                   | MAGN  | 7.407332E-18 | 1.154482E-02 | 3.332242E-15 |
|                   | PHASE | 1.889126E+02 | 1.796772E+02 | 1.890774E+02 |
|                   | PHASE | 8.935550E+00 | 8.698010E+00 | 1.809890E+02 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: MODAL FREQUENCY RESPONSE

LOADCASE 2

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.9500E+00)

| GRID ID           |      | T1           | T2           | T3           |
|-------------------|------|--------------|--------------|--------------|
|                   |      | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |      |              |              |              |
| 1                 | MAGN | 0.000000E+00 | 0.000000E+00 | 9.997017E-01 |

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|                |               |                          |     |
|----------------|---------------|--------------------------|-----|
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EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.0500E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.671093E+01 |
|                   | MAGN  | 0.000000E+00 | 7.183150E-03 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.540289E+02 |
|                   | PHASE | 0.000000E+00 | 1.736330E+02 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1000E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.886258E+01 |
|                   | MAGN  | 0.000000E+00 | 8.169900E-03 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.532345E+02 |
|                   | PHASE | 0.000000E+00 | 1.728400E+02 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.1500E+00)

| GRID ID           |      | T1           | T2           | T3           |
|-------------------|------|--------------|--------------|--------------|
|                   |      | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |      |              |              |              |
| 1                 | MAGN | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
|   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 2.163688E+01 |
|   | MAGN  | 0.000000E+00 | 9.442401E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.522087E+02 |
|   | PHASE | 0.000000E+00 | 1.718155E+02 | 0.000000E+00 |

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 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2000E+00)

| GRID ID |       | T1                | T2           | T3           |
|---------|-------|-------------------|--------------|--------------|
|         |       | R1                | R2           | R3           |
|         |       | (MAGNITUDE/PHASE) |              |              |
| 1       | MAGN  | 0.000000E+00      | 0.000000E+00 | 1.000000E+00 |
|         | MAGN  | 0.000000E+00      | 0.000000E+00 | 0.000000E+00 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 3.599994E+02 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 0.000000E+00 |
| 5       | MAGN  | 0.000000E+00      | 0.000000E+00 | 2.534523E+01 |
|         | MAGN  | 0.000000E+00      | 1.114365E-02 | 0.000000E+00 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 3.508341E+02 |
|         | PHASE | 0.000000E+00      | 1.704421E+02 | 0.000000E+00 |

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 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.2500E+00)

| GRID ID |       | T1                | T2           | T3           |
|---------|-------|-------------------|--------------|--------------|
|         |       | R1                | R2           | R3           |
|         |       | (MAGNITUDE/PHASE) |              |              |
| 1       | MAGN  | 0.000000E+00      | 0.000000E+00 | 1.000000E+00 |
|         | MAGN  | 0.000000E+00      | 0.000000E+00 | 0.000000E+00 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 3.599994E+02 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 0.000000E+00 |
| 5       | MAGN  | 0.000000E+00      | 0.000000E+00 | 3.054304E+01 |
|         | MAGN  | 0.000000E+00      | 1.352878E-02 | 0.000000E+00 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 3.488986E+02 |
|         | PHASE | 0.000000E+00      | 1.685080E+02 | 0.000000E+00 |

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 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM

EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.3000E+00)

| GRID ID           |       | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|-------------------|-------|--------------|--------------|--------------|
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 3.831632E+01 |
|                   | MAGN  | 0.000000E+00 | 1.709689E-02 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.459798E+02 |
|                   | PHASE | 0.000000E+00 | 1.655905E+02 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.3500E+00)

| GRID ID           |       | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|-------------------|-------|--------------|--------------|--------------|
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 5.105785E+01 |
|                   | MAGN  | 0.000000E+00 | 2.294854E-02 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.411099E+02 |
|                   | PHASE | 0.000000E+00 | 1.607219E+02 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.4000E+00)

| GRID ID           |      | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|-------------------|------|--------------|--------------|--------------|
| (MAGNITUDE/PHASE) |      |              |              |              |
| 1                 | MAGN | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |





EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.5500E+00)

| GRID ID           |       | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|-------------------|-------|--------------|--------------|--------------|
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 9.423014E+01 |
|                   | MAGN  | 0.000000E+00 | 4.357602E-02 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.171540E+02 |
|                   | PHASE | 0.000000E+00 | 3.677125E+01 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.6000E+00)

| GRID ID           |       | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|-------------------|-------|--------------|--------------|--------------|
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 6.072992E+01 |
|                   | MAGN  | 0.000000E+00 | 2.828039E-02 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 2.029680E+02 |
|                   | PHASE | 0.000000E+00 | 2.258650E+01 | 0.000000E+00 |

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ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.6500E+00)

| GRID ID           |      | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|-------------------|------|--------------|--------------|--------------|
| (MAGNITUDE/PHASE) |      |              |              |              |
| 1                 | MAGN | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |

|  |       |              |              |              |
|--|-------|--------------|--------------|--------------|
|  | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|  | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|  | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 4.390747E+01 |
|   | MAGN  | 0.000000E+00 | 2.058828E-02 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.964279E+02 |
|   | PHASE | 0.000000E+00 | 1.604779E+01 | 0.000000E+00 |

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1GENESIS  VERSION  7.2                      DATE 12-13-2002  TIME 12:50  PAGE   41
ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM
EXAMPLE USING LAGRANGE MULTIPLIER METHOD
SOLUTION USING:  DIRECT FREQUENCY RESPONSE                      LOADCASE      3

G R I D    D Y N A M I C    D I S P L A C E M E N T S    (FREQUENCY =  9.7000E+00)

GRID ID          T1              T2              T3
                  R1              R2              R3

                        (MAGNITUDE/PHASE)

1  MAGN  0.000000E+00    0.000000E+00    1.000000E+00
   MAGN  0.000000E+00    0.000000E+00    0.000000E+00
   PHASE 0.000000E+00    0.000000E+00    3.599994E+02
   PHASE 0.000000E+00    0.000000E+00    0.000000E+00

5  MAGN  0.000000E+00    0.000000E+00    3.420778E+01
   MAGN  0.000000E+00    1.615028E-02    0.000000E+00
   PHASE 0.000000E+00    0.000000E+00    1.927591E+02
   PHASE 0.000000E+00    1.238022E+01    0.000000E+00

```

```

1GENESIS  VERSION  7.2                      DATE 12-13-2002  TIME 12:50  PAGE   42
ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM
EXAMPLE USING LAGRANGE MULTIPLIER METHOD
SOLUTION USING:  DIRECT FREQUENCY RESPONSE                      LOADCASE      3

G R I D    D Y N A M I C    D I S P L A C E M E N T S    (FREQUENCY =  9.7500E+00)

GRID ID          T1              T2              T3
                  R1              R2              R3

                        (MAGNITUDE/PHASE)

1  MAGN  0.000000E+00    0.000000E+00    1.000000E+00
   MAGN  0.000000E+00    0.000000E+00    0.000000E+00
   PHASE 0.000000E+00    0.000000E+00    3.599994E+02
   PHASE 0.000000E+00    0.000000E+00    0.000000E+00

5  MAGN  0.000000E+00    0.000000E+00    2.797294E+01
   MAGN  0.000000E+00    1.329661E-02    0.000000E+00
   PHASE 0.000000E+00    0.000000E+00    1.904286E+02
   PHASE 0.000000E+00    1.005106E+01    0.000000E+00

```

```

1GENESIS  VERSION  7.2                      DATE 12-13-2002  TIME 12:50  PAGE   43
ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM

```

EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.8000E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 2.364820E+01 |
|                   | MAGN  | 0.000000E+00 | 1.131681E-02 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.888219E+02 |
|                   | PHASE | 0.000000E+00 | 8.445633E+00 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 44

ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.8500E+00)

| GRID ID           |       | T1           | T2           | T3           |
|-------------------|-------|--------------|--------------|--------------|
|                   |       | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |       |              |              |              |
| 1                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |
|                   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5                 | MAGN  | 0.000000E+00 | 0.000000E+00 | 2.047950E+01 |
|                   | MAGN  | 0.000000E+00 | 9.866047E-03 | 0.000000E+00 |
|                   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.876487E+02 |
|                   | PHASE | 0.000000E+00 | 7.273663E+00 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 45

ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD

SOLUTION USING: DIRECT FREQUENCY RESPONSE

LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.9000E+00)

| GRID ID           |      | T1           | T2           | T3           |
|-------------------|------|--------------|--------------|--------------|
|                   |      | R1           | R2           | R3           |
| (MAGNITUDE/PHASE) |      |              |              |              |
| 1                 | MAGN | 0.000000E+00 | 0.000000E+00 | 1.000000E+00 |

|   |       |              |              |              |
|---|-------|--------------|--------------|--------------|
|   | MAGN  | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 3.599994E+02 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 5 | MAGN  | 0.000000E+00 | 0.000000E+00 | 1.806088E+01 |
|   | MAGN  | 0.000000E+00 | 8.758632E-03 | 0.000000E+00 |
|   | PHASE | 0.000000E+00 | 0.000000E+00 | 1.867549E+02 |
|   | PHASE | 0.000000E+00 | 6.381176E+00 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 46  
 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 9.9500E+00)

| GRID ID |       | T1                | T2           | T3           |
|---------|-------|-------------------|--------------|--------------|
|         |       | R1                | R2           | R3           |
|         |       | (MAGNITUDE/PHASE) |              |              |
| 1       | MAGN  | 0.000000E+00      | 0.000000E+00 | 1.000000E+00 |
|         | MAGN  | 0.000000E+00      | 0.000000E+00 | 0.000000E+00 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 3.599994E+02 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 0.000000E+00 |
| 5       | MAGN  | 0.000000E+00      | 0.000000E+00 | 1.615557E+01 |
|         | MAGN  | 0.000000E+00      | 7.886222E-03 | 0.000000E+00 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 1.860516E+02 |
|         | PHASE | 0.000000E+00      | 5.679166E+00 | 0.000000E+00 |

1GENESIS VERSION 7.2 DATE 12-13-2002 TIME 12:50 PAGE 47  
 ENFORCED DYNAMIC DISPLACEMENT OF BUILT-IN END OF A CANTILEVER BEAM  
 EXAMPLE USING LAGRANGE MULTIPLIER METHOD  
 SOLUTION USING: DIRECT FREQUENCY RESPONSE LOADCASE 3

GRID DYNAMIC DISPLACEMENTS (FREQUENCY = 1.0000E+01)

| GRID ID |       | T1                | T2           | T3           |
|---------|-------|-------------------|--------------|--------------|
|         |       | R1                | R2           | R3           |
|         |       | (MAGNITUDE/PHASE) |              |              |
| 1       | MAGN  | 0.000000E+00      | 0.000000E+00 | 1.000000E+00 |
|         | MAGN  | 0.000000E+00      | 0.000000E+00 | 0.000000E+00 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 3.599994E+02 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 0.000000E+00 |
| 5       | MAGN  | 0.000000E+00      | 0.000000E+00 | 1.461655E+01 |
|         | MAGN  | 0.000000E+00      | 7.181528E-03 | 0.000000E+00 |
|         | PHASE | 0.000000E+00      | 0.000000E+00 | 1.854838E+02 |
|         | PHASE | 0.000000E+00      | 5.112696E+00 | 0.000000E+00 |

1 \*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.37 Random Response of a Simple Spring/Mass/Damper Structure

### Example ID:

A037

### Analysis Data:

GRID, CELAS2 CDAMP2, CONM2, SPC, FREQ1, RLOAD1, DAREA, TABLED1, RANDOM, RANDT1, TABRND1

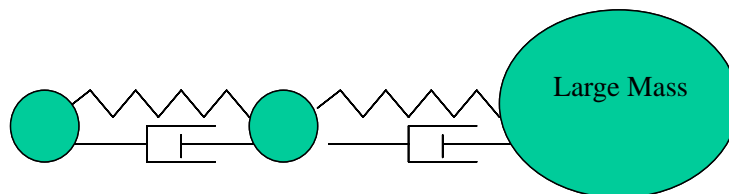
### Special Features Used:

Random response using large mass method.

### Problem Statement:

A simple structure assembled with elastics, damping and mass elements. Calculate the random responses of the system subjected to random loads.

### Structural Analysis Model:



### Analysis Model Description:

1. Three mass connected with two springs and two dampers.

### Random Analysis Data Relationships:



### Special Modeling Techniques:

The Large Mass Method is used to apply an enforced dynamic motion.

## Reference Solution:

None.

## Calculated Solution:

1. Power spectral density functions (PSDF) ,  $S_j(\omega)$
2. Autocorrelation function (ATOC),  $R_j(t)$
3. Root mean square (RMS),  $u_j$
4. Cumulative root mean square (CRMS),  $u_j(\omega)$
5. Number of zero crossings,  $N_0$

The above analyses are calculate for the following user requested responses:

1. Displacements
2. Velocities
3. Accelerations
4. Forces in CELAS2 and CDAMP2
5. Stresses CELAS2

The above results are printed in the A037.out file.

---

## 2.37.1 Input Data

```
ID A037
SOL COMPAT1
CEND
$
TITLE=RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE
SUBTITLE=LARGE MASS METHOD TO ENFORCE DISPLACEMENTS AT BASE
LINE=64,80
ECHO=NONE
$
FREQUENCY = 1
RANDOM=1
$
SUBCASE 1
    LABEL = FORCE IN GRID 2
    DLOAD = 1
    DISP(SORT2,RPRINT,PSDF,CRMS) = ALL
    VELO(SORT2,RPRINT,PSDF,CRMS) = ALL
    ACCE(SORT2,RPRINT,PSDF,CRMS) = ALL
    FORCE(SORT2,RPRINT,PSDF,CRMS) = ALL
    STRESS(SORT2,RPRINT,PSDF,CRMS) = ALL
$
SUBCASE 2
    LABEL = ENFORCED DISPLACEMENT AT GRID 1 (LARGE MASS
    LOCATION)
    DLOAD = 2
$
BEGIN BULK
$
$ --- ANALYSIS DATA
$
GRID1          0.00.0          0.0          23456
GRID2          1.00.0          0.0          23456
GRID3          2.00.0          0.0          23456
$
CONM21 1          1.0+8
CONM22 2          1.0
CONM23 3          1.0
$
CELAS2  4          5.0E61121          1.0
CELAS2  5          1.0E72131          1.0
```



```

$
CDAMP2  6      10.001121
CDAMP2  7      50.002131
$
$   FREQUENCIES
$
FREQ1   1      200.010.020
$
$ LOAD DEFINITION FOR DYNAMIC LOADCASE 1
$
DAREA102      1100.0
RLOAD1110      10
TABLED110
+1.0 1.0      10000.01.0      ENDT
$
$ LOAD DEFINITION FOR DYNAMIC LOADCASE 2
$
DAREA201      11.0E8
RLOAD1220      20
TABLED4200.0    1.01.0      10000.0
+0.0 0.0      40.0ENDT
$
$RANDOM ANALYSIS DATA
$
RANDPS11      11.0E5      30
RANDPS12      21.0E-4      30
RANDPS11      22.0      30
$
TABRND130
+0.0 0.0      199.990.0      200.0      1.0      400.0
      1.0
+400.010.0      10000.00.0      ENDT
$
RANDT11100      0.0.1
$
ENDDATA

```

---

## 2.37.2 Output Data

1

```
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N  E      S      I      S
G  GG  EEEE   N N N  EEEE   SSSS   I      SSSS
G  G   E      N  NN  E      S      I      S
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 8.0

BUILD CODE 200512131710

PROJECT NAME: A037  
RUN STARTED: Dec 13, 2005 18:22  
SYSTEM TYPE: ix86 (or compatible) Windows NT  
LENVEC: 11000000

LICENSED TO: Vanderplaats R&D, Inc.  
LICENSE EXPIRES: Dec 31, 2020

INFORMATIONAL MESSAGE FROM SUBROUTINE GN12E2

CELAS1 AND PELAS DATA WERE CREATED TO REPLACE CELAS2  
DATA. THE ID USED FOR CELAS1 IS THE SAME AS THE CELAS2.  
THE ID OF PELAS COULD BE EITHER THE SAME AS THE CELAS2  
OR AN ARBITRARY NUMBER.  
THE ID USED.  
NOTE: THE USEFULNESS OF THE PELAS ID IS THAT WITH IT

THE PROPERTIES OF THE CELAS2 ELEMENT CAN BE DESIGNED  
BY REFERENCING THE PELAS.

TOTAL NUMBER OF PELAS CREATED FOR CELAS2: 2  
TOTAL NUMBER OF PELAS CREATED FOR CELAS2 THAT USE CELAS2 ID: 2  
TOTAL NUMBER OF PELAS CREATED FOR CELAS2 THAT DO NOT USE CELAS2 ID: 0

INFORMATIONAL MESSAGE FROM SUBROUTINE GN12D2

CDAMP1 AND PDAMP DATA WERE CREATED TO REPLACE CDAMP2  
DATA. THE ID USED FOR CDAMP1 IS THE SAME AS THE CDAMP2.  
THE ID OF PDAMP COULD BE EITHER THE SAME AS THE CDAMP2  
OR AN ARBITRARY NUMBER.  
THE ID USED.

NOTE: THE USEFULNESS OF THE PDAMP ID IS THAT WITH IT  
THE PROPERTIES OF THE CDAMP2 ELEMENT CAN BE DESIGNED  
BY REFERENCING THE PDAMP.

TOTAL NUMBER OF PDAMP CREATED FOR CDAMP2: 2  
TOTAL NUMBER OF PDAMP CREATED FOR CDAMP2 THAT USE CDAMP2 ID: 2  
TOTAL NUMBER OF PDAMP CREATED FOR CDAMP2 THAT DO NOT USE CDAMP2 ID: 0

1GENESIS VERSION 8.0 DATE 12-13-2005 TIME 18:22 PAGE 1  
RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE  
LARGE MASS METHOD TO ENFORCE DISPLACEMENTS AT BASE

#### ANALYSIS PROBLEM SUMMARY

|                                     |   |
|-------------------------------------|---|
| NUMBER OF GRID POINTS:              | 3 |
| NUMBER OF CELAS1/2 ELEMENTS:        | 2 |
| NUMBER OF CONM2 ELEMENTS:           | 3 |
| NUMBER OF CDAMP1/2 ELEMENTS:        | 2 |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 7 |
| NUMBER OF ELEMENT PROPERTIES:       | 4 |
| NUMBER OF DEGREES OF FREEDOM:       | 3 |

#### LOAD CASES SUMMARY

|                                      |   |
|--------------------------------------|---|
| NUMBER OF DIRECT DYNAMIC LOAD CASES: | 2 |
| TOTAL NUMBER OF LOAD CASES:          | 2 |

1GENESIS VERSION 8.0 DATE 12-13-2005 TIME 18:22 PAGE 2  
RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE  
LARGE MASS METHOD TO ENFORCE DISPLACEMENTS AT BASE

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS SUMMARY)\*  
\*\*\*\*\*

1GENESIS VERSION 8.0 DATE 12-13-2005 TIME 18:22 PAGE 3  
RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE  
LARGE MASS METHOD TO ENFORCE DISPLACEMENTS AT BASE

# M A S S / V O L U M E   S U M M A R Y

```

SYSTEM MASS                1.000000E+08
SYSTEM VOLUME              0.000000E+00
1GENESIS  VERSION  8.0                      DATE 12-13-2005  TIME 18:22  PAGE    4
RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE
LARGE MASS METHOD TO ENFORCE DISPLACEMENTS AT BASE

```

# E N C L O S I N G   B O X   S U M M A R Y

```

LENGTH X  LENGTH Y  LENGTH Z  AREA XY  AREA YZ  AREA XZ  VOLUME XYZ
2.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00

1GENESIS  VERSION  8.0                      DATE 12-13-2005  TIME 18:22  PAGE    5
RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE
LARGE MASS METHOD TO ENFORCE DISPLACEMENTS AT BASE

```

```

*****
*   D E S I G N   C Y C L E           0 (ANALYSIS)*
*****

```

```

1GENESIS  VERSION  8.0                      DATE 12-13-2005  TIME 18:22  PAGE    6
RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE
LARGE MASS METHOD TO ENFORCE DISPLACEMENTS AT BASE
FORCE IN GRID 2                                LOADCASE          1

```

-----  
Dynamic responses have been deleted to save space  
-----

```

1GENESIS  VERSION  8.0                      DATE 12-13-2005  TIME 18:22  PAGE  132
RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

```

```

                                RANDOM          1
POINT-ID =                      1

```

D I S P L A C E M E N T   V E C T O R  
( P O W E R S P E C T R A L D E N S I T Y F U N C T I O N )

| FREQUENCY    | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|--------------|------|--------------|--------------|--------------|
| 2.000000E+02 | G    | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 | G    | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 | G    | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 | G    | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 | G    | 1.026599E-04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 | G    | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 | G    | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 | G    | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 | G    | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 1.026598E-04 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

1GENESIS VERSION 8.0 DATE 12-13-2005 TIME 18:22 PAGE 133  
RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 2

D I S P L A C E M E N T V E C T O R  
( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY      | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------------|------|--------------|--------------|--------------|
| 2.000000E+02 G |      | 6.446210E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 1.162229E-03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G |      | 2.910726E-03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G |      | 2.071692E-02 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G |      | 3.234677E-02 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G |      | 2.787265E-03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G |      | 8.986428E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G |      | 4.182093E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G |      | 2.318012E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G |      | 1.422467E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G |      | 9.324227E-05 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 6.396847E-05 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 4.534328E-05 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 3.291870E-05 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 2.432090E-05 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 1.819596E-05 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 1.372970E-05 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 1.041094E-05 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 7.907064E-06 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 5.994981E-06 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 4.521221E-06 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 3

DISPLACEMENT VECTOR  
( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY      | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------------|------|--------------|--------------|--------------|
| 2.000000E+02 G |      | 9.090424E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 1.703836E-03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G |      | 4.448100E-03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G |      | 3.309664E-02 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G |      | 5.418760E-02 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G |      | 4.912111E-03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G |      | 1.671880E-03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G |      | 8.244351E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G |      | 4.861441E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G |      | 3.187587E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G |      | 2.243112E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G |      | 1.660578E-04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G |      | 1.277374E-04 | 0.000000E+00 | 0.000000E+00 |

|                |              | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|----------------|--------------|--------------|--------------|--------------|
| 3.300000E+02 G | 1.012684E-04 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 8.227267E-05 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 6.821438E-05 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 5.754452E-05 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 4.927465E-05 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 4.275074E-05 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 3.752657E-05 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 3.328962E-05 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

# DISPLACEMENT VECTOR ( ROOT MEAN SQUARE )

| POINT-ID | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------|------|--------------|--------------|--------------|
| 1        | G    | 1.432898E-01 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2        | G    | 7.890210E-01 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3        | G    | 1.015272E+00 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

# DISPLACEMENT VECTOR ( NUMBER OF ZERO CROSSINGS )

| POINT-ID | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------|------|--------------|--------------|--------------|
| 1        | G    | 3.055323E+02 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2        | G    | 2.368747E+02 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3        | G    | 2.378739E+02 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 1

# DISPLACEMENT VECTOR ( CUMULATIVE ROOT MEAN SQUARE )

| FREQUENCY | TYPE | T1 | T2 | T3 |
|-----------|------|----|----|----|
|-----------|------|----|----|----|

|                | R1           | R2           | R3           |
|----------------|--------------|--------------|--------------|
| 2.000000E+02 G | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G | 3.204057E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G | 4.531221E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G | 5.549589E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G | 6.408114E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G | 7.164489E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G | 7.848305E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G | 8.477139E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 9.062442E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 9.612171E-02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 1.013212E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 1.062666E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 1.109918E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 1.155239E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 1.198848E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 1.240926E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 1.281623E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 1.321067E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 1.359366E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 1.396616E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 1.432898E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 2

DISPLACEMENT VECTOR

( CUMULATIVE ROOT MEAN SQUARE )

| FREQUENCY      | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------------|------|--------------|--------------|--------------|
| 2.000000E+02 G |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 9.504867E-02 | 0.000000E+00 | 0.000000E+00 |



|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G | 1.714614E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G | 3.841058E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G | 6.425385E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G | 7.671544E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G | 7.790734E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G | 7.832877E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 7.853596E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 7.865494E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 7.872975E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 7.877966E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 7.881434E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 7.883916E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 7.885731E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 7.887079E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 7.888091E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 7.888855E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 7.889436E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 7.889876E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 7.890210E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM

1

POINT-ID =

3

DISPLACEMENT VECTOR  
( CUMULATIVE ROOT MEAN SQUARE )

| FREQUENCY      | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------------|------|--------------|--------------|--------------|
| 2.000000E+02 G |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 1.142996E-01 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G |      | 2.093420E-01 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G |      | 4.811941E-01 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G | 8.172937E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G | 9.815639E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G | 9.981921E-01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G | 1.004425E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 1.007681E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 1.009676E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 1.011020E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 1.011985E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 1.012711E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 1.013276E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 1.013728E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 1.014099E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 1.014410E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 1.014673E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 1.014899E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 1.015097E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 1.015272E+00 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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 RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 1  
 V E L O C I T Y V E C T O R  
 ( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY    | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|--------------|------|--------------|--------------|--------------|
| 2.000000E+02 | G    | 1.621139E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 | G    | 1.787305E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 | G    | 1.961578E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 | G    | 2.143955E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 | G    | 2.334442E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 | G    | 2.533030E+02 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G | 2.739725E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G | 2.954526E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 3.177433E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 3.408445E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 3.647563E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 3.894786E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 4.150116E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 4.413551E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 4.685092E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 4.964738E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 5.252490E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 5.548348E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 5.852311E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 6.164381E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 6.484556E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 2

V E L O C I T Y V E C T O R

( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY      | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------------|------|--------------|--------------|--------------|
| 2.000000E+02 G |      | 1.017945E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 2.023438E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G |      | 5.561687E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G |      | 4.326539E+04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G |      | 7.355515E+04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G |      | 6.877302E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G |      | 2.398245E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G |      | 1.203596E+03 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 7.174498E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 4.722781E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 3.312952E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 2.426885E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 1.833043E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 1.415241E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 1.109934E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 8.799758E+01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 7.024666E+01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 5.626694E+01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 4.507567E+01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 3.599787E+01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 2.855850E+01 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 3  
VELOCITY VECTOR  
( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY      | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------------|------|--------------|--------------|--------------|
| 2.000000E+02 G |      | 1.435502E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 2.966375E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G |      | 8.499230E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G |      | 6.911930E+04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G |      | 1.232203E+05 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G |      | 1.212015E+04 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G |      | 4.461814E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G |      | 2.372705E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G |      | 1.504668E+03 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G |      | 1.058322E+03 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 7.969907E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 6.300028E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 5.163900E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 4.353732E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 3.754682E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 3.298920E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 2.944210E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 2.663095E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 2.437084E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 2.253346E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 2.102754E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

# VELOCITY VECTOR ( ROOT MEAN SQUARE )

| POINT-ID | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------|------|--------------|--------------|--------------|
| 1        | G    | 1.015272E+00 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2        | G    | 1.015272E+00 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3        | G    | 1.015272E+00 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

# VELOCITY VECTOR ( NUMBER OF ZERO CROSSINGS )

| POINT-ID | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------|------|--------------|--------------|--------------|
| 1        | G    | 2.378739E+02 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2        | G    | 2.378739E+02 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3        | G    | 2.378739E+02 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

POINT-ID = 1  
 V E L O C I T Y V E C T O R  
 ( CUMULATIVE ROOT MEAN SQUARE )

| FREQUENCY    | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|--------------|------|--------------|--------------|--------------|
| 2.000000E+02 | G    | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 | G    | 4.128223E+01 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 | G    | 5.982193E+01 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 | G    | 7.504285E+01 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 | G    | 8.871656E+01 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 | G    | 1.015104E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 | G    | 1.137574E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 | G    | 1.256498E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 | G    | 1.373093E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 | G    | 1.488180E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 | G    | 1.602336E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 | G    | 1.715983E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 | G    | 1.829438E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 | G    | 1.942942E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 | G    | 2.056686E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 | G    | 2.170817E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 | G    | 2.285456E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 | G    | 2.400698E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 | G    | 2.516622E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 | G    | 2.633291E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 | G    | 2.750757E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

POINT-ID = 2  
 V E L O C I T Y V E C T O R  
 ( CUMULATIVE ROOT MEAN SQUARE )

| FREQUENCY    | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|--------------|------|--------------|--------------|--------------|
| 2.000000E+02 | G    | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 | G    | 1.233163E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 | G    | 2.305050E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 | G    | 5.452228E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 | G    | 9.388134E+02 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 | G    | 1.132931E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 | G    | 1.153218E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 | G    | 1.161000E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 | G    | 1.165129E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 | G    | 1.167679E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 | G    | 1.169398E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 | G    | 1.170624E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 | G    | 1.171534E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 | G    | 1.172227E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 | G    | 1.172765E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 | G    | 1.173189E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 | G    | 1.173526E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 | G    | 1.173796E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 | G    | 1.174012E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 | G    | 1.174184E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 | G    | 1.174322E+03 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 3  
VELOCITY VECTOR  
( CUMULATIVE ROOT MEAN SQUARE )

| FREQUENCY    | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|--------------|------|--------------|--------------|--------------|
| 2.000000E+02 | G    | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G | 1.483556E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G | 2.816690E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G | 6.836886E+02 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G | 1.195461E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G | 1.451148E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G | 1.479439E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G | 1.490943E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 1.497431E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 1.501704E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 1.504789E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 1.507158E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 1.509059E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 1.510635E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 1.511976E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 1.513142E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 1.514173E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 1.515098E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 1.515940E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 1.516713E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 1.517431E+03 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 1

A C C E L E R A T I O N V E C T O R

( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY      | TYPE | T1           | T2           | T3           |
|----------------|------|--------------|--------------|--------------|
|                |      | R1           | R2           | R3           |
| 2.000000E+02 G |      | 2.560000E+08 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 3.111696E+08 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G |      | 3.748095E+08 | 0.000000E+00 | 0.000000E+00 |



|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G | 4.477454E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G | 5.308420E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G | 6.250001E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G | 7.311617E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G | 8.503057E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 9.834497E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 1.131650E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 1.296000E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 1.477634E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 1.677722E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 1.897474E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 2.138138E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 2.401000E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 2.687386E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 2.998658E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 3.336218E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 3.701506E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 4.096000E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 2

A C C E L E R A T I O N V E C T O R  
( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY      | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------------|------|--------------|--------------|--------------|
| 2.000000E+02 G |      | 1.607474E+09 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 3.522802E+09 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G |      | 1.062702E+10 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G |      | 9.035579E+10 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G |      | 1.672612E+11 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G | 1.696906E+10 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G | 6.400294E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G | 3.463923E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 2.220584E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 1.568027E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 1.177111E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 9.207299E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 7.410240E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 6.084403E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 5.065413E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 4.255657E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 3.594102E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 3.041001E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 2.569621E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 2.161552E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 1.803911E+08 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

POINT-ID = 3

A C C E L E R A T I O N V E C T O R  
( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY      | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------------|------|--------------|--------------|--------------|
| 2.000000E+02 G |      | 2.266854E+09 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 G |      | 5.164453E+09 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 G |      | 1.623995E+10 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 G |      | 1.443493E+11 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 G |      | 2.801976E+11 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 G |      | 2.990527E+10 | 0.000000E+00 | 0.000000E+00 |
|                |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 G |      | 1.190743E+10 | 0.000000E+00 | 0.000000E+00 |

|                |              |              |              |
|----------------|--------------|--------------|--------------|
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 G | 6.828590E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 G | 4.657112E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 G | 3.513771E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 G | 2.831754E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 G | 2.390152E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 G | 2.087553E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 G | 1.871757E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 G | 1.713526E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 G | 1.595393E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 G | 1.506376E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 G | 1.439295E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 G | 1.389305E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 G | 1.353059E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 G | 1.328215E+09 | 0.000000E+00 | 0.000000E+00 |
|                | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

# A C C E L E R A T I O N V E C T O R ( R O O T M E A N S Q U A R E )

| POINT-ID | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------|------|--------------|--------------|--------------|
| 1        | G    | 5.636831E+05 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2        | G    | 1.757267E+06 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3        | G    | 2.286349E+06 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

# A C C E L E R A T I O N V E C T O R ( N U M B E R O F Z E R O C R O S S I N G S )

| POINT-ID | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|----------|------|--------------|--------------|--------------|
| 1        | G    | 3.423308E+02 | 0.000000E+00 | 0.000000E+00 |
|          |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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      2      G      2.400047E+02      0.000000E+00      0.000000E+00
      3      G      2.430016E+02      0.000000E+00      0.000000E+00
      0.000000E+00      0.000000E+00      0.000000E+00
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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

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                                RANDOM                                1
POINT-ID =                      1
      A C C E L E R A T I O N    V E C T O R
      ( C U M U L A T I V E R O O T M E A N S Q U A R E )

```

| FREQUENCY    | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|--------------|------|--------------|--------------|--------------|
| 2.000000E+02 | G    | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 | G    | 5.325268E+04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 | G    | 7.915645E+04 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 | G    | 1.018750E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 | G    | 1.235777E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 | G    | 1.450885E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 | G    | 1.668277E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 | G    | 1.890471E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 | G    | 2.119141E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 | G    | 2.355485E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 | G    | 2.600410E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 | G    | 2.854637E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 | G    | 3.118754E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 | G    | 3.393262E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 | G    | 3.678591E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 | G    | 3.975122E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 | G    | 4.283199E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 | G    | 4.603131E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 | G    | 4.935205E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 | G    | 5.279689E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 | G    | 5.636831E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

RANDOM 1

POINT-ID = 2

A C C E L E R A T I O N V E C T O R  
( C U M U L A T I V E R O O T M E A N S Q U A R E )

| FREQUENCY    | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|--------------|------|--------------|--------------|--------------|
| 2.000000E+02 | G    | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 | G    | 1.601605E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 | G    | 3.104843E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 | G    | 7.754448E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 | G    | 1.374554E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 | G    | 1.676470E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 | G    | 1.710964E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 | G    | 1.725317E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 | G    | 1.733534E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 | G    | 1.738990E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 | G    | 1.742932E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 | G    | 1.745938E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 | G    | 1.748316E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 | G    | 1.750244E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 | G    | 1.751836E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 | G    | 1.753166E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 | G    | 1.754285E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 | G    | 1.755230E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 | G    | 1.756029E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 | G    | 1.756703E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 | G    | 1.757267E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

RANDOM 1

POINT-ID = 3

| FREQUENCY    | TYPE | T1<br>R1     | T2<br>R2     | T3<br>R3     |
|--------------|------|--------------|--------------|--------------|
| 2.000000E+02 | G    | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.100000E+02 | G    | 1.927603E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.200000E+02 | G    | 3.797085E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.300000E+02 | G    | 9.732034E+05 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.400000E+02 | G    | 1.752101E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.500000E+02 | G    | 2.149505E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.600000E+02 | G    | 2.197598E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.700000E+02 | G    | 2.218810E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.800000E+02 | G    | 2.231714E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 2.900000E+02 | G    | 2.240848E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.000000E+02 | G    | 2.247916E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.100000E+02 | G    | 2.253716E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.200000E+02 | G    | 2.258678E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.300000E+02 | G    | 2.263056E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.400000E+02 | G    | 2.267013E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.500000E+02 | G    | 2.270659E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.600000E+02 | G    | 2.274072E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.700000E+02 | G    | 2.277308E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.800000E+02 | G    | 2.280411E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 3.900000E+02 | G    | 2.283415E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |
| 4.000000E+02 | G    | 2.286349E+06 | 0.000000E+00 | 0.000000E+00 |
|              |      | 0.000000E+00 | 0.000000E+00 | 0.000000E+00 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM                      1

ELEMENT-ID =                 6

F O R C E S   I N   S C A L A R   D A M P E R S

( POWER SPECTRAL DENSITY FUNCTION )

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 5.453018E+04 | 2.100000E+02 | 1.262520E+05 |
| 2.200000E+02 | 4.126058E+05 | 2.300000E+02 | 3.873134E+06 |
| 2.400000E+02 | 8.022849E+06 | 2.500000E+02 | 9.191536E+05 |
| 2.600000E+02 | 3.938394E+05 | 2.700000E+02 | 2.430640E+05 |
| 2.800000E+02 | 1.781116E+05 | 2.900000E+02 | 1.439948E+05 |
| 3.000000E+02 | 1.239155E+05 | 3.100000E+02 | 1.112498E+05 |
| 3.200000E+02 | 1.029229E+05 | 3.300000E+02 | 9.733413E+04 |
| 3.400000E+02 | 9.357666E+04 | 3.500000E+02 | 9.110141E+04 |
| 3.600000E+02 | 8.955787E+04 | 3.700000E+02 | 8.871218E+04 |
| 3.800000E+02 | 8.840232E+04 | 3.900000E+02 | 8.851228E+04 |
| 4.000000E+02 | 8.895648E+04 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 7  
FORCES IN SCALAR DAMPERS  
( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY    | FORCE        | FREQUENCY    | FORCE        |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 8.948829E+04 | 2.100000E+02 | 2.247728E+05 |
| 2.200000E+02 | 7.757273E+05 | 2.300000E+02 | 7.536114E+06 |
| 2.400000E+02 | 1.592802E+07 | 2.500000E+02 | 1.844594E+06 |
| 2.600000E+02 | 7.943929E+05 | 2.700000E+02 | 4.912777E+05 |
| 2.800000E+02 | 3.603287E+05 | 2.900000E+02 | 2.916306E+05 |
| 3.000000E+02 | 2.515123E+05 | 3.100000E+02 | 2.266770E+05 |
| 3.200000E+02 | 2.109567E+05 | 3.300000E+02 | 2.011548E+05 |
| 3.400000E+02 | 1.954784E+05 | 3.500000E+02 | 1.928639E+05 |
| 3.600000E+02 | 1.926561E+05 | 3.700000E+02 | 1.944439E+05 |
| 3.800000E+02 | 1.979714E+05 | 3.900000E+02 | 2.030863E+05 |
| 4.000000E+02 | 2.097101E+05 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 4  
FORCES IN SCALAR SPRINGS  
( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY    | FORCE        | FREQUENCY    | FORCE        |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 8.632909E+09 | 2.100000E+02 | 1.812925E+10 |
| 2.200000E+02 | 5.398465E+10 | 2.300000E+02 | 4.636466E+11 |
| 2.400000E+02 | 8.820362E+11 | 2.500000E+02 | 9.312973E+10 |
| 2.600000E+02 | 3.689375E+10 | 2.700000E+02 | 2.111414E+10 |
| 2.800000E+02 | 1.438655E+10 | 2.900000E+02 | 1.084254E+10 |
| 3.000000E+02 | 8.718934E+09 | 3.100000E+02 | 7.330883E+09 |
| 3.200000E+02 | 6.364910E+09 | 3.300000E+02 | 5.660014E+09 |
| 3.400000E+02 | 5.126134E+09 | 3.500000E+02 | 4.709440E+09 |
| 3.600000E+02 | 4.376017E+09 | 3.700000E+02 | 4.103553E+09 |
| 3.800000E+02 | 3.876830E+09 | 3.900000E+02 | 3.685145E+09 |
| 4.000000E+02 | 3.520772E+09 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 5  
FORCES IN SCALAR SPRINGS  
( POWER SPECTRAL DENSITY FUNCTION )

| FREQUENCY    | FORCE        | FREQUENCY    | FORCE        |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 2.266765E+09 | 2.100000E+02 | 5.164229E+09 |
| 2.200000E+02 | 1.623918E+10 | 2.300000E+02 | 1.443418E+11 |
| 2.400000E+02 | 2.801817E+11 | 2.500000E+02 | 2.990343E+10 |
| 2.600000E+02 | 1.190663E+10 | 2.700000E+02 | 6.828099E+09 |
| 2.800000E+02 | 4.656751E+09 | 2.900000E+02 | 3.513479E+09 |
| 3.000000E+02 | 2.831502E+09 | 3.100000E+02 | 2.389926E+09 |
| 3.200000E+02 | 2.087342E+09 | 3.300000E+02 | 1.871555E+09 |
| 3.400000E+02 | 1.713331E+09 | 3.500000E+02 | 1.595200E+09 |
| 3.600000E+02 | 1.506184E+09 | 3.700000E+02 | 1.439101E+09 |
| 3.800000E+02 | 1.389107E+09 | 3.900000E+02 | 1.352856E+09 |
| 4.000000E+02 | 1.328005E+09 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

F O R C E S I N S C A L A R D A M P E R S  
( R O O T M E A N S Q U A R E )

| ELEMENT-ID | FORCE        | ELEMENT-ID | FORCE        |
|------------|--------------|------------|--------------|
| 6          | 1.239356E+04 | 7          | 1.745373E+04 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

F O R C E S I N S C A L A R S P R I N G S  
( R O O T M E A N S Q U A R E )

| ELEMENT-ID | FORCE        | ELEMENT-ID | FORCE        |
|------------|--------------|------------|--------------|
| 4          | 4.067176E+06 | 5          | 2.286282E+06 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

F O R C E S I N S C A L A R D A M P E R S  
( N U M B E R O F Z E R O C R O S S I N G S )

| ELEMENT-ID | FORCE        | ELEMENT-ID | FORCE        |
|------------|--------------|------------|--------------|
| 6          | 2.478490E+02 | 7          | 2.487480E+02 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

F O R C E S I N S C A L A R S P R I N G S  
( N U M B E R O F Z E R O C R O S S I N G S )

| ELEMENT-ID | FORCE        | ELEMENT-ID | FORCE        |
|------------|--------------|------------|--------------|
| 4          | 2.424897E+02 | 5          | 2.430012E+02 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 6

F O R C E S I N S C A L A R D A M P E R S  
( C U M U L A T I V E R O O T M E A N S Q U A R E )



| FREQUENCY    | FORCE        | FREQUENCY    | FORCE        |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 0.000000E+00 | 2.100000E+02 | 9.507422E+02 |
| 2.200000E+02 | 1.641719E+03 | 2.300000E+02 | 4.629291E+03 |
| 2.400000E+02 | 7.712622E+03 | 2.500000E+02 | 6.687131E+03 |
| 2.600000E+02 | 2.563523E+03 | 2.700000E+02 | 1.785240E+03 |
| 2.800000E+02 | 1.451779E+03 | 2.900000E+02 | 1.269639E+03 |
| 3.000000E+02 | 1.157938E+03 | 3.100000E+02 | 1.084889E+03 |
| 3.200000E+02 | 1.035349E+03 | 3.300000E+02 | 1.001160E+03 |
| 3.400000E+02 | 9.775250E+02 | 3.500000E+02 | 9.614405E+02 |
| 3.600000E+02 | 9.509247E+02 | 3.700000E+02 | 9.446170E+02 |
| 3.800000E+02 | 9.415504E+02 | 3.900000E+02 | 9.410178E+02 |
| 4.000000E+02 | 9.424886E+02 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 7  
FORCES IN SCALAR DAMPERS  
( CUMULATIVE ROOT MEAN SQUARE )

| FREQUENCY    | FORCE        | FREQUENCY    | FORCE        |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 0.000000E+00 | 2.100000E+02 | 1.253517E+03 |
| 2.200000E+02 | 2.236907E+03 | 2.300000E+02 | 6.446817E+03 |
| 2.400000E+02 | 1.083177E+04 | 2.500000E+02 | 9.427297E+03 |
| 2.600000E+02 | 3.633780E+03 | 2.700000E+02 | 2.536136E+03 |
| 2.800000E+02 | 2.064114E+03 | 2.900000E+02 | 1.806062E+03 |
| 3.000000E+02 | 1.648490E+03 | 3.100000E+02 | 1.546802E+03 |
| 3.200000E+02 | 1.479769E+03 | 3.300000E+02 | 1.435979E+03 |
| 3.400000E+02 | 1.408759E+03 | 3.500000E+02 | 1.393958E+03 |
| 3.600000E+02 | 1.388882E+03 | 3.700000E+02 | 1.391722E+03 |
| 3.800000E+02 | 1.401238E+03 | 3.900000E+02 | 1.416577E+03 |
| 4.000000E+02 | 1.437149E+03 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 4  
FORCES IN SCALAR SPRINGS  
( CUMULATIVE ROOT MEAN SQUARE )

| FREQUENCY    | FORCE        | FREQUENCY    | FORCE        |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 0.000000E+00 | 2.100000E+02 | 3.658016E+05 |
| 2.200000E+02 | 6.004747E+05 | 2.300000E+02 | 1.608775E+06 |
| 2.400000E+02 | 2.593919E+06 | 2.500000E+02 | 2.208129E+06 |
| 2.600000E+02 | 8.062999E+05 | 2.700000E+02 | 5.385538E+05 |
| 2.800000E+02 | 4.213122E+05 | 2.900000E+02 | 3.551702E+05 |
| 3.000000E+02 | 3.127423E+05 | 3.100000E+02 | 2.832832E+05 |
| 3.200000E+02 | 2.616854E+05 | 3.300000E+02 | 2.452038E+05 |
| 3.400000E+02 | 2.322305E+05 | 3.500000E+02 | 2.217614E+05 |
| 3.600000E+02 | 2.131373E+05 | 3.700000E+02 | 2.059079E+05 |
| 3.800000E+02 | 1.997552E+05 | 3.900000E+02 | 1.944481E+05 |
| 4.000000E+02 | 1.898151E+05 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 5

F O R C E S   I N   S C A L A R   S P R I N G S  
( C U M U L A T I V E   R O O T   M E A N   S Q U A R E )

| FREQUENCY    | FORCE        | FREQUENCY    | FORCE        |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 0.000000E+00 | 2.100000E+02 | 1.927562E+05 |
| 2.200000E+02 | 3.271349E+05 | 2.300000E+02 | 8.960497E+05 |
| 2.400000E+02 | 1.456921E+06 | 2.500000E+02 | 1.245161E+06 |
| 2.600000E+02 | 4.572215E+05 | 2.700000E+02 | 3.060623E+05 |
| 2.800000E+02 | 2.396342E+05 | 2.900000E+02 | 2.021173E+05 |
| 3.000000E+02 | 1.781154E+05 | 3.100000E+02 | 1.615776E+05 |
| 3.200000E+02 | 1.496212E+05 | 3.300000E+02 | 1.406934E+05 |
| 3.400000E+02 | 1.338827E+05 | 3.500000E+02 | 1.286188E+05 |
| 3.600000E+02 | 1.245273E+05 | 3.700000E+02 | 1.213530E+05 |
| 3.800000E+02 | 1.189166E+05 | 3.900000E+02 | 1.170894E+05 |
| 4.000000E+02 | 1.157775E+05 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM                      1

ELEMENT-ID =                      4  
S T R E S S E S   I N   S C A L A R   S P R I N G S  
( P O W E R   S P E C T R A L   D E N S I T Y   F U N C T I O N )

| FREQUENCY    | STRESS       | FREQUENCY    | STRESS       |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 8.632909E+09 | 2.100000E+02 | 1.812925E+10 |
| 2.200000E+02 | 5.398465E+10 | 2.300000E+02 | 4.636466E+11 |
| 2.400000E+02 | 8.820362E+11 | 2.500000E+02 | 9.312973E+10 |
| 2.600000E+02 | 3.689375E+10 | 2.700000E+02 | 2.111414E+10 |
| 2.800000E+02 | 1.438655E+10 | 2.900000E+02 | 1.084254E+10 |
| 3.000000E+02 | 8.718934E+09 | 3.100000E+02 | 7.330883E+09 |
| 3.200000E+02 | 6.364910E+09 | 3.300000E+02 | 5.660014E+09 |
| 3.400000E+02 | 5.126134E+09 | 3.500000E+02 | 4.709440E+09 |
| 3.600000E+02 | 4.376017E+09 | 3.700000E+02 | 4.103553E+09 |
| 3.800000E+02 | 3.876830E+09 | 3.900000E+02 | 3.685145E+09 |
| 4.000000E+02 | 3.520772E+09 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM                      1

ELEMENT-ID =                      5  
S T R E S S E S   I N   S C A L A R   S P R I N G S  
( P O W E R   S P E C T R A L   D E N S I T Y   F U N C T I O N )

| FREQUENCY    | STRESS       | FREQUENCY    | STRESS       |
|--------------|--------------|--------------|--------------|
| 2.000000E+02 | 2.266765E+09 | 2.100000E+02 | 5.164229E+09 |
| 2.200000E+02 | 1.623918E+10 | 2.300000E+02 | 1.443418E+11 |
| 2.400000E+02 | 2.801817E+11 | 2.500000E+02 | 2.990343E+10 |
| 2.600000E+02 | 1.190663E+10 | 2.700000E+02 | 6.828099E+09 |
| 2.800000E+02 | 4.656751E+09 | 2.900000E+02 | 3.513479E+09 |
| 3.000000E+02 | 2.831502E+09 | 3.100000E+02 | 2.389926E+09 |
| 3.200000E+02 | 2.087342E+09 | 3.300000E+02 | 1.871555E+09 |
| 3.400000E+02 | 1.713331E+09 | 3.500000E+02 | 1.595200E+09 |
| 3.600000E+02 | 1.506184E+09 | 3.700000E+02 | 1.439101E+09 |
| 3.800000E+02 | 1.389107E+09 | 3.900000E+02 | 1.352856E+09 |
| 4.000000E+02 | 1.328005E+09 |              |              |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

S T R E S S E S   I N   S C A L A R   S P R I N G S  
( R O O T M E A N S Q U A R E )

|            |              |            |              |
|------------|--------------|------------|--------------|
| ELEMENT-ID | STRESS       | ELEMENT-ID | STRESS       |
| 4          | 4.067176E+06 | 5          | 2.286282E+06 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

S T R E S S E S   I N   S C A L A R   S P R I N G S  
( N U M B E R O F Z E R O C R O S S I N G S )

|            |              |            |              |
|------------|--------------|------------|--------------|
| ELEMENT-ID | STRESS       | ELEMENT-ID | STRESS       |
| 4          | 2.424897E+02 | 5          | 2.430012E+02 |

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RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 4

S T R E S S E S   I N   S C A L A R   S P R I N G S  
( C U M U L A T I V E R O O T M E A N S Q U A R E )

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| FREQUENCY    | STRESS       | FREQUENCY    | STRESS       |
| 2.000000E+02 | 0.000000E+00 | 2.100000E+02 | 3.658016E+05 |
| 2.200000E+02 | 6.004747E+05 | 2.300000E+02 | 1.608775E+06 |
| 2.400000E+02 | 2.593919E+06 | 2.500000E+02 | 2.208129E+06 |
| 2.600000E+02 | 8.062999E+05 | 2.700000E+02 | 5.385538E+05 |
| 2.800000E+02 | 4.213122E+05 | 2.900000E+02 | 3.551702E+05 |
| 3.000000E+02 | 3.127423E+05 | 3.100000E+02 | 2.832832E+05 |
| 3.200000E+02 | 2.616854E+05 | 3.300000E+02 | 2.452038E+05 |
| 3.400000E+02 | 2.322305E+05 | 3.500000E+02 | 2.217614E+05 |
| 3.600000E+02 | 2.131373E+05 | 3.700000E+02 | 2.059079E+05 |
| 3.800000E+02 | 1.997552E+05 | 3.900000E+02 | 1.944481E+05 |
| 4.000000E+02 | 1.898151E+05 |              |              |

1GENESIS VERSION 8.0 DATE 12-13-2005 TIME 18:22 PAGE 173

RANDOM RESPONSE OF A SIMPLE ELAS/DAMP/MASS STRUCTURE

RANDOM 1

ELEMENT-ID = 5

S T R E S S E S   I N   S C A L A R   S P R I N G S  
( C U M U L A T I V E R O O T M E A N S Q U A R E )

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| FREQUENCY    | STRESS       | FREQUENCY    | STRESS       |
| 2.000000E+02 | 0.000000E+00 | 2.100000E+02 | 1.927562E+05 |
| 2.200000E+02 | 3.271349E+05 | 2.300000E+02 | 8.960497E+05 |
| 2.400000E+02 | 1.456921E+06 | 2.500000E+02 | 1.245161E+06 |
| 2.600000E+02 | 4.572215E+05 | 2.700000E+02 | 3.060623E+05 |
| 2.800000E+02 | 2.396342E+05 | 2.900000E+02 | 2.021173E+05 |
| 3.000000E+02 | 1.781154E+05 | 3.100000E+02 | 1.615776E+05 |
| 3.200000E+02 | 1.496212E+05 | 3.300000E+02 | 1.406934E+05 |
| 3.400000E+02 | 1.338827E+05 | 3.500000E+02 | 1.286188E+05 |
| 3.600000E+02 | 1.245273E+05 | 3.700000E+02 | 1.213530E+05 |
| 3.800000E+02 | 1.189166E+05 | 3.900000E+02 | 1.170894E+05 |
| 4.000000E+02 | 1.157775E+05 |              |              |

\*\*\*\*\* THERE WERE 4 INFORMATION MESSAGES \*\*\*\*\*



---

## 2.38 Random Response of a Cantilever Plate

### Example ID:

A038

### Analysis Data:

GRID, CQUAD4, SPC, EIGR, FREQ1, RLOAD1, DAREA, TABLED1, TABDMP1, RANDPS, RANDOM, TABRND1

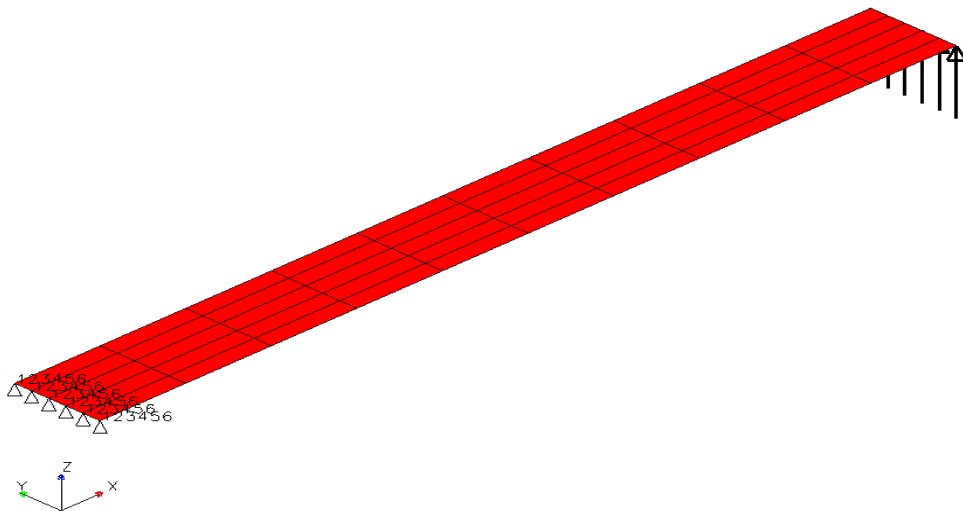
### Special Features Used:

Random response analysis.

### Problem Statement:

Find the random responses of a simple cantilevered plate assembled with quadrilateral elements.

### Structural Analysis Model:



### Analysis Model Description:

1. A simple cantilevered plate modeled with 50 quadrilateral elements.

## Random Analysis Data Relationships:

RANDOM → RANDPS → TABRND1

## Special Modeling Techniques:

None.

## Reference Solution:

None.

## Calculated Solution:

1. Power spectral density functions (PSDF) ,  $S_j(\omega)$
2. Root mean square (RMS),  $u_j$
3. Cumulative root mean square (CRMS),  $u_j(\omega)$
4. Number of zero crossings,  $N_0$

The above analyses are calculate for the following user requested responses:

1. Displacements
2. Velocities
3. Accelerations
4. Stresses CQUAD4

The above results are printed in the A038.out file.

---

## 2.38.1 Input Data

```
ID A038
SOL COMPAT1
CEND
$
TITLE = RANDOM RESPONSE OF A CANTILEVERED PLATE
LINE = 64,80
ECHO = NONE
SET 1 = 44
SET 2 = 21
$
FREQUENCY = 40
RANDOM = 30
$
SUBCASE 1
    LABEL = DYNAMIC LOADING AT THE FREE END OF THE PLATE
    SPC      = 1
    METHOD    = 1
    DLOAD    = 10
    SDAMPING = 20
    DISP(SORT2,RPRINT,PSDF,CRMS) = 1
    VELO(SORT2,RPRINT,PSDF,CRMS) = 1
    ACCE(SORT2,RPRINT,PSDF,CRMS) = 1
    STRESS(SORT2,RPRINT,PSDF,CRMS) = 2
$
BEGIN BULK
$
$ LOAD DEFINITION FOR DYNAMIC LOADING
$
RLOAD1      10      11      12
DAREA       11      66      3-.200000      11      3-.200000
DAREA       11      55      3-.400000      44      3-.400000
DAREA       11      33      3-.400000      22      3-.400000
TABLED1     12
+           2.0      1.0      200.0      1.0      ENDT
TABDMP1     20
+           2.0      0.05      200.0      0.05      ENDT
FREQ1       40      2.0      2.0      99
$
$ RANDOM ANALYSIS DATA
$
RANDPS      30      1      1      1.0      0.0      31
TABRND1     31
+           1.999      0.01      2.0      0.475      200.0      0.475      200.1      0.01+
+           ENDT
$
$ EIGEN VALUE DATA
$
EIGR        1      LAN      400.0000
```

```

$
$ LOADING DATA
$
FORCE          2      66      .2000000      0.0      0.0-1.00000
FORCE          2      11      .2000000      0.0      0.0-1.00000
FORCE          2      55      .4000000      0.0      0.0-1.00000
FORCE          2      44      .4000000      0.0      0.0-1.00000
FORCE          2      33      .4000000      0.0      0.0-1.00000
FORCE          2      22      .4000000      0.0      0.0-1.00000
$
$ BOUNDARY CONDITIONS
$
SPC             1      56  123456      0.0
SPC             1      45  123456      0.0
SPC             1      34  123456      0.0
SPC             1      23  123456      0.0
SPC             1      12  123456      0.0
SPC             1       1  123456      0.0
$
$ PROPERTIES
$
PSHELL          1      17.000000      1      1
$
$ MATERIAL
$
MAT1            168900.00      .30000002.8500-6
$
$ GRIDS
$
GRID            1           0.0      0.0      0.0
GRID            2      15.00000      0.0      0.0
GRID            3      30.00000      0.0      0.0
GRID            4      45.00000      0.0      0.0
GRID            5      60.00000      0.0      0.0
GRID            6      75.00000      0.0      0.0
GRID            7      90.00000      0.0      0.0
GRID            8     105.0000      0.0      0.0
GRID            9     120.0000      0.0      0.0
GRID           10     135.0000      0.0      0.0
GRID           11     150.0000      0.0      0.0
GRID           12           0.03.000000      0.0
GRID           13     15.000003.000000      0.0
GRID           14     30.000003.000000      0.0
GRID           15     45.000003.000000      0.0
GRID           16     60.000003.000000      0.0
GRID           17     75.000003.000000      0.0
GRID           18     90.000003.000000      0.0
GRID           19    105.00003.000000      0.0
GRID           20    120.00003.000000      0.0
GRID           21    135.00003.000000      0.0
GRID           22    150.00003.000000      0.0

```



|             |    |   |                    |     |    |    |     |
|-------------|----|---|--------------------|-----|----|----|-----|
| GRID        | 23 |   | 0.06.000000        | 0.0 |    |    |     |
| GRID        | 24 |   | 15.000006.000000   | 0.0 |    |    |     |
| GRID        | 25 |   | 30.000006.000000   | 0.0 |    |    |     |
| GRID        | 26 |   | 45.000006.000000   | 0.0 |    |    |     |
| GRID        | 27 |   | 60.000006.000000   | 0.0 |    |    |     |
| GRID        | 28 |   | 75.000006.000000   | 0.0 |    |    |     |
| GRID        | 29 |   | 90.000006.000000   | 0.0 |    |    |     |
| GRID        | 30 |   | 105.000006.000000  | 0.0 |    |    |     |
| GRID        | 31 |   | 120.000006.000000  | 0.0 |    |    |     |
| GRID        | 32 |   | 135.000006.000000  | 0.0 |    |    |     |
| GRID        | 33 |   | 150.000006.000000  | 0.0 |    |    |     |
| GRID        | 34 |   | 0.09.000000        | 0.0 |    |    |     |
| GRID        | 35 |   | 15.000009.000000   | 0.0 |    |    |     |
| GRID        | 36 |   | 30.000009.000000   | 0.0 |    |    |     |
| GRID        | 37 |   | 45.000009.000000   | 0.0 |    |    |     |
| GRID        | 38 |   | 60.000009.000000   | 0.0 |    |    |     |
| GRID        | 39 |   | 75.000009.000000   | 0.0 |    |    |     |
| GRID        | 40 |   | 90.000009.000000   | 0.0 |    |    |     |
| GRID        | 41 |   | 105.000009.000000  | 0.0 |    |    |     |
| GRID        | 42 |   | 120.000009.000000  | 0.0 |    |    |     |
| GRID        | 43 |   | 135.000009.000000  | 0.0 |    |    |     |
| GRID        | 44 |   | 150.000009.000000  | 0.0 |    |    |     |
| GRID        | 45 |   | 0.012.000000       | 0.0 |    |    |     |
| GRID        | 46 |   | 15.0000012.000000  | 0.0 |    |    |     |
| GRID        | 47 |   | 30.0000012.000000  | 0.0 |    |    |     |
| GRID        | 48 |   | 45.0000012.000000  | 0.0 |    |    |     |
| GRID        | 49 |   | 60.0000012.000000  | 0.0 |    |    |     |
| GRID        | 50 |   | 75.0000012.000000  | 0.0 |    |    |     |
| GRID        | 51 |   | 90.0000012.000000  | 0.0 |    |    |     |
| GRID        | 52 |   | 105.0000012.000000 | 0.0 |    |    |     |
| GRID        | 53 |   | 120.0000012.000000 | 0.0 |    |    |     |
| GRID        | 54 |   | 135.0000012.000000 | 0.0 |    |    |     |
| GRID        | 55 |   | 150.0000012.000000 | 0.0 |    |    |     |
| GRID        | 56 |   | 0.015.000000       | 0.0 |    |    |     |
| GRID        | 57 |   | 15.0000015.000000  | 0.0 |    |    |     |
| GRID        | 58 |   | 30.0000015.000000  | 0.0 |    |    |     |
| GRID        | 59 |   | 45.0000015.000000  | 0.0 |    |    |     |
| GRID        | 60 |   | 60.0000015.000000  | 0.0 |    |    |     |
| GRID        | 61 |   | 75.0000015.000000  | 0.0 |    |    |     |
| GRID        | 62 |   | 90.0000015.000000  | 0.0 |    |    |     |
| GRID        | 63 |   | 105.0000015.000000 | 0.0 |    |    |     |
| GRID        | 64 |   | 120.0000015.000000 | 0.0 |    |    |     |
| GRID        | 65 |   | 135.0000015.000000 | 0.0 |    |    |     |
| GRID        | 66 |   | 150.0000015.000000 | 0.0 |    |    |     |
| \$          |    |   |                    |     |    |    |     |
| \$ ELEMENTS |    |   |                    |     |    |    |     |
| \$          |    |   |                    |     |    |    |     |
| CQUAD4      | 1  | 1 | 1                  | 2   | 13 | 12 | 0.0 |
| CQUAD4      | 2  | 1 | 2                  | 3   | 14 | 13 | 0.0 |
| CQUAD4      | 3  | 1 | 3                  | 4   | 15 | 14 | 0.0 |
| CQUAD4      | 4  | 1 | 4                  | 5   | 16 | 15 | 0.0 |

|         |    |   |    |    |    |    |     |
|---------|----|---|----|----|----|----|-----|
| CQUAD4  | 5  | 1 | 5  | 6  | 17 | 16 | 0.0 |
| CQUAD4  | 6  | 1 | 6  | 7  | 18 | 17 | 0.0 |
| CQUAD4  | 7  | 1 | 7  | 8  | 19 | 18 | 0.0 |
| CQUAD4  | 8  | 1 | 8  | 9  | 20 | 19 | 0.0 |
| CQUAD4  | 9  | 1 | 9  | 10 | 21 | 20 | 0.0 |
| CQUAD4  | 10 | 1 | 10 | 11 | 22 | 21 | 0.0 |
| CQUAD4  | 11 | 1 | 12 | 13 | 24 | 23 | 0.0 |
| CQUAD4  | 12 | 1 | 13 | 14 | 25 | 24 | 0.0 |
| CQUAD4  | 13 | 1 | 14 | 15 | 26 | 25 | 0.0 |
| CQUAD4  | 14 | 1 | 15 | 16 | 27 | 26 | 0.0 |
| CQUAD4  | 15 | 1 | 16 | 17 | 28 | 27 | 0.0 |
| CQUAD4  | 16 | 1 | 17 | 18 | 29 | 28 | 0.0 |
| CQUAD4  | 17 | 1 | 18 | 19 | 30 | 29 | 0.0 |
| CQUAD4  | 18 | 1 | 19 | 20 | 31 | 30 | 0.0 |
| CQUAD4  | 19 | 1 | 20 | 21 | 32 | 31 | 0.0 |
| CQUAD4  | 20 | 1 | 21 | 22 | 33 | 32 | 0.0 |
| CQUAD4  | 21 | 1 | 23 | 24 | 35 | 34 | 0.0 |
| CQUAD4  | 22 | 1 | 24 | 25 | 36 | 35 | 0.0 |
| CQUAD4  | 23 | 1 | 25 | 26 | 37 | 36 | 0.0 |
| CQUAD4  | 24 | 1 | 26 | 27 | 38 | 37 | 0.0 |
| CQUAD4  | 25 | 1 | 27 | 28 | 39 | 38 | 0.0 |
| CQUAD4  | 26 | 1 | 28 | 29 | 40 | 39 | 0.0 |
| CQUAD4  | 27 | 1 | 29 | 30 | 41 | 40 | 0.0 |
| CQUAD4  | 28 | 1 | 30 | 31 | 42 | 41 | 0.0 |
| CQUAD4  | 29 | 1 | 31 | 32 | 43 | 42 | 0.0 |
| CQUAD4  | 30 | 1 | 32 | 33 | 44 | 43 | 0.0 |
| CQUAD4  | 31 | 1 | 34 | 35 | 46 | 45 | 0.0 |
| CQUAD4  | 32 | 1 | 35 | 36 | 47 | 46 | 0.0 |
| CQUAD4  | 33 | 1 | 36 | 37 | 48 | 47 | 0.0 |
| CQUAD4  | 34 | 1 | 37 | 38 | 49 | 48 | 0.0 |
| CQUAD4  | 35 | 1 | 38 | 39 | 50 | 49 | 0.0 |
| CQUAD4  | 36 | 1 | 39 | 40 | 51 | 50 | 0.0 |
| CQUAD4  | 37 | 1 | 40 | 41 | 52 | 51 | 0.0 |
| CQUAD4  | 38 | 1 | 41 | 42 | 53 | 52 | 0.0 |
| CQUAD4  | 39 | 1 | 42 | 43 | 54 | 53 | 0.0 |
| CQUAD4  | 40 | 1 | 43 | 44 | 55 | 54 | 0.0 |
| CQUAD4  | 41 | 1 | 45 | 46 | 57 | 56 | 0.0 |
| CQUAD4  | 42 | 1 | 46 | 47 | 58 | 57 | 0.0 |
| CQUAD4  | 43 | 1 | 47 | 48 | 59 | 58 | 0.0 |
| CQUAD4  | 44 | 1 | 48 | 49 | 60 | 59 | 0.0 |
| CQUAD4  | 45 | 1 | 49 | 50 | 61 | 60 | 0.0 |
| CQUAD4  | 46 | 1 | 50 | 51 | 62 | 61 | 0.0 |
| CQUAD4  | 47 | 1 | 51 | 52 | 63 | 62 | 0.0 |
| CQUAD4  | 48 | 1 | 52 | 53 | 64 | 63 | 0.0 |
| CQUAD4  | 49 | 1 | 53 | 54 | 65 | 64 | 0.0 |
| CQUAD4  | 50 | 1 | 54 | 55 | 66 | 65 | 0.0 |
| ENDDATA |    |   |    |    |    |    |     |

---

## 2.38.2 Output Data

1

```
      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G          E          NN      N      E          S          I          S
G      GG      EEEEE      N      N      N      EEEEE      SSSS      I          SSSS
G      G      E          N      NN      E          S          I          S
      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 10.1

BUILD CODE r106                      BETA

PROJECT NAME: A038  
RUN STARTED: Dec 4, 2008 10:05  
SYSTEM TYPE: Linux 2.6.18.8-0.9-default x86\_64 4-CPU (LP64)  
LENVEC: 40000000  
IOBUFF: 256,64K  
THREADS: 1

LICENSED TO: VR&D  
LICENSE EXPIRES: Apr 17, 2009

WARNING MESSAGE FROM GN1245:  
LOADS AND/OR BOUNDARY CONDITIONS AND/OR EIGENVALUE DATA  
STATEMENTS ARE NOT REFERENCED.  
A SUMMARY OF THEM ARE GIVEN BELOW.  
THE NON REFERENCED BULK DATA STATEMENTS WERE IGNORED.

| DATA STATEMENT | REFERENCED | TOTAL |
|----------------|------------|-------|
|----------------|------------|-------|

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 1  
RANDOM RESPONSE OF A CANTILEVERED PLATE

## ANALYSIS PROBLEM SUMMARY

NUMBER OF GRID POINTS: 66  
NUMBER OF CQUAD4->PSHELL ELEMENTS: 50  
TOTAL NUMBER OF NON RIGID ELEMENTS: 50  
NUMBER OF ELEMENT PROPERTIES: 1  
NUMBER OF MATERIALS: 1  
NUMBER OF DEGREES OF FREEDOM: 360

## LOAD CASES SUMMARY

NUMBER OF USER FREQUENCY LOAD CASES: 0  
NUMBER OF INTERNAL FREQUENCY LOAD CASES: 1  
NUMBER OF MODAL DYNAMIC LOAD CASES: 1  
TOTAL NUMBER OF LOAD CASES: 2

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 2  
RANDOM RESPONSE OF A CANTILEVERED PLATE

## LANCZOS EIGENSOLUTION STATISTICS

| SHIFT<br>COUNT | SHIFT<br>VALUE | FREQUENCY<br>(CYC./TIME) | EVALUES<br>BELOW | NEW<br>FOUND | TERM.<br>CODE | CPU<br>TIME |
|----------------|----------------|--------------------------|------------------|--------------|---------------|-------------|
| 1              | 6.316547E+06   | 4.000000E+02             | 10               | 0            | 10            | 0.00        |
| 2              | -1.000000E+01  | -5.032921E-01            | 0                | 10           | 2             | 0.01        |

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 3  
RANDOM RESPONSE OF A CANTILEVERED PLATE

\*\*\*\*\*  
\* D E S I G N C Y C L E 0 (ANALYSIS SUMMARY)\*  
\*\*\*\*\*

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 4  
RANDOM RESPONSE OF A CANTILEVERED PLATE

## M A S S / V O L U M E S U M M A R Y

SYSTEM MASS 4.488750E-02  
SYSTEM VOLUME 1.575000E+04  
SYSTEM MASS/VOLUME 2.850000E-06

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 5  
RANDOM RESPONSE OF A CANTILEVERED PLATE

# ENCLOSING BOX SUMMARY

|           |           |           |           |           |           |            |
|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| LENGTH X  | LENGTH Y  | LENGTH Z  | AREA XY   | AREA YZ   | AREA XZ   | VOLUME XYZ |
| 1.500E+02 | 1.500E+01 | 0.000E+00 | 2.250E+03 | 0.000E+00 | 0.000E+00 | 0.000E+00  |

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 6  
RANDOM RESPONSE OF A CANTILEVERED PLATE

MODES FOR METHOD OF LOADCASE 1 LOADCASE 2

## EIGENVALUES

| MODE | CYCLES       | EIGENVALUE   | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|--------------|--------------|---------------------|--------------------------|
| 1    | 7.848049E+00 | 2.431550E+03 | 4.931075E+01 | 1.000000E+00        | 2.431550E+03             |
| 2    | 1.672115E+01 | 1.103805E+04 | 1.050621E+02 | 1.000000E+00        | 1.103805E+04             |
| 3    | 4.936079E+01 | 9.618868E+04 | 3.101430E+02 | 1.000000E+00        | 9.618868E+04             |
| 4    | 1.026158E+02 | 4.157080E+05 | 6.447542E+02 | 1.000000E+00        | 4.157080E+05             |
| 5    | 1.216394E+02 | 5.841281E+05 | 7.642827E+02 | 1.000000E+00        | 5.841281E+05             |
| 6    | 1.400379E+02 | 7.741959E+05 | 8.798840E+02 | 1.000000E+00        | 7.741959E+05             |
| 7    | 2.601806E+02 | 2.672451E+06 | 1.634763E+03 | 1.000000E+00        | 2.672451E+06             |
| 8    | 2.803592E+02 | 3.103055E+06 | 1.761549E+03 | 1.000000E+00        | 3.103055E+06             |
| 9    | 2.812544E+02 | 3.122901E+06 | 1.767173E+03 | 1.000000E+00        | 3.122901E+06             |
| 10   | 3.676873E+02 | 5.337244E+06 | 2.310248E+03 | 1.000000E+00        | 5.337244E+06             |
| 11   | 5.682706E+02 | 1.274882E+07 | 3.570549E+03 | 1.000000E+00        | 1.274882E+07             |

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 7  
RANDOM RESPONSE OF A CANTILEVERED PLATE

\*\*\*\*\*  
\* DESIGN CYCLE 0 (ANALYSIS)\*  
\*\*\*\*\*

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 8  
RANDOM RESPONSE OF A CANTILEVERED PLATE

DYNAMIC LOADING AT THE FREE END OF THE PLATE LOADCASE 1

-----  
Dynamic responses have been deleted to save space  
-----

-----Porsions of this file has been removed to save space-----

1GENESIS VERSION 10.1 DATE 12-04-2008 TIME 10:05 PAGE 408  
RANDOM RESPONSE OF A CANTILEVERED PLATE

RANDOM 30

+00

\*\*\*\*\* THERE WERE

1 WARNING MESSAGES \*\*\*\*\*

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*

---

## 2.39 Rigid Body Modes of Doubly Curved Shells

### Example ID:

A039

### Analysis Data:

GRID, CQUAD4, CWELD, EIGR

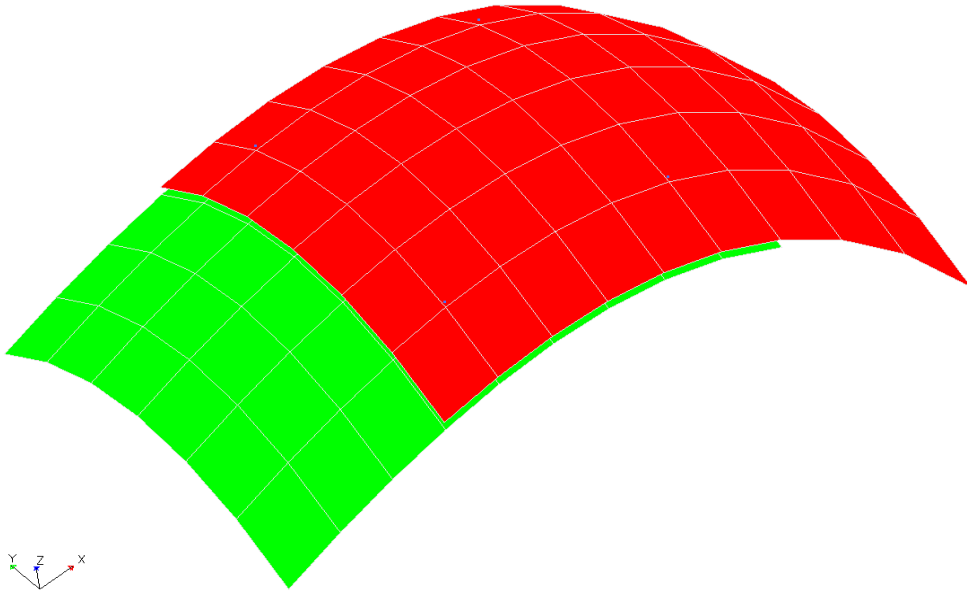
### Special Features Used:

CWELD connector elements

### Problem Statement:

Verify the six rigid body modes of two doubly curved shells connected by WELD elements of type PARTPAT, ELPAT, ELEMID, GRIDID and ALIGN.

### Structural Analysis Model:



### Analysis Model Description:

1. Two doubly curved shells with 48 quadrilateral elements each connected by 4 welds.

**Special Modeling Techniques:**

None.

**Reference Solution:**

None.

**Calculated Solution:**

1. Rigid and elastic modes of structure

The above results are printed in the A039.out file.

---

## 2.39.1 Input Data

```
$-----$
$ Data written by Design Studio for Genesis 10.1 (r521)
$ Exported on 2009-03-31 17:42
$-----$
$
$ Executive Control
$
POST = PUNCH
SOL COMPAT1
CEND
$
$ Solution Control
$
LINE = 64,80
ECHO = UNSORT(PARAM,DOPT)
$
$ Loadcase definitions
$
SUBCASE 10
  LABEL = Loadcase 10
  METHOD = 1
  MODTRK=NO
  DISP = POST,ALL
  ESE = POST,ALL
BEGIN BULK
$
$ Parameters
$
$
$ Property definitions
$
$HMCOLOR COMP      1      1
PSHELL      1      11.000000      1      1
$HMCOLOR COMP      2      2
PSHELL      2      11.000000      1      1
$HMCOLOR COMP      6      3
PWELD      6      12.000000      SPOT
$HMCOLOR COMP      7      5
PSHELL      7      21.000000      2      2
$HMCOLOR COMP      8      6
PSHELL      8      21.000000      2      2
$HMCOLOR COMP     12      7
PWELD     12      22.000000      SPOT
$HMCOLOR COMP     13      1
PSHELL     13      31.000000      3      3
$HMCOLOR COMP     14      2
PSHELL     14      31.000000      3      3
$HMCOLOR COMP     18      3
PWELD     18      32.000000
SWLDPRM      PRTSW      0 NREDIA      2 GSMOVE      2 GMCHK      1
+          GSTOL      0.0 PROJTOL      0.2 GS PROJ      20.0
$
```



```

$ Material definitions
$
$HMNAME MAT      1 "MAT1"
MAT1             1207000.0      .30000007.8000-9
$HMNAME MAT      2 "MAT1"
MAT1             2207000.0      .30000007.8000-9
$HMNAME MAT      3 "MAT1"
MAT1             3207000.0      .30000007.8000-9
$
$ Load definitions
$
$HMNAME LOADCOL   1 "Method 1"
EIGR              1      SMS      10000.00
$
$ Boundary Condition definitions
$
$
$ Grid points
$
GRID              5      90.00000      0.070.08133
GRID              7      0.060.0000070.01790
GRID              9      90.0000060.0000070.09919
GRID             10      0.0      0.070.00000
GRID             77      10.00000      0.075.07760
GRID             78      20.00000      0.079.20670
GRID             79      30.00000      0.082.34758
GRID             80      40.00000      0.084.34912
GRID             81      50.00000      0.085.06771
GRID             82      60.00000      0.084.30118
GRID             83      70.00000      0.081.79037
GRID             84      80.00000      0.077.20472
GRID             85      0.010.0000078.36184
GRID             86      10.0000010.0000083.38850
GRID             87      20.0000010.0000087.51803
GRID             88      30.0000010.0000090.65744
GRID             89      40.0000010.0000092.65994
GRID             90      50.0000010.0000093.37844
GRID             91      60.0000010.0000092.61201
GRID             92      70.0000010.0000090.10146
GRID             93      80.0000010.0000085.51624
GRID             94      90.0000010.0000078.40251
GRID             95      0.020.0000083.34978
GRID             96      10.0000020.0000088.37678
GRID             97      20.0000020.0000092.50993
GRID             98      30.0000020.0000095.64416
GRID             99      40.0000020.0000097.64646
.
.
.
GRID            1000      120.000040.00000-98.2621
GRID            1001      30.0000050.00000-47.3552
GRID            1002      40.0000050.00000-45.3511
GRID            1003      50.0000050.00000-44.6355
GRID            1004      60.0000050.00000-45.4034
GRID            1005      70.0000050.00000-47.9115

```

|                        |      |    |                           |     |     |     |     |
|------------------------|------|----|---------------------------|-----|-----|-----|-----|
| GRID                   | 1006 |    | 80.0000050.00000-52.4981  |     |     |     |     |
| GRID                   | 1007 |    | 90.0000050.00000-59.6061  |     |     |     |     |
| GRID                   | 1008 |    | 100.0000050.00000-69.8869 |     |     |     |     |
| GRID                   | 1009 |    | 110.0000050.00000-83.9798 |     |     |     |     |
| GRID                   | 1010 |    | 120.0000050.00000-103.243 |     |     |     |     |
| GRID                   | 1011 |    | 40.0000060.00000-53.6619  |     |     |     |     |
| GRID                   | 1012 |    | 50.0000060.00000-52.9464  |     |     |     |     |
| GRID                   | 1013 |    | 60.0000060.00000-53.7143  |     |     |     |     |
| GRID                   | 1014 |    | 70.0000060.00000-56.2221  |     |     |     |     |
| GRID                   | 1015 |    | 80.0000060.00000-60.8082  |     |     |     |     |
| GRID                   | 1016 |    | 90.0000060.00000-67.9156  |     |     |     |     |
| GRID                   | 1017 |    | 100.0000060.00000-78.1953 |     |     |     |     |
| GRID                   | 1018 |    | 110.0000060.00000-92.2868 |     |     |     |     |
| \$                     |      |    |                           |     |     |     |     |
| \$ Element definitions |      |    |                           |     |     |     |     |
| \$                     |      |    |                           |     |     |     |     |
| CQUAD4                 | 109  | 1  | 146                       | 147 | 156 | 155 | 0.0 |
| CQUAD4                 | 110  | 1  | 147                       | 148 | 157 | 156 | 0.0 |
| CQUAD4                 | 111  | 1  | 148                       | 149 | 158 | 157 | 0.0 |
| CQUAD4                 | 112  | 1  | 149                       | 150 | 159 | 158 | 0.0 |
| CQUAD4                 | 113  | 1  | 150                       | 151 | 160 | 159 | 0.0 |
| CQUAD4                 | 114  | 1  | 151                       | 152 | 161 | 160 | 0.0 |
| CQUAD4                 | 115  | 1  | 152                       | 153 | 162 | 161 | 0.0 |
| CQUAD4                 | 116  | 1  | 153                       | 154 | 163 | 162 | 0.0 |
| CQUAD4                 | 117  | 1  | 154                       | 143 | 164 | 163 | 0.0 |
| CQUAD4                 | 118  | 1  | 155                       | 156 | 166 | 165 | 0.0 |
| CQUAD4                 | 119  | 1  | 156                       | 157 | 167 | 166 | 0.0 |
| CQUAD4                 | 120  | 1  | 157                       | 158 | 168 | 167 | 0.0 |
| .                      |      |    |                           |     |     |     |     |
| .                      |      |    |                           |     |     |     |     |
| .                      |      |    |                           |     |     |     |     |
| CQUAD4                 | 100  | 2  | 125                       | 126 | 135 | 7   | 0.0 |
| CQUAD4                 | 101  | 2  | 126                       | 127 | 136 | 135 | 0.0 |
| CQUAD4                 | 102  | 2  | 127                       | 128 | 137 | 136 | 0.0 |
| CQUAD4                 | 103  | 2  | 128                       | 129 | 138 | 137 | 0.0 |
| CQUAD4                 | 104  | 2  | 129                       | 130 | 139 | 138 | 0.0 |
| CQUAD4                 | 105  | 2  | 130                       | 131 | 140 | 139 | 0.0 |
| CQUAD4                 | 106  | 2  | 131                       | 132 | 141 | 140 | 0.0 |
| CQUAD4                 | 107  | 2  | 132                       | 133 | 142 | 141 | 0.0 |
| CQUAD4                 | 108  | 2  | 133                       | 134 | 9   | 142 | 0.0 |
| CWELD                  | 209  | 6  | 300PARTPAT                |     |     |     |     |
| +                      | 1    | 2  |                           |     |     |     |     |
| CWELD                  | 210  | 6  | 301ELPAT                  |     |     |     |     |
| +                      | 146  | 95 |                           |     |     |     |     |
| CWELD                  | 211  | 6  | 302ELPAT                  |     |     |     |     |
| +                      | 122  | 71 |                           |     |     |     |     |
| CWELD                  | 212  | 6  | 303PARTPAT                |     |     |     |     |
| +                      | 1    | 2  |                           |     |     |     |     |
| CQUAD4                 | 321  | 7  | 449                       | 450 | 459 | 458 | 0.0 |
| CQUAD4                 | 322  | 7  | 450                       | 451 | 460 | 459 | 0.0 |
| CQUAD4                 | 323  | 7  | 451                       | 452 | 461 | 460 | 0.0 |
| CQUAD4                 | 324  | 7  | 452                       | 453 | 462 | 461 | 0.0 |
| CQUAD4                 | 325  | 7  | 453                       | 454 | 463 | 462 | 0.0 |
| CQUAD4                 | 326  | 7  | 454                       | 455 | 464 | 463 | 0.0 |
| CQUAD4                 | 327  | 7  | 455                       | 456 | 465 | 464 | 0.0 |

|         |     |     |       |        |      |     |     |
|---------|-----|-----|-------|--------|------|-----|-----|
| CQUAD4  | 328 | 7   | 456   | 457    | 466  | 465 | 0.0 |
| CQUAD4  | 329 | 7   | 457   | 446    | 467  | 466 | 0.0 |
| CQUAD4  | 330 | 7   | 458   | 459    | 469  | 468 | 0.0 |
| .       |     |     |       |        |      |     |     |
| .       |     |     |       |        |      |     |     |
| .       |     |     |       |        |      |     |     |
| CQUAD4  | 315 | 8   | 431   | 432    | 441  | 440 | 0.0 |
| CQUAD4  | 316 | 8   | 432   | 433    | 442  | 441 | 0.0 |
| CQUAD4  | 317 | 8   | 433   | 434    | 443  | 442 | 0.0 |
| CQUAD4  | 318 | 8   | 434   | 435    | 444  | 443 | 0.0 |
| CQUAD4  | 319 | 8   | 435   | 436    | 445  | 444 | 0.0 |
| CQUAD4  | 320 | 8   | 436   | 437    | 312  | 445 | 0.0 |
| CWELD   | 421 | 12  | 803   | GRIDID |      | QQ  |     |
| +       | 449 | 450 | 459   | 458    |      |     |     |
| +       | 382 | 383 | 392   | 391    |      |     |     |
| CWELD   | 422 | 12  | 804   | GRIDID |      | QQ  |     |
| +       | 498 | 499 | 508   | 447    |      |     |     |
| +       | 431 | 432 | 441   | 440    |      |     |     |
| CWELD   | 423 | 12  | 805   | ELEMID |      |     |     |
| +       | 326 | 275 |       |        |      |     |     |
| CWELD   | 424 | 12  | 806   | ELEMID |      |     |     |
| +       | 371 | 320 |       |        |      |     |     |
| CQUAD4  | 533 | 13  | 952   | 953    | 962  | 961 | 0.0 |
| CQUAD4  | 534 | 13  | 953   | 954    | 963  | 962 | 0.0 |
| CQUAD4  | 535 | 13  | 954   | 955    | 964  | 963 | 0.0 |
| CQUAD4  | 536 | 13  | 955   | 956    | 965  | 964 | 0.0 |
| CQUAD4  | 537 | 13  | 956   | 957    | 966  | 965 | 0.0 |
| CQUAD4  | 538 | 13  | 957   | 958    | 967  | 966 | 0.0 |
| CQUAD4  | 539 | 13  | 958   | 959    | 968  | 967 | 0.0 |
| CQUAD4  | 540 | 13  | 959   | 960    | 969  | 968 | 0.0 |
| .       |     |     |       |        |      |     |     |
| .       |     |     |       |        |      |     |     |
| .       |     |     |       |        |      |     |     |
| CQUAD4  | 520 | 14  | 926   | 927    | 937  | 936 | 0.0 |
| CQUAD4  | 521 | 14  | 927   | 928    | 938  | 937 | 0.0 |
| CQUAD4  | 522 | 14  | 928   | 929    | 939  | 938 | 0.0 |
| CQUAD4  | 523 | 14  | 929   | 930    | 940  | 939 | 0.0 |
| CQUAD4  | 524 | 14  | 931   | 932    | 941  | 813 | 0.0 |
| CQUAD4  | 525 | 14  | 932   | 933    | 942  | 941 | 0.0 |
| CQUAD4  | 526 | 14  | 933   | 934    | 943  | 942 | 0.0 |
| CQUAD4  | 527 | 14  | 934   | 935    | 944  | 943 | 0.0 |
| CQUAD4  | 528 | 14  | 935   | 936    | 945  | 944 | 0.0 |
| CQUAD4  | 529 | 14  | 936   | 937    | 946  | 945 | 0.0 |
| CQUAD4  | 530 | 14  | 937   | 938    | 947  | 946 | 0.0 |
| CQUAD4  | 531 | 14  | 938   | 939    | 948  | 947 | 0.0 |
| CQUAD4  | 532 | 14  | 939   | 940    | 815  | 948 | 0.0 |
| CWELD   | 633 | 18  | ALIGN |        | 962  | 895 |     |
| CWELD   | 634 | 18  | ALIGN |        | 1002 | 935 |     |
| CWELD   | 635 | 18  | ALIGN |        | 966  | 899 |     |
| CWELD   | 636 | 18  | ALIGN |        | 1006 | 939 |     |
| ENDDATA |     |     |       |        |      |     |     |

---

## 2.39.2 Output Data

1

```

      GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
G      E      NN      N      E      S      I      S
G      GG      EEEE      N      N      N      EEEEE      SSSS      I      SSSS
G      G      E      N      NN      E      S      I      S
GGGG      EEEEE      N      N      EEEEE      SSSSS      IIIII      SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 10.2

BUILD CODE 200904030915 BETA

```

PROJECT NAME: A039
RUN STARTED: Apr  3, 2009 09:16
SYSTEM TYPE: Linux 2.6.25.5-1.1-default x86_64 4-CPU (LP64)
LENVEC:      250000000
IOBUFF:      256,64K
THREADS:      1
```

### ANALYSIS PROBLEM SUMMARY

```

NUMBER OF GRID POINTS:      428
NUMBER OF CQUAD4->PSHELL ELEMENTS:      324
NUMBER OF CWELD ELEMENTS:      12
TOTAL NUMBER OF NON RIGID ELEMENTS:      336
NUMBER OF ELEMENT PROPERTIES:      9
NUMBER OF MATERIALS:      3
NUMBER OF DEGREES OF FREEDOM:      2520
```

# LOAD CASES SUMMARY

NUMBER OF USER FREQUENCY LOAD CASES: 1  
TOTAL NUMBER OF LOAD CASES: 1

1GENESIS VERSION 10.2 DATE 04-03-2009 TIME 9:16 PAGE 4

\*\*\*\*\*  
\* DESIGN CYCLE 0 (ANALYSIS SUMMARY)\*  
\*\*\*\*\*

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## MASS / VOLUME SUMMARY

SYSTEM MASS 3.328003E-04  
SYSTEM VOLUME 4.266671E+04  
SYSTEM MASS/VOLUME 7.800000E-09

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## ENCLOSING BOX SUMMARY

| LENGTH X  | LENGTH Y  | LENGTH Z  | AREA XY   | AREA YZ   | AREA XZ   | VOLUME XYZ |
|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| 1.200E+02 | 6.000E+01 | 2.136E+02 | 7.200E+03 | 1.282E+04 | 2.563E+04 | 1.538E+06  |

1GENESIS VERSION 10.2 DATE 04-03-2009 TIME 9:16 PAGE 7

LOADCASE 10 LOADCASE 10

## EIGENVALUES

| MODE | CYCLES       | EIGENVALUE    | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|---------------|--------------|---------------------|--------------------------|
| 1    | 5.878905E-03 | -1.364434E-03 | 3.693825E-02 | 1.000000E+00        | -1.364434E-03            |
| 2    | 3.312458E-03 | -4.331721E-04 | 2.081279E-02 | 1.000000E+00        | -4.331721E-04            |
| 3    | 3.075418E-03 | -3.733945E-04 | 1.932342E-02 | 1.000000E+00        | -3.733945E-04            |
| 4    | 2.245860E-03 | -1.991248E-04 | 1.411116E-02 | 1.000000E+00        | -1.991248E-04            |
| 5    | 1.751814E-03 | -1.211534E-04 | 1.100697E-02 | 1.000000E+00        | -1.211534E-04            |
| 6    | 1.602762E-03 | -1.014140E-04 | 1.007045E-02 | 1.000000E+00        | -1.014140E-04            |
| 7    | 1.498613E-03 | -8.866227E-05 | 9.416065E-03 | 1.000000E+00        | -8.866227E-05            |
| 8    | 5.285808E-04 | 1.103018E-05  | 3.321171E-03 | 1.000000E+00        | 1.103018E-05             |
| 9    | 1.034052E-03 | 4.221281E-05  | 6.497138E-03 | 1.000000E+00        | 4.221281E-05             |
| 10   | 1.911360E-03 | 1.442264E-04  | 1.200943E-02 | 1.000000E+00        | 1.442264E-04             |
| 11   | 2.463215E-03 | 2.395324E-04  | 1.547684E-02 | 1.000000E+00        | 2.395324E-04             |
| 12   | 2.978914E-03 | 3.503287E-04  | 1.871707E-02 | 1.000000E+00        | 3.503287E-04             |
| 13   | 4.522433E-03 | 8.074282E-04  | 2.841528E-02 | 1.000000E+00        | 8.074282E-04             |
| 14   | 4.609521E-03 | 8.388250E-04  | 2.896248E-02 | 1.000000E+00        | 8.388250E-04             |

```

15  6.304370E-03  1.569073E-03  3.961153E-02  1.000000E+00  1.569073E-03
16  3.979237E-02  6.251141E-02  2.500228E-01  1.000000E+00  6.251141E-02
17  1.902518E-01  1.428950E+00  1.195387E+00  1.000000E+00  1.428950E+00
18  2.896874E-01  3.312982E+00  1.820160E+00  1.000000E+00  3.312982E+00
19  4.608776E+02  8.385539E+06  2.895779E+03  1.000000E+00  8.385539E+06
20  4.933177E+02  9.607560E+06  3.099606E+03  1.000000E+00  9.607560E+06
21  5.033549E+02  1.000249E+07  3.162672E+03  1.000000E+00  1.000249E+07
22  5.097313E+02  1.025752E+07  3.202736E+03  1.000000E+00  1.025752E+07
23  5.164023E+02  1.052776E+07  3.244651E+03  1.000000E+00  1.052776E+07
24  5.206990E+02  1.070368E+07  3.271648E+03  1.000000E+00  1.070368E+07
25  8.404569E+02  2.788629E+07  5.280747E+03  1.000000E+00  2.788629E+07
26  8.711762E+02  2.996206E+07  5.473761E+03  1.000000E+00  2.996206E+07
27  8.824563E+02  3.074300E+07  5.544637E+03  1.000000E+00  3.074300E+07
28  1.070319E+03  4.522577E+07  6.725011E+03  1.000000E+00  4.522577E+07
29  1.103356E+03  4.806084E+07  6.932593E+03  1.000000E+00  4.806084E+07
30  1.126401E+03  5.008941E+07  7.077387E+03  1.000000E+00  5.008941E+07
31  1.461045E+03  8.427268E+07  9.180015E+03  1.000000E+00  8.427268E+07
32  1.506711E+03  8.962309E+07  9.466947E+03  1.000000E+00  8.962309E+07
33  1.514124E+03  9.050704E+07  9.513519E+03  1.000000E+00  9.050704E+07
34  1.525955E+03  9.192702E+07  9.587858E+03  1.000000E+00  9.192702E+07
35  1.534064E+03  9.290661E+07  9.638808E+03  1.000000E+00  9.290661E+07
36  1.593267E+03  1.002159E+08  1.001079E+04  1.000000E+00  1.002159E+08
1GENESIS  VERSION 10.2  DATE 04-03-2009  TIME 9:16  PAGE 8

```

```

*****
*   D E S I G N   C Y C L E           0 (ANALYSIS)*
*****
1          *****  END OF OUTPUT  *****

```

---

## 2.40 Rigid Body Modes of Double Hat Structure

### Example ID:

A040

### Analysis Data:

GRID, CQUAD4, CWELD, EIGR

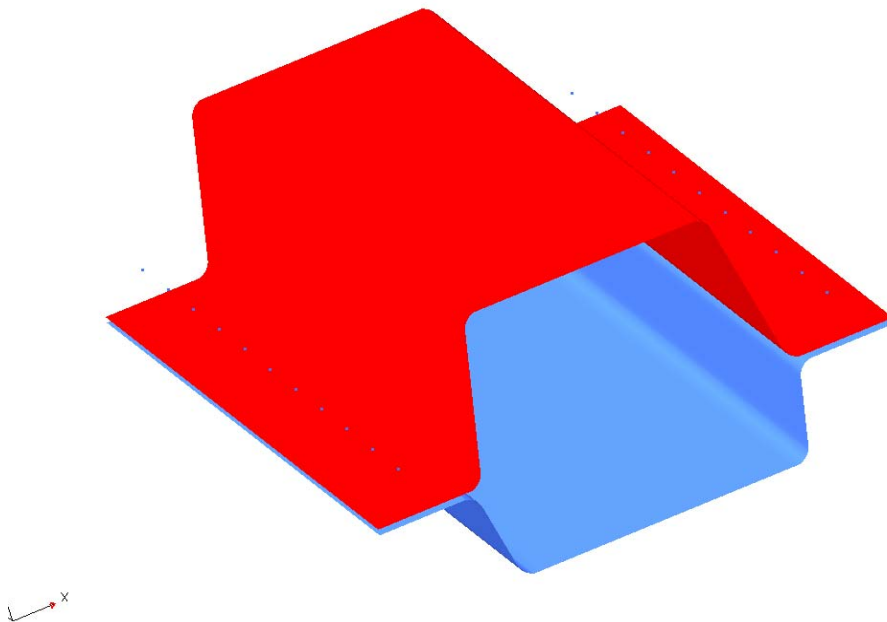
### Special Features Used:

CWELD connector elements

### Problem Statement:

Verify the six rigid body modes of a double hat structure connected by WELD elements of type PARTPAT.

### Structural Analysis Model:



### Analysis Model Description:

1. Double hat structure with 25088 quadrilateral elements connected by 22 welds.

**Special Modeling Techniques:**

None.

**Reference Solution:**

None.

**Calculated Solution:**

1. Rigid and elastic modes of structure

The above results are printed in the A040.out file.



---

## 2.40.1 Input Data

```
$-----$
--$
$ Data written by Design Studio for Genesis 10.1 (r521)
$ Exported on 2009-03-31 15:02
$-----$
--$
$
$ Executive Control
$
POST = PUNCH
DIAG = 85
SOL COMPAT1
CEND
$
$ Solution Control
$
LINE = 64,80
ECHO = UNSORT(PARAM,DOPT)
PERTURBATION = POST
SHAPE = POST
$
$ Loadcase definitions
$
SUBCASE 2
    LABEL = Loadcase 2
    METHOD = 1
    MODTRK=NO
    DISP = POST,ALL
    ESE = POST,ALL
BEGIN BULK
$
$ Parameters
$
$
$ Property definitions
$
$HMNAME COMP      1 "Top"
$HMCOLOR COMP      1      1
PSHELL            1      11.000000      1      1
$HMNAME COMP      3 "Bottom"
$HMCOLOR COMP      3      3
PSHELL            3      11.000000      1      1
$HMCOLOR COMP      6      3
PWELD             6      15.000000
SWLDPRM          PRTSW      0 NREDIA      2 GSMOVE      2 GMCHK      1
+                GSTOL      0.0 PROJTOL      0.2 GS PROJ      20.0
$
$ Material definitions
```

```

$
$HMNAME MAT          1 "MAT1 1 (fix me)"
MAT1          1207000.0          .30000007.8000-9
$
$ Load definitions
$
$HMNAME LOADCOL      1 "Method 1"
EIGR          1      SMS          30000.00          12
$
$ Boundary Condition definitions
$
$
$ Grid points
$
GRID          1          23.00000          0.0          0.0
GRID          2          0.0          0.0          0.0
GRID          3          26.00000          0.03.000000
GRID          4          33.00000          0.039.00000
GRID          5          36.00000          0.042.00000
GRID          6          63.00000          0.042.00000
GRID          16          0.05.00000          0.0
GRID          17          23.000005.00000          0.0
GRID          18          26.000005.0000003.000000
GRID          19          33.000005.00000039.00000
GRID          20          36.000005.00000042.00000
GRID          21          63.000005.00000042.00000
GRID          22          0.03.75000          0.0
GRID          23          0.02.50000          0.0
GRID          24          0.01.25000          0.0
GRID          25          1.9166675.00000          0.0
GRID          26          1.9166673.75000          0.0
GRID          27          1.9166672.50000          0.0
GRID          28          1.9166671.25000          0.0
GRID          29          1.916667          0.0          0.0
GRID          30          3.8333335.00000          0.0
GRID          31          3.8333333.75000          0.0
GRID          32          3.8333332.50000          0.0
GRID          33          3.8333331.25000          0.0
GRID          34          3.833333          0.0          0.0
GRID          35          5.7500005.00000          0.0
.
.
.
GRID          29600          65.52564156.2500-25.5000
GRID          29601          68.05128156.2500-25.5000
GRID          29602          70.57692156.2500-25.5000
GRID          29605          63.00000155.0000-25.5000
GRID          29607          68.05128155.0000-25.5000
GRID          29608          115.00005.00000010.00000
GRID          29609          115.000020.0000010.00000
GRID          29610          115.000035.0000010.00000

```

|                        |       |   |                             |
|------------------------|-------|---|-----------------------------|
| GRID                   | 29611 |   | 115.000050.0000010.00000    |
| GRID                   | 29612 |   | 115.000065.0000010.00000    |
| GRID                   | 29613 |   | 115.000080.0000010.00000    |
| GRID                   | 29614 |   | 115.000095.0000010.00000    |
| GRID                   | 29615 |   | 115.0000110.000010.00000    |
| GRID                   | 29616 |   | 115.0000125.000010.00000    |
| GRID                   | 29617 |   | 115.0000140.000010.00000    |
| GRID                   | 29618 |   | 115.0000155.000010.00000    |
| GRID                   | 29619 |   | 10.0000020.0000010.00000    |
| GRID                   | 29620 |   | 10.0000035.0000010.00000    |
| GRID                   | 29621 |   | 10.0000050.0000010.00000    |
| GRID                   | 29622 |   | 10.0000065.0000010.00000    |
| GRID                   | 29623 |   | 10.0000080.0000010.00000    |
| GRID                   | 29624 |   | 10.0000095.0000010.00000    |
| GRID                   | 29625 |   | 10.00000110.000010.00000    |
| GRID                   | 29626 |   | 10.00000125.000010.00000    |
| GRID                   | 29627 |   | 10.00000140.000010.00000    |
| GRID                   | 29628 |   | 10.00000155.000010.00000    |
| GRID                   | 29631 |   | 10.000005.00000010.00000    |
| \$                     |       |   |                             |
| \$ Element definitions |       |   |                             |
| \$                     |       |   |                             |
| CQUAD4                 | 1     | 1 | 16 22 26 25 0.0             |
| CQUAD4                 | 2     | 1 | 22 23 27 26 0.0             |
| CQUAD4                 | 3     | 1 | 23 24 28 27 0.0             |
| CQUAD4                 | 4     | 1 | 24 2 29 28 0.0              |
| CQUAD4                 | 5     | 1 | 25 26 31 30 0.0             |
| CQUAD4                 | 6     | 1 | 26 27 32 31 0.0             |
| CQUAD4                 | 7     | 1 | 27 28 33 32 0.0             |
| CQUAD4                 | 8     | 1 | 28 29 34 33 0.0             |
| CQUAD4                 | 9     | 1 | 30 31 36 35 0.0             |
| CQUAD4                 | 10    | 1 | 31 32 37 36 0.0             |
| .                      |       |   |                             |
| .                      |       |   |                             |
| .                      |       |   |                             |
| CQUAD4                 | 25190 | 3 | 29339 29591 29597 29351 0.0 |
| CQUAD4                 | 25191 | 3 | 29591 29592 29598 29597 0.0 |
| CQUAD4                 | 25192 | 3 | 29592 29593 29599 29598 0.0 |
| CQUAD4                 | 25193 | 3 | 29593 29594 29600 29599 0.0 |
| CQUAD4                 | 25194 | 3 | 29594 29595 29601 29600 0.0 |
| CQUAD4                 | 25195 | 3 | 29595 29529 29602 29601 0.0 |
| CQUAD4                 | 25196 | 3 | 29351 29597 29114 28850 0.0 |
| CQUAD4                 | 25197 | 3 | 29597 29598 29115 29114 0.0 |
| CQUAD4                 | 25198 | 3 | 29598 29599 29605 29115 0.0 |
| CQUAD4                 | 25199 | 3 | 29599 29600 29117 29605 0.0 |
| CQUAD4                 | 25200 | 3 | 29600 29601 29607 29117 0.0 |
| CQUAD4                 | 25201 | 3 | 29601 29602 29088 29607 0.0 |
| CWELD                  | 30001 | 6 | 29608PARTPAT                |
| +                      | 1     | 3 |                             |
| CWELD                  | 30002 | 6 | 29609PARTPAT                |
| +                      | 1     | 3 |                             |

|         |       |   |              |
|---------|-------|---|--------------|
| CWELD   | 30003 | 6 | 29610PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30004 | 6 | 29611PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30005 | 6 | 29612PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30006 | 6 | 29613PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30007 | 6 | 29614PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30008 | 6 | 29615PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30009 | 6 | 29616PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30010 | 6 | 29617PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30011 | 6 | 29618PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30012 | 6 | 29619PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30013 | 6 | 29620PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30014 | 6 | 29621PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30015 | 6 | 29622PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30016 | 6 | 29623PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30017 | 6 | 29624PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30018 | 6 | 29625PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30019 | 6 | 29626PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30020 | 6 | 29627PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30021 | 6 | 29628PARTPAT |
| +       | 1     | 3 |              |
| CWELD   | 30022 | 6 | 29631PARTPAT |
| +       | 1     | 3 |              |
| ENDDATA |       |   |              |

---

## 2.40.2 Output Data

1

```
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
G      E      NN  N  E      S      I      S
G  GG  EEEE   N N N  EEEE   SSSS   I      SSSS
G  G  E      N  NN  E      S      I      S
GGGG  EEEEE  N   N  EEEEE  SSSSS  IIIII  SSSSS
```

STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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VERSION 10.2

BUILD CODE 200904030915 BETA

```
PROJECT NAME: DH2
RUN STARTED: Apr  3, 2009 09:17
SYSTEM TYPE: Linux 2.6.25.5-1.1-default x86_64 4-CPU (LP64)
LENVEC:      250000000
IOBUFF:      256,64K
THREADS:      1
```

```
FOR DEVELOPMENT TESTING ONLY
*** NOT FOR DISTRIBUTION ***
```

```
1GENESIS  VERSION 10.2
      PAGE      1
```

```
DATE 04-03-2009  TIME  9:17
```

\*\*\* INPUT DATA ECHO \*\*\*

--- EXECUTIVE & SOLUTION CONTROL SECTION ---

```
$-----$
  ---$
$ Data written by Design Studio for Genesis 10.1 (r521)
$ Exported on 2009-03-31 15:02
$-----$
  ---$

$
$ Executive Control
$
POST = PUNCH
DIAG = 85
SOL COMPAT1
CEND

$
$ Solution Control
$
LINE = 64,80
ECHO = UNSORT(PARAM,DOPT)
PERTURBATION = POST
SHAPE = POST
$
$ Loadcase definitions
$
SUBCASE 2
  LABEL = Loadcase 2
  METHOD = 1
  MODTRK=NO
  DISP = POST,ALL
  ESE = POST,ALL
1GENESIS VERSION 10.2
  PAGE      2
                                         DATE 04-03-2009 TIME 9:17
```

--- BULKDATA SECTION ---

```
BEGIN BULK
ENDDATA

1GENESIS VERSION 10.2
  PAGE      3
                                         DATE 04-03-2009 TIME 9:17
```

# ANALYSIS PROBLEM SUMMARY

|                                     |        |
|-------------------------------------|--------|
| NUMBER OF GRID POINTS:              | 26010  |
| NUMBER OF CQUAD4->PSHELL ELEMENTS:  | 25088  |
| NUMBER OF CWELD ELEMENTS:           | 22     |
| TOTAL NUMBER OF NON RIGID ELEMENTS: | 25110  |
| NUMBER OF ELEMENT PROPERTIES:       | 3      |
| NUMBER OF MATERIALS:                | 1      |
| NUMBER OF DEGREES OF FREEDOM:       | 155928 |

# LOAD CASES SUMMARY

|                                      |   |
|--------------------------------------|---|
| NUMBER OF USER FREQUENCY LOAD CASES: | 1 |
| TOTAL NUMBER OF LOAD CASES:          | 1 |

|          |              |                 |           |
|----------|--------------|-----------------|-----------|
| 1GENESIS | VERSION 10.2 | DATE 04-03-2009 | TIME 9:17 |
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\*\*\*\*\*  
 \* D E S I G N C Y C L E        0 (ANALYSIS SUMMARY)\*  
 \*\*\*\*\*

|          |              |                 |           |
|----------|--------------|-----------------|-----------|
| 1GENESIS | VERSION 10.2 | DATE 04-03-2009 | TIME 9:17 |
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# MASS / VOLUME SUMMARY

|                    |              |
|--------------------|--------------|
| SYSTEM MASS        | 4.447218E-04 |
| SYSTEM VOLUME      | 5.701561E+04 |
| SYSTEM MASS/VOLUME | 7.800000E-09 |

|          |              |                 |           |
|----------|--------------|-----------------|-----------|
| 1GENESIS | VERSION 10.2 | DATE 04-03-2009 | TIME 9:17 |
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# ENCLOSING BOX SUMMARY

| LENGTH X  | LENGTH Y  | LENGTH Z  | AREA XY   | AREA YZ   | AREA XZ   | VOLUME XYZ |
|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| 1.262E+02 | 1.600E+02 | 6.750E+01 | 2.019E+04 | 1.080E+04 | 8.516E+03 |            |
| 1.363E+06 |           |           |           |           |           |            |

|          |              |                 |           |
|----------|--------------|-----------------|-----------|
| 1GENESIS | VERSION 10.2 | DATE 04-03-2009 | TIME 9:17 |
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## E I G E N V A L U E S

| MODE | CYCLES       | EIGENVALUE    | RADIANS      | GENERALIZED<br>MASS | GENERALIZED<br>STIFFNESS |
|------|--------------|---------------|--------------|---------------------|--------------------------|
| 1    | 4.556492E-02 | -8.196358E-02 | 2.862928E-01 | 1.000000E+00        | -                        |
|      | 8.196358E-02 |               |              |                     |                          |
| 2    | 3.236136E-02 | -4.134408E-02 | 2.033324E-01 | 1.000000E+00        | -                        |
|      | 4.134408E-02 |               |              |                     |                          |
| 3    | 2.095993E-02 | -1.734361E-02 | 1.316951E-01 | 1.000000E+00        | -                        |
|      | 1.734361E-02 |               |              |                     |                          |
| 4    | 9.380188E-03 | -3.473624E-03 | 5.893746E-02 | 1.000000E+00        | -                        |
|      | 3.473624E-03 |               |              |                     |                          |
| 5    | 2.443385E-02 | 2.356914E-02  | 1.535224E-01 | 1.000000E+00        |                          |
|      | 2.356914E-02 |               |              |                     |                          |
| 6    | 8.059826E-02 | 2.564549E-01  | 5.064138E-01 | 1.000000E+00        |                          |
|      | 2.564549E-01 |               |              |                     |                          |
| 7    | 2.840831E+02 | 3.186035E+06  | 1.784947E+03 | 1.000000E+00        |                          |
|      | 3.186035E+06 |               |              |                     |                          |
| 8    | 3.260877E+02 | 4.197866E+06  | 2.048869E+03 | 1.000000E+00        |                          |
|      | 4.197866E+06 |               |              |                     |                          |
| 9    | 3.350886E+02 | 4.432808E+06  | 2.105424E+03 | 1.000000E+00        |                          |
|      | 4.432808E+06 |               |              |                     |                          |
| 10   | 3.725899E+02 | 5.480522E+06  | 2.341052E+03 | 1.000000E+00        |                          |
|      | 5.480522E+06 |               |              |                     |                          |
| 11   | 6.589418E+02 | 1.714170E+07  | 4.140254E+03 | 1.000000E+00        |                          |
|      | 1.714170E+07 |               |              |                     |                          |
| 12   | 7.062279E+02 | 1.969017E+07  | 4.437361E+03 | 1.000000E+00        |                          |
|      | 1.969017E+07 |               |              |                     |                          |

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\*\*\*\*\*  
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\*\*\*\*\*

1

\*\*\*\*\* END OF OUTPUT \*\*\*\*\*





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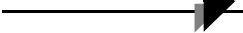
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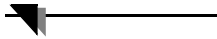
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